DESIGN AND CONSTRUCTION
OF
HEAT ENGINES

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BY

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DEDICATION

To the man who esteems excellence
in his calling of greater value
than his remuneration

PREFACE

The object of this book is to supply in one volume the material most essential to the well-equipped, independent designer of heat engines, and to give this material in the form most convenient for use in class room and practical work, by a separate treatment of the different phases of the subject.

Contributory to the contents are the author's note books on engine design covering his practice and observation for over twenty years; revisions and additions necessary to adapt the notes to his teaching work for the past ten years; material from the best technical books and periodicals necessary to fill the gaps in first-hand information, and to add breadth and character to the work; and data and drawings from some of the best designers and builders of heat engines in the United States—giving a practical, commercial touch.

In the derivation of working formulas, elaborate and abstruse methods have been avoided as much as possible, but no important element which permits of practical treatment has been omitted; also, while reverting to fundamental principles and making the formulas as rational as possible, their limitations are pointed out, and they are usually brought to a form suitable for direct application.

Illustrations of none but the most excellent designs are provided. These are necessarily limited on account of space, but qualitative design, and certain details and auxiliaries may be studied in books of a more descriptive character, and in builders' catalogs which are always obtainable.

The numbering of formulas begins with each chapter, and manufacturers' material is usually placed in the latter part of the chapter to facilitate the revisions necessary to keep the work up to date. The notation for a chapter is listed at its beginning except for short, scattered discussions, when it is given only in the text.

Steam tables and other tables found in all handbooks are omitted; they are more convenient to use with the formulas of this book if in a separate cover.

A few references are given at the ends of chapters to extend the scope of the work.

The process of absorption by contact with associates in office, shop and college, and habitual gleaning from technical periodicals, books and catalogs long before the writing of a book was thought of, renders the placing of credit difficult, but when possible it has been done.

The author is very grateful to the manufacturers and engineers who have furnished illustrations and data, and to authors and publishers who have permitted the use of material from their publications. Thanks are also due my wife, Luella V. Ninde, whose assistance and encouragement have made the book possible.

W. E. N.

Syracuse University,
January, 1920.

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DESIGN AND CONSTRUCTION OF HEAT ENGINES

PART I—THE HEAT ENGINE

CHAPTER I

STATUS OF THE HEAT ENGINE

It is roughly estimated that 85 per cent. of the power utilized in the United States for manufacturing plants, central power plants and electric-railway power stations is furnished by heat engines, the remaining 15 per cent. being water power. This does not include locomotives, steamships or any form of self-propelled vehicles, which would increase the heat-engine percentage still more.

It has also been estimated that the total available water power in the United States approximates the present output of steam power. However, by the time this power is developed it is probable that the demand will have so increased that the power developed by the heat engine in its varied forms will be as greatly in excess as at the present time. The importance of heat engines is then obvious and their manufacture will continue to be an important industry until our fuel supply is exhausted or some radical discovery displaces them.

Heat engines may be divided into three important classes which are commercially successful today, viz.: 1. The steam engine; 2, the steam turbine; and 3, the internal-combustion engine. The last named may properly be divided into two commercial forms: (*a*) The gas engine and (*b*) the oil engine, the former using gaseous, the latter liquid fuel. The gas turbine is not yet recognized in the commercial field and will not be further mentioned.

Heat engines may also be classified as: 1. Prime movers; and 2, reversed heat engines, such as the compression refrigerating machine and the air compressor. Prime movers only will be considered.

The relative importance of the different classes is an open question

but it is probable that each has its particular field of usefulness which in some cases cannot be as well provided for by the others.

Statements were current in the technical press a few years ago that little was to be expected in the way of improved economy for the reciprocating steam engine, and its passing was predicted. That the limit of improvement had been nearly reached may, at that time have been true across the sea, but since then in this country, the application of superheated steam, the uniflow principle and the locomobile, together with improvements in materials and some details of construction which have made possible higher piston and rotative speeds in the larger units, have greatly improved the steam engine, so that it is holding its own in all-around efficiency and reliability with other forms. The steam locomotive, that once proverbially wasteful machine, now equals in economic performance some of the best stationary steam plants of a few years ago.

But the reciprocating engine has limits in the size of its units, and beyond this the steam turbine is best adapted. The turbine, with its high rotative speed and uniform rotation is well fitted to drive alternating-current generators of large capacity. The floor space is also less than for a reciprocating engine of the same power.

The internal-combustion engine has attained the highest efficiency of all heat engines, and this is probably responsible for the prediction that the days of the steam engine were numbered. Notwithstanding all this, the internal-combustion engine furnishes at the present time but a small percentage of the output of stationary power plants, but there seems little doubt but that this ratio will be increased. It is a reciprocating engine, and in this particular may be classed with the steam engine.

Power, in the issue of Nov. 17, 1914, contains the statement that the total power of automobile engines manufactured the previous year was equivalent to twice the potential power of Niagara Falls, or 13,500,000 horsepower. This, coupled with the fact that, notwithstanding the increasing demands for large turbine units, the reciprocating engine is still largely in excess, makes it probable that the passing of the reciprocating engine is not imminent.

It is not intended to compare the merits of the different types of prime movers, or the different designs of a given type, as practically all when well designed, constructed and operated give good results in their respective fields. The determination of the best machine for a given set of conditions is often much involved and is outside the scope of this work. It is undoubtedly true that the periodic popularity of the various machines is not always based upon a profound knowledge of their actual merits.

CHAPTER II

THE POWER PLANT

The Power Plant includes the prime mover and all other appliances and accessories necessary for the production of power, for manufacturing plants, central stations, ship propulsion, locomotives and automobiles, or for any form of activity requiring power. The development of the power plant has been such that the selection and arrangement of its various apparatus has become a highly specialized profession. The work of the power-plant designer and of the designers of the plant apparatus is interdependent, the latter making possible great advancement in plant efficiency, while on the other hand the demands of the plant designer have been an incentive to the designer of power apparatus.

Power-plant design is not within the range of this book, and as the development of power apparatus is in a state of continual progress, descriptions of different designs will not be attempted, the task of keeping pace with the substantial progress in heat-engine design being thought sufficient; however, the steam cycle is not completed in the engine or turbine, and in view of future references to the different phases of the cycle, the path of the steam through the plant will be traced.

Steam is generated in the boiler by the combustion of fuel in the boiler furnace. From thence it is conducted through piping to the turbine or engine where it does work. It is then exhausted, sometimes into the atmosphere, which is very wasteful; sometimes into pipes for heating purposes or for some industrial process, and it is often exhausted into a condenser. There it comes into contact directly, as in the jet condenser, or indirectly, as in the surface condenser, with cold water. The heat is withdrawn from the steam and it condenses back to water, and due to the great difference between the volume of a given weight of water and of steam, a partial vacuum is formed, often approaching within three-quarters of a pound of a perfect vacuum. With either type of condenser the water is pumped out, together with any air which entered with the steam or water.

Water must be pumped into the boiler, either by the boiler feed pump or the injector, to replace the water used by the engine, and this completes the cycle. In most large plants the water is passed through a feed-water heater before going to the boiler. This is heated by the

exhaust from the auxiliaries or main engine, or sometimes by the hot gases of the flue leading from the furnace to the chimney; in the latter case it is called an economizer by common usage.

If a surface condenser is used, the condensed steam may be pumped back into the boiler, and barring leakage losses, a nearly continuous cycle with the same water may be effected, approaching the theoretical cycle.

With the internal-combustion engine the thermal cycle is complete in the engine cylinder. There are, however, numerous accessories. For the gas engine there must be the gas producer, and the scrubbers for cleaning the gas; with the oil engine, apparatus for handling the oil must be provided; air compressors are also necessary for some types and vaporizers or carbureters for others. These may be considered as part of the power-plant equipment, but are not part of the engine proper, although in many cases they are furnished by the engine builder.

CHAPTER III

THE STEAM ENGINE

1. The Steam Engine.—The history of the steam engine is full of interest, and should form a part of the reading of every steam engineer. In the popular mind the profession of mechanical engineering is closely associated with the steam engine, and in fact owes much to it as an important factor in the separation of this branch of science from the general field of engineering, or civil engineering, which formerly included the several now distinct branches.

In order to reserve space for present-day practice, it was thought best to refer to the works of Rankine, Thurston, Ewing and others, and to eliminate from this text practically all historical matter.

2. Mechanism.—With the exception of comparatively small pumps, all practical reciprocating engines employ for the conversion of heat energy into useful work, the simple slider-crank mechanism. This mechanism also resolves reciprocating into rotary motion, or *vice versa* in the case of reversed heat engines such as compressors. For engine purposes the slider-crank mechanism is the simplest and most economical from the standpoint of both construction and operation, and although attempts have been made to improve upon it, they have been fruitless. Neglecting friction, which in some cases is as low as 4 per cent. of the entire power developed in the cylinder, the efficiency is 100 per cent., as proven in Par. 100, Chap. XVI.

A diagram of the slider-crank mechanism as applied to the steam engine is shown in Fig. 1. The mechanism proper consists of the frame (*f*), the crank (*e*), the connecting rod (*d*) and the crosshead (*c*), sliding upon the guides which restrict its movement to a straight line. The piston (*a*) and piston rod (*b*) may be considered as an extension to the crosshead and serve to transmit the pressure of the steam to it.

Ports for the admission and outlet of steam are shown at 1, 2, 3 and 4. In some engines the same ports are used for inlet and outlet. The control of the steam passing into and from the cylinder is effected by valves which are actuated by an eccentric, a form of crank, which is located on the engine shaft. In Fig. 1, valves 1 and 2 are inlet valves, while 3 and 4 are outlet or exhaust valves.

To further illustrate the action of the steam engine as a whole, Figs.

2 to 9 are given. Contrary to the usual practice of introducing valve action with a single-valve engine, it is thought that less confusion will accompany the conception of a separate valve controlling each operation concerned with the handling of steam during the engine cycle, which

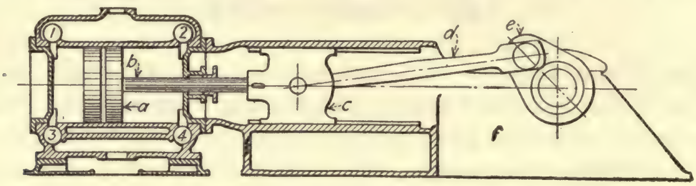

Fig. 1

with the steam engine is effected in one revolution of the crank; consequently, a four-valve engine of the nonreleasing type is shown.

Figure 2 is the side of the engine showing the crank and connecting rod. Figure 3 shows the valve-gear side with the flywheel removed.

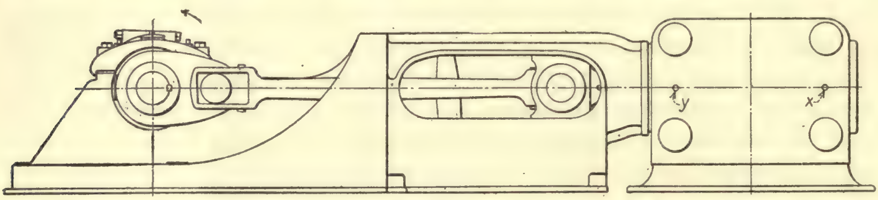

Fig. 2.

Figure 4 shows the valve-gear side with the cylinder in section, the rods and levers operating the gear being shown by dotted lines, as are also the crank and connecting rod.

Figures 5, and 7 to 9 show valves, with center lines only of wrist

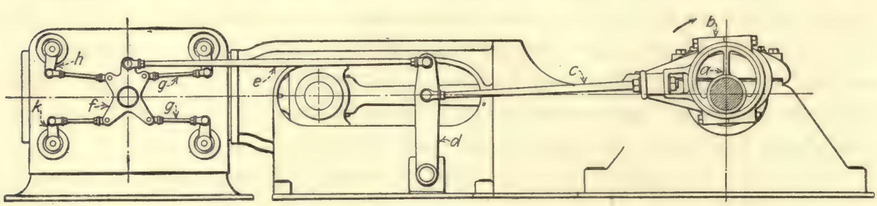

Fig. 3.

plate arms, levers, crank and eccentric, in order to show the different positions more clearly. This forms a valve diagram, treated more fully in Chap. XX.

In practice, with this type of gear, the eccentric changes position in relation to the crank, under the control of the governor located on the

shaft, as the load on the engine is varied, but for simplicity, this is neglected in the present treatment, and the eccentric is assumed in a fixed position on the shaft. All of these details, and the exact action of the valves, are fully treated in Chap. XX.

In Fig. 3, (*a*) is the eccentric, which by means of the eccentric strap (*b*) and eccentric rod (*c*) connects with the rocker or carrier (*d*). The reach rod (*e*) connects to the wrist plate (*f*). Links (*g*) connect the wrist plate to the valve levers (*h*) and (*k*).

It is plainly seen that as the crank revolves, a rocking motion is given to the wrist plate, which in turn causes the valve levers to oscillate through a given angle. The valve levers being keyed to the valve stems, impart motion to the valves, opening and closing the ports, thus controlling the steam supply to the cylinder.

The valve gear is shown in its central position, and the crank is at the head-end dead center. When the valves are properly set this would not be the case; the eccentric would be advanced for this position of the crank as shown in Fig. 4.

Figure 4 is a sectional view of the same engine. The end of the cylinder toward the crank is called the *crank end*, and the opposite end the *head end*. This is preferable to the terms front and back, as with stationary engines opinion is not unanimous as to which is front and which back; thus to avoid confusion, head and crank end will be used, the ports and everything dealing with the steam entering or leaving the ports at either end of the cylinder being designated accordingly.

The ports and valves controlling the incoming steam are commonly known as steam ports and valves, while for the outgoing steam they are exhaust ports and valves; steam and exhaust in general refer to incoming and outgoing steam respectively.

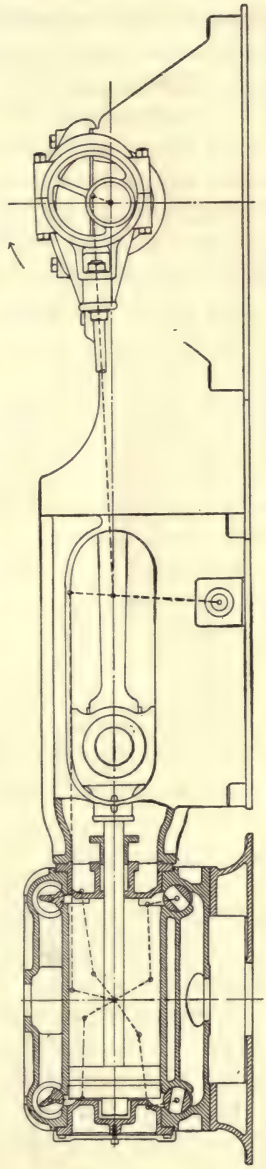

Fig. 4.

Then ports 1 and 2 are the steam ports for the head and crank ends respectively, while 3 and 4 are the exhaust ports.

The piston rod passes through a stuffing box in the cylinder head at the crank end, which prevents leakage of steam from this end of the cylinder. One or more packing rings, to be described in Chap. XXIII, keep the steam from leaking past the piston. In Fig. 4 the piston is at the head end of the stroke and the crank is on its head-end *dead center*, so called because the steam can have no turning action on it when in this position, and the engine would stop if it were not for the momentum of the flywheel, which carries the crank past the dead center.

When the crank is on dead center, a space is left between the piston and the cylinder head, so that there will be no danger of striking in case of wear in the pin joints of the connecting rod and the main bearing. The distance between the piston and cylinder head is called the clearance distance or mechanical clearance. The volume of this space, including the volume of the ports up to the valve seats is called the clearance volume; the ratio of this volume at one end of the cylinder, to the volume swept through by the piston during one stroke is commonly called the clearance, and is usually given in per cent. In the ordinary steam engine this varies from 3 to 12 per cent., and it is usually desirable to keep it as small as possible; however, in some forms of steam engine, and in the gas engine, it must have a certain definite value, which in some cases may be much greater than the values given.

On the top of the cylinder is the steam chest, forming a passage from the steam-pipe connection to the steam valves at each end. The throttle valve or stop valve is connected to the steam-chest flange. The valves are of the rotary type, having an oscillating motion. The exhaust passage at the bottom of the cylinder connects the exhaust valve at each end with the exhaust pipe.

Admission.—Figure 4 shows the piston at the head end of the stroke, with the crank on dead center. The head-end steam valve is open by a small amount, this opening being called the *lead*. This means that the opening of the valve, or *admission*, occurred a little before the crank reached the dead center. The purpose of this is to allow full steam pressure in the clearance space, and to have the valve partly open, before the piston starts its stroke, in order that there may be no delay in the steam entering the cylinder and following the piston without loss of pressure. The higher the rotative speed of the engine, the more is lead necessary, high-speed engines having proportionately more lead than low-speed engines.

The head-end steam valve being open, steam will enter and force the

piston toward the crank end of the cylinder. It will also be seen that the crank-end exhaust valve is open, allowing steam in that end of the cylinder to escape to the exhaust pipe.

As the crank rotates in a clockwise direction, and the eccentric is somewhat more than 90 deg. in advance of the crank, it is obvious that before the crank reaches an upright position, the wrist plate will swing to its extreme right position, giving a maximum opening to the head-end steam valve and the crank-end exhaust valve, both of which start to close as the crank continues to rotate.

Cut-off.—The closing of the steam valve is known as *cut-off*, and this event for the head end is shown in Fig. 5. This is the first event since the beginning of the stroke under consideration. The crank-end steam valve and head-end exhaust valve have remained closed so far, and the

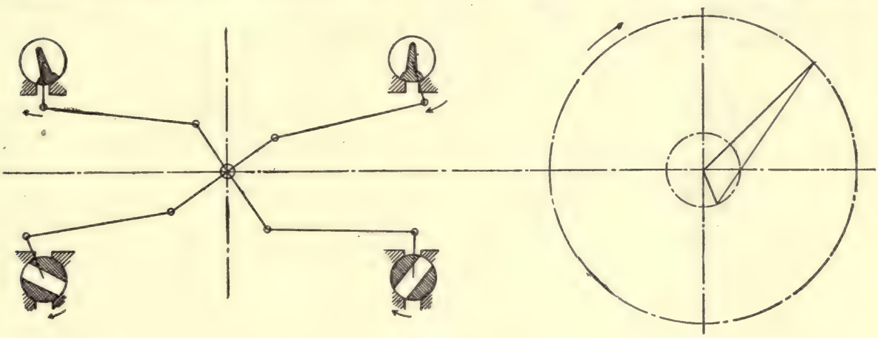

FIG. 5.

crank-end exhaust valve is still open. Cut-off is usually designated in the fraction of the stroke accomplished before cut-off occurs, as one-half cut-off, one-third cut-off, etc.

Early steam engines received steam during the entire stroke, exhausting it to the atmosphere when at its maximum pressure, which was wasteful. The cutting off of steam before the stroke is completed was introduced by James Watt in order to utilize the expansive energy of the steam. After cut-off the steam expands, continuing to exert pressure against the piston until the end of the stroke. The pressure, however, decreases as expansion continues, resulting in a decreased mean pressure during the stroke, and to offset this a larger cylinder must be used. But the larger cylinder with cut-off before the completion of the stroke, always uses less steam for a given horsepower than the older method. This will be demonstrated in Chap. XII. The earlier the cut-off the greater the expansion, and if carried so far that the pressure drops to

that of the atmosphere or condenser into which the engine exhausts, the expansion is said to be complete. Theoretically, this would secure the maximum economy, but for practical reasons to be explained in Chap. IX, the best economy demands a cut-off which does not result in complete expansion.

The ratio of the total volume of steam on the working side of the piston at the completion of the stroke, to the volume at cut-off when expansion begins, is called the *ratio of expansion*. For the purpose of avoiding a more advanced discussion of valves and valve gears at this place, the cut-off shown in Fig. 5 is late in the stroke, or such as would obtain with a heavy load on the engine.

Brief mention of the indicator diagram may be of advantage at this point. The steam-engine indicator is an instrument for determining the steam pressure acting upon the piston during its stroke. This instrument traces diagrams similar to Fig. 6, which shows conventional dia-

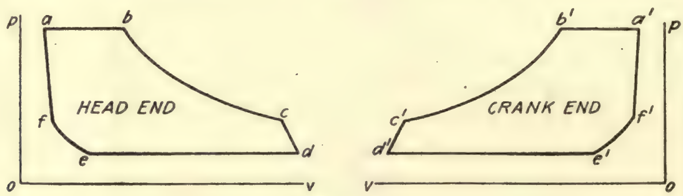

Fig. 6.

grams with which the different events of the stroke may more easily be explained than with actual diagrams taken from the engine. Horizontal measurements represent points along the piston travel, while vertical measurements represent pressure. The pressure at (a) is the *initial pressure*, at (c) the *terminal pressure*, along line d–e the *back pressure* and at (f) the *compression pressure*. In common parlance, this means pressure measured from atmospheric pressure (14.7 lb. per sq. in.), or *gage pressure*, but it is more satisfactory, and is necessary for calculation to measure it from perfect vacuum, or in *absolute pressure*.

Line o–v represents zero pressure, and pressures measured from it are absolute pressures. Line o–p is the zero volume line, and all volumes are measured from it. The distance p–a represents the clearance volume previously mentioned. This space is full of steam when the piston starts its stroke.

In practice the indicator is attached at (x), Fig. 2, in the clearance space for the head-end diagram, so that this diagram gives the pressure acting on the left side of the piston during the two strokes forming the

cycle. Likewise the indicator is attached at (y) for the crank-end diagram and gives the pressure at the right of the piston. It will be assumed that an indicator is attached at each end, drawing both diagrams at the same time, which is practiced in high class tests.

Thus far the piston has moved from (a) to (b), drawing the line a–b, or *steam line* of the head-end diagram, and a portion of line d'–e', or *back pressure line* of the crank-end diagram.

Compression.—Continuing the rotation of the crank a little further to the position of Fig. 7, causes the closure of the crank-end exhaust valve. This completes the back pressure line d'–e' of the crank-end diagram, and draws a portion of the expansion curve b–c of the head-end diagram, a curve which will be discussed in later chapters. The closing of the exhaust

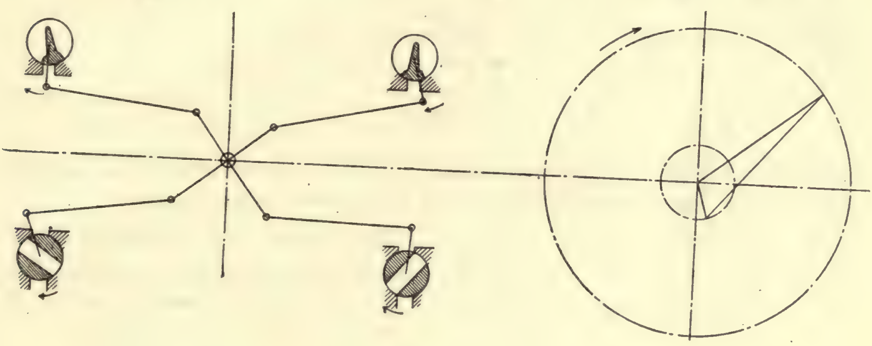

FIG. 7.

valve is commonly known as *compression,* and is expressed as the fraction of the stroke completed up to the closure of the exhaust valve, as nine-tenths compression.

As with cut-off, the early engine closed the exhaust valve at the end of the stroke, but with higher speeds it was found that an engine was apt to run more quietly if the exhaust valves closed before the stroke was complete, retaining a certain amount of steam and compressing it in the clearance space to form a cushion which gradually takes up the lost motion in the pin joints. The necessity of compression has been questioned, so any further remarks along this line are left until Chap. XVI. The effect of compression is discussed in Chaps. IX, XII and XVI.

Release.—After compression, all valves are closed. A further rotation of the crank starts the head-end exhaust valve open, which is shown in Fig. 8; this is known as *release.* This completes the expansion line b–c of the head-end diagram and draws a portion of the crank-end compression curve e'–f' of the crank-end diagram. Release is usually earlier

in the stroke than compression but nearly always occurs before the stroke is complete in order that the steam may leave the cylinder sufficiently to have its pressure lowered to that of the atmosphere or condenser before the piston starts the next stroke; otherwise the back pressure is

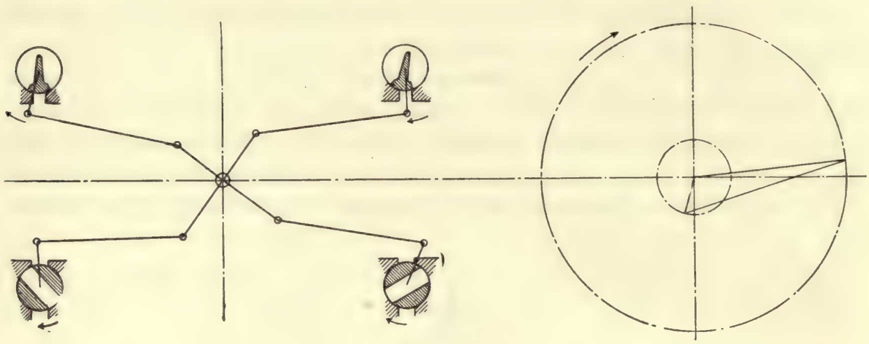

FIG. 8.

high at the beginning of the stroke, reducing the effective work on the piston. As with lead, release must be earlier with higher speeds.

A little further rotation of the crank opens the crank-end steam valve. This completes crank-end compression curve e'–f' and draws a portion of head-end exhaust line c–d. The stroke is completed in Fig. 9,

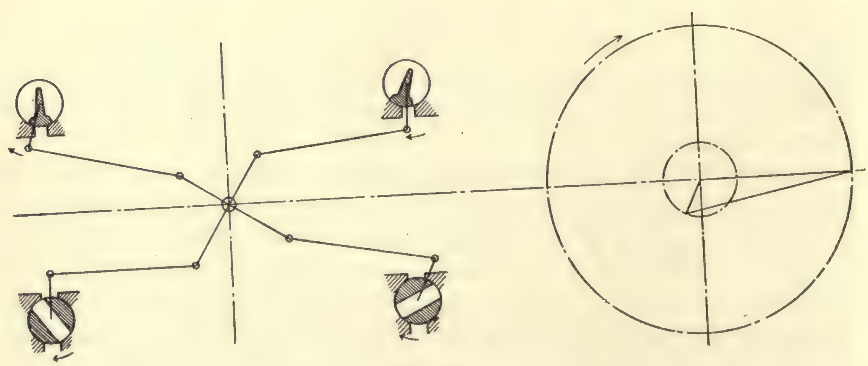

FIG. 9.

in which the crank is on the crank-end dead center. This completes the head-end exhaust line c–d and draws the crank-end admission line f'–a'. These two lines are shown as straight lines in Fig. 6, but they vary in form in actual diagrams and the junctions of the different lines are not sharply defined.

Completing the revolution to head-end dead center produces consecutively, crank-end cut-off, head-end compression, crank-end release and head-end admission, completing the cycle, and drawing the line $a'b'c'd'$ of the crank-end diagram and *defa* of the head-end diagram.

The principle of operation is practically the same in all steam engines, except that some accomplish all four events in both ends of the cylinder with a single valve; exception may also be made in the case where the piston has the function of a valve, as in the uniflow engine.

3. Types and Classification.—The development of the steam engine, extending as it has over a comparatively long period, has resulted in a large variety of designs, and, eliminating all but those at present being placed upon the market by recognized builders still leaves a goodly number. But little attempt has been made at general standardization, although there is considerable similarity in a given type as manufactured by different builders, due no doubt to a sort of mutual influence, those having the most originality and initiative leading the way.

A logical classification is difficult, as the engine may belong to several classes. With this in mind, terms in common use which express some feature of design will be arranged under different headings, and a few of the least obvious briefly described.

Classification According to General Form.

- Horizontal or vertical.
- Single-acting or double-acting.
- Side-crank or center-crank.
- Right-hand or left-hand.
- Run over or run under.
- Belt forward or belt back.

Single-acting or Double-acting.—A single-acting engine uses but one end of the cylinder, the head end, while the crank end is open. This obviates the necessity of a crank-end cylinder head and a stuffing box, consequently the crosshead may be eliminated and its function performed by the piston, which is made unusually long to provide ample wearing surface. Crossheads are sometimes used on single-acting engines.

Single-acting steam engines usually have two cylinders, one of them being a low-pressure cylinder, forming a compound engine, in some cases. The cranks are opposite, or 180 degrees apart. With steam engines, this type is usually vertical and its advantage is reduced head room.

Side-crank or Center-crank.—In this country, most steam engines of large and medium size are of the side-crank class. This is shown in diagram in Fig. 10. The shaft has but one bearing located in the engine frame, the other, called the outer bearing or outboard bearing, being a

separate bearing, usually having no connection to the engine frame, but bolted separately to the foundation.

Some small engines are also built with the side crank, and of course, some large engines have the center crank. Figure 11 shows a center-crank engine in diagram. Sometimes the outer bearing is omitted, a belt

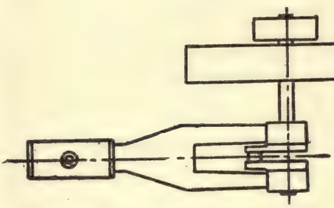

FIG. 10.

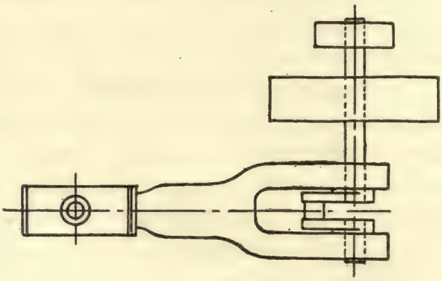

FIG. 11.

wheel being overhung from the main bearing on one side, while the governor wheel is on the other.

The advantage claimed for the center-crank engine is that of a beam supported at the ends with a load in the middle, over a cantilever beam loaded at the end; but where the outer bearing is used, as it always is in large engines, the difficulty and uncertainty of keeping three bearings in perfect alinement makes the advantage dubious, and as there is no difficulty in the practical design of the side-crank type, it is used almost entirely in this country, not only for steam engines, but for large gas engines

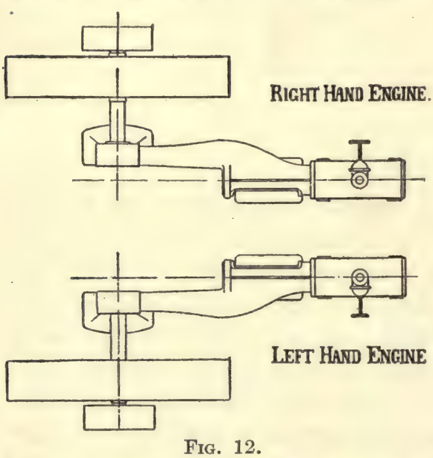

FIG. 12.

Right-hand or Left-hand.—This classification has not a universal application except in the case of horizontal side-crank engines. With these, standing at the cylinder end and looking toward the crank, the engine is right-hand if the main bearing is at the right of the center line of engine, and left-hand if the bearing is at the left. This is illustrated in Fig. 12.

Center-crank engines may be classified as right- or left-hand according to the valve-gear or governor side, and vertical engines are so classified

for convenience in ordering; in such cases directions are usually given in the builders' catalogs.

Run Over or Under.—This is a functional classification with most engines, it being possible to run them either over or under by adjusting the valve gear. This classification is a definite one only with horizontal engines, in which case, if the top of the wheel moves from an observer standing at the cylinder, the engine runs over; if it comes toward him it runs under. This is illustrated by Fig. 13.

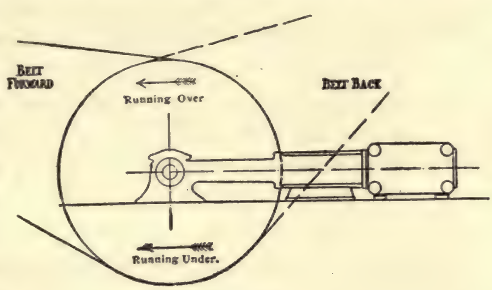

Fig. 13.

As with the designation right- and left-hand, the terms run over and under are sometimes applied to vertical engines by special direction in the builders' catalogs.

It is true that some engines, as the marine type, are built to run in one direction, the bearing surface of the crosshead shoe being much greater on one side than the other. The smaller surface comes into play

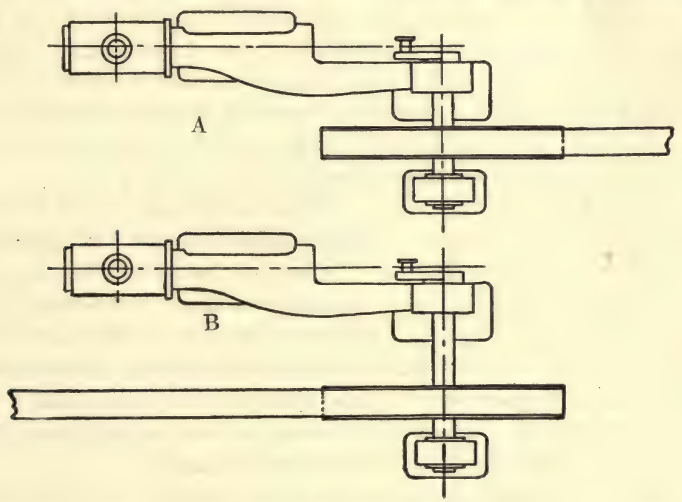

Fig. 14.

only while the engine is reversing, which is a very small portion of the time.

Belt Forward or Back.—While this might appear to be entirely functional, it often affects the location of the wheel, and therefore the length

of the shaft, which in turn sometimes affects the necessary diameter. In Fig. 14, A shows the belt going forward, and B going back. In the latter, space must be left between the belt and the engine cylinder so that the operating engineer may get at the valve gear and throttle valve.

Classification According to Valve Gear.
- Single-valve.
- Four-valve.
- Gridiron-valve.
- Poppet-valve.
- Throttling.
- Automatic cut-off.
- Reversing.

Throttling Engine.—All stationary engines are controlled by governors, which limit the speed fluctuation to a small fraction under changing loads, within the capacity of the engine. The governor and valve gear are treated in later chapters, but their effect upon the steam supply may be considered here. The throttling engine governs in the simplest way and requires the least mechanism. The governor is attached to a throttle valve in the steam pipe. If the load on the engine decreases, the engine tends to speed up, and in doing so the governor weights fly out by centrifugal force, and being connected to the controlling valve in the steam pipe, partly closes it, throttling the steam, which cannot get through the valve fast enough to keep up pressure as it follows the piston, therefore less work is done, the steam pressure being adapted to the load by the governor. If the load is increased, the engine tends to slow down, and this opens the valve, increasing the steam pressure.

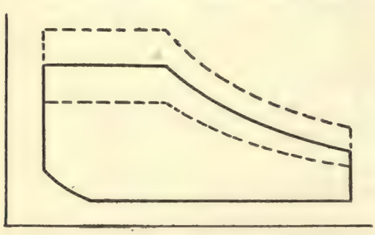

Fig. 15.

The throttling engine is usually a single-valve engine with either a flat "D"-valve or a piston valve. The eccentric is fixed to the shaft and the valve always has the same travel; therefore the cut-off, compression, etc. are always the same and the controlling device is really not a part of the engine proper.

The effect of the governor upon the steam pressure within the cylinder may be shown by an indicator diagram, Fig. 15, in which the full lines represent the normal load on the engine, and the dotted lines a greater and a lesser load. During the time the steam is being driven from the cylinder, it is open to the atmosphere, so the back pressure line and compression curve are the same for all loads.

Throttling engines are usually small engines and their economy is comparatively poor. This is attributable in part, however, to lack of refinement in other features of design common in a good many small engines. Within reasonable limits of load variation this method of governing possesses some practical advantages. An economical cut-off may be maintained at all loads, and for loads less than the maximum the throttling of the steam tends to reduce the amount of moisture due to partial condensation, which will be explained in Chap. IX. Governing by throttling is further discussed in Chap. XII, Par. 58.

Automatic Cut-off Engine.—These engines are controlled by changing the cut-off, allowing smaller or greater quantities of steam of constant pressure to enter the cylinder to meet the load requirements. The Corliss engine was one of the earliest forms of this class, and one of the most important for medium and large powers. The gear of this engine is

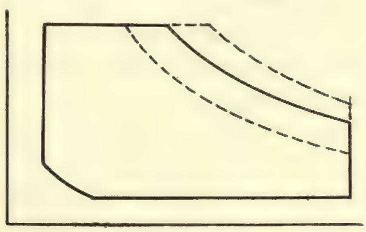

FIG. 16.

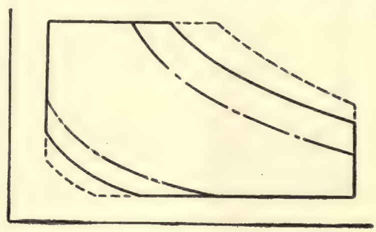

FIG. 17.

known as a releasing gear, and sometimes as a drop-cut-off gear. There are four valves as previously stated, two steam and two exhaust valves, which receive their motion directly from a wrist plate as explained in Par. 2. The eccentric is fixed to the engine shaft, giving a constant movement to the wrist-plate and valve levers connected thereto by the links. The movement of the exhaust valves is constant, and the exhaust events, compression and release, are always the same.

This gear is described in Chap. XX.

The releasing principle has also been applied to four-valve engines with gridiron steam valves, and also to poppet-valve engines.

The indicator diagram for the engine just described is shown in Fig. 16, the full lines being for rated load and the dotted lines for a greater and smaller load.

Another method of regulating the steam supply by change of cut-off is that of changing the position of the eccentric upon the shaft, which is effected by a shaft governor to be described in Chap. XIX. The diameter of the circle upon which the eccentric center travels is usually changed, but not in all engines, and the angular position of the eccentric relative to

the crank is always changed upon change of load. This method is applied to engines fitted with all the variety of valves previously mentioned. When the single valve is used, or a four-valve engine with a single eccentric such as was employed in describing steam-engine operation in Par. 1, the change in eccentric position necessarily affects all events of the stroke as shown in Fig. 17, the full lines being for rated load as before, and the dotted lines for greater and lesser loads. Two styles of dotted lines are used to associate the expansion curve with its corresponding compression curve. It will be noticed that shortening the cut-off is accompanied by an earlier compression, reducing the diagram area on two sides, while for long cut-off, the area is increased on two sides. This necessitates less change of the gear for a given change of load than if the compression were constant. For engines having exhaust valves operated by a separate eccentric, the compression and release are constant, and the diagram is that of Fig. 16.

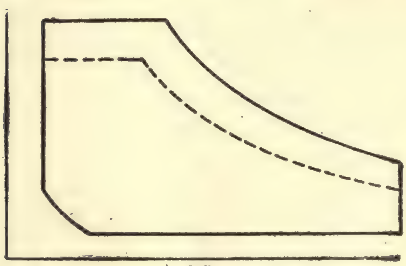

Fig. 18.

A combination of throttling and automatic cut-off is employed in some engines with good economy, especially at light loads, when the reduction of pressure due to throttling, with its consequent drying effect, makes unnecessary quite so short a cut-off as would be required if only the cut-off were altered. This is shown by the indicator diagram of Fig. 18, in which full lines are for normal or rated load and dotted lines for a lighter load.

Classification According to the Use of Steam.
- Simple.
- Compound and multiple-expansion.
- Condensing.
- Noncondensing.

Simple.—A simple engine receives steam at or near boiler pressure, which, after doing work in the engine cylinder, is exhausted to the atmosphere, a condenser, into an exhaust heating system or utilized for some industrial process. A simple engine may have more than one cylinder, such as a duplex or twin engine, but each receives steam from the same source at the same pressure; at any rate, no cylinder receives steam from the other. Most modern simple engines have a single cylinder.

Compound and Multiple-expansion.—It has been already stated in

Par. 2 that the use of the expansive energy of steam is conducive to economy, but that too great an expansion in one cylinder, or too early a cut-off is not economical, so that the ratio of expansion is necessarily limited in a single cylinder. If a cut-off within practical economical limits is effected in one cylinder, and the steam expanded to some pressure between that of the steam and exhaust mains, and if the exhaust from this cylinder is piped to a cylinder of greater volume, having a cut-off also within economical limits, and such that the volume of the cylinder up to cut-off is approximately equal to the volume of the first cylinder, the steam would expand again from this intermediate pressure, which is the initial pressure for the second cylinder, to some terminal pressure somewhat higher than the exhaust-main pressure, the latter being the back pressure of the second cylinder. Then *the total ratio of expansion*, from the volume at cut-off in the first cylinder to the final volume in the second, will be approximately the product of the ratios of expansion in the two cylinders. The exhaust from the second cylinder is usually piped to a condenser, but sometimes to the atmosphere, or the steam may be used for some purpose as with the simple engine.

Thus, by the use of two cylinders of different size, but so connected as to deliver power to the same shaft, economical cut-off may be combined with a high expansion ratio, greatly increasing the economy. The first cylinder is known as the high-pressure cylinder and the second as the low-pressure cylinder. As the piston stroke is the same in both cylinders, the volumes of the cylinders are proportional to the piston areas acted upon by the steam. The mean pressures acting on the two pistons are about proportional to the piston areas inversely, therefore the work done in the two cylinders is nearly equal. Such an engine is called a *compound engine*. Although in a general sense this term covers engines in which expansion takes place in three or four cylinders, it is usually employed for a double-expansion engine as just described, and when expansion occurs in three or four stages, the engines are known as triple-expansion engines or quadruple-expansion engines.

The exhaust from the high-pressure cylinder is piped to a vessel called a *receiver*, from which the steam supply for the low-pressure cylinder is taken. The influence of the receiver is to prevent excessive fluctuation of pressure during the passage of steam from the high-pressure to the low-pressure cylinder. In triple-expansion engines the first cylinder is the high-pressure, the second the intermediate and the third the low-pressure cylinder. There is a receiver between the high and intermediate, and another between the intermediate and low-pressure cylinders. In quadruple-expansion engines, the second cylinder is the

first intermediate and the third, the second intermediate. A receiver is placed between each cylinder as before, there being three receivers with a quadruple-expansion engine.

In marine engines, in order to secure better balancing and a more uniform turning effort on the crank shaft, the low-pressure cylinders of compound and triple-expansion engines are sometimes replaced by two cylinders with a combined capacity of the single cylinder. These two cylinders take steam from the same receiver and exhaust to a common condenser, and therefore belong to the same stage. Such engines are known respectively as three-cylinder compounds and four-cylinder triple-expansion engines.

Multiple-expansion engines other than double-expansion or compounds are used comparatively little except in marine and pumping service. There is little gain over a compound in economy when the ratio of cylinder volumes is properly chosen, and the use of superheated steam has still further reduced such gain.

The most common types of compound engines are the *cross-compound* and the *tandem-compound*. The cross-compound consists of two engines, one high-pressure and one low-pressure, parallel to each other and connected to the same shaft. To provide a more uniform turning effort on the shaft, the cranks are placed 90 degrees apart, although this is not necessarily the best angle to produce this result. This subject is discussed in Chap. XVIII.

A plan drawing of a cross-compound engine with its receiver and piping is shown in Fig. 461, Chap. XXXIV.

A tandem-compound engine has its cylinders in line, the pistons being fastened to a common piston rod. The cylinders are connected to a single engine frame, and all other parts are as for a single engine. There is a single crank and the turning effort is not as uniform as with a cross-compound engine. A tandem engine is more compact and is usually considered to be better adapted to high speed.

Noncondensing and Condensing.—Any steam engine exhausting into the atmosphere, or any higher pressure, such as a heating system, is known as a noncondensing engine. If the exhaust pressure is below that of the atmosphere, a condenser must be used and the engine is known as a condensing engine. This classification affects the design of the simple engine but little; in the compound, however, the size of the low-pressure cylinder, and the ratio of cylinder diameters are both greater for the condensing engine, but it is not uncommon for a condensing engine to run noncondensing for a portion of the time.

Condensing engines show a greater economy than noncondensing

engines, due to the removal of back pressure, and to the greater ratio of expansion permitted, especially in compound engines.

Classification According to Service for which Designed.—Among these are the following:

Locomotive.
Marine engine.
Hoisting engine.
Pumping engine.
Rolling-mill engine.
Others might be added to the list.

4. The Uniflow Engine.—This engine was designed by Prof. Stumpf, and is manufactured by several firms in the United States. It has no

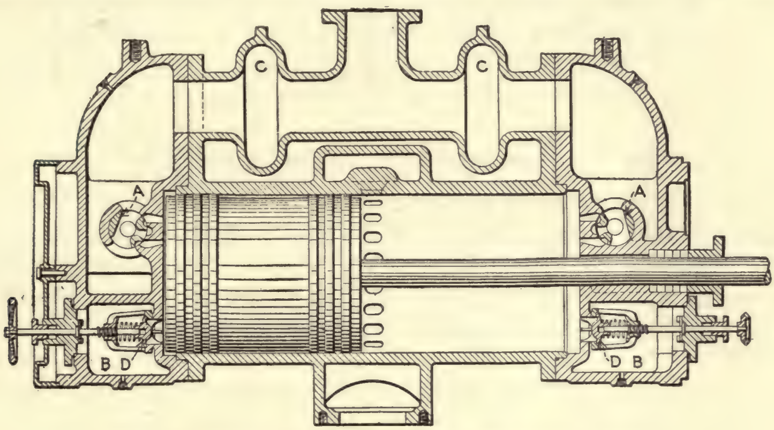

FIG. 19.—Nordberg uniflow cylinder with Corliss valves.

exhaust valves, the piston performing this function near the end of the stroke. The steam thus flows in but one direction at each end of the cylinder, the period of exhaust being greatly decreased, thus lessening the cooling effect of the low-pressure steam upon the cylinder walls and reducing condensation.

FIG. 20.—Indicator diagrams for Universal uniflow engine.

A section of the cylinder of the Nordberg uniflow engine is shown in Fig. 19. The valves D act as automatic relief valves when the vacuum is accidentally broken, increasing the clearance space by the volume of

space *B*. Otherwise, as compression begins at about $\frac{1}{10}$ stroke, the compression pressure would be much above the initial pressure. These valves are backed off by the hand wheels when it is desired to run the engine noncondensing

Uniflow engines are mostly made with poppet valves.

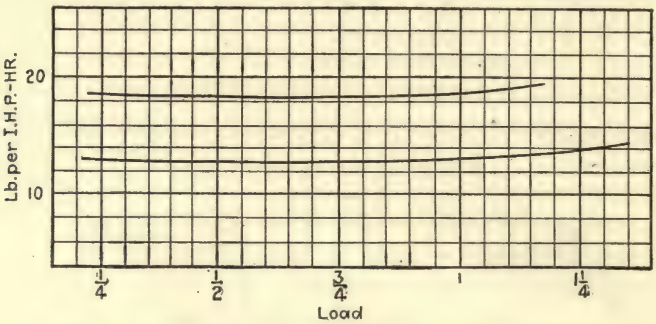

FIG. 21.—Steam consumption of Universal uniflow engine.

Indicator diagrams for condensing and noncondensing operation for a Universal Uniflow engine are shown in Fig. 20. This engine has an auxiliary exhaust valve which closes about 0.7 stroke, and is in operation when the engine works noncondensing. Fig. 21 shows the steam consumption of the Universal Uniflow engine.

CHAPTER IV

THE STEAM TURBINE

5. According to historical records, the first self-acting machine operated by steam was a turbine, but it was not until a comparatively recent date that the development of the steam turbine into a commercially successful machine was undertaken. Naturally, under the present state of the mechanical arts, its advance was much more rapid than that of the steam engine, until now it equals in economy and far exceeds in unit capacity, the best steam engines.

The expansion of steam in the cylinder of a reciprocating steam engine, while essential to economy, is not a necessity to practical operation; this is obvious from certain types of steam pumps which receive steam at boiler pressure throughout the stroke, and even from high-grade engines successfully carrying overloads which require steam admission nearly the entire stroke with very little expansion in the cylinder. But the turbine depends upon the velocity of the steam, which may only be effected by expanding it through properly formed nozzles from a higher to a lower pressure, the form of the nozzles depending upon the lower pressure against which they discharge. With nozzles properly designed and constructed, the steam issues in well-formed jets at high velocity, which impinge upon blades or vanes on the turbine wheel, causing it to turn and do useful work.

The operation is therefore dependent upon two principles, the conversion of heat into kinetic energy being thermal and the utilization of kinetic energy for work on the turbine wheel, mechanical.

6. Impulse and Reaction.—Assume three fixed vanes of different form as in Fig. 22, and that jets of steam or other fluids of velocity V enter and leave the vanes as indicated by the arrows. Further assume that the velocity V, the angle α and the weight of fluid flowing in a given time is the same in each case, and that the flow is frictionless.

The force P_A of Fig. 22–A, required to prevent the horizontal displacement of the vane to the right is said to be due to the *impulse* of the jet. The force P_B of Fig. 22–B is due to the *reaction* of the jet and is equal to P_A. The force P_C in Fig. 22–C is due to both impulse and reaction and is equal to the sum of P_A and P_B. The general principle is the same

whether the vanes are fixed or moving, and whether the velocity of the jet relative to the surface of the vane is constant or variable.

In most turbines there are more than one set of moving vanes and at least one set of fixed vanes. The fixed vanes serve the purpose of nozzles as far as directing the steam is concerned, and will be so designated when they compose openings through a diagram dividing the turbine into compartments. When they are attached to the turbine casing and are of a form similar to the moving vanes they are called *guides*, and all moving vanes are called *blades*.

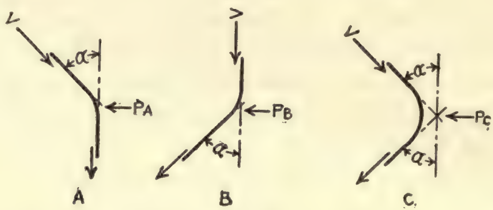

FIG. 22.

7. Commercial Classification.—Although all practical steam turbines utilize both impulse and reaction, the greater use of one principle or the other has led to the general classification of *impulse turbines* and *reaction turbines*. All turbines in which expansion occurs only in the nozzles are called impulse turbines, while in reaction turbines, expansion also occurs in the blade passages.

Considering only designs largely used, impulse turbines may be further classified as follows:

$$\text{Impulse}\begin{cases}\text{Simple} \\ \text{Compound}\begin{cases}\text{Velocity-stage.} \\ \text{Pressure-stage.}\end{cases}\end{cases}$$

Reaction turbines are always compound or multi-stage.

8. Simple Impulse Turbines.—In these turbines the steam expands in the nozzles from steam-chest pressure to the exhaust pressure, which is atmospheric for non-condensing and vacuum for condensing turbines.

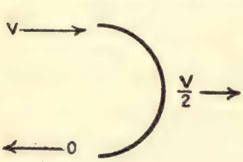

FIG. 23.

If a turbine blade of semicircular section, moving in the direction of the arrow in Fig. 23 receives a steam jet flowing in the same direction of velocity V, it is obvious that if the velocity of the blade were $V/2$, the relative velocity of steam to blade would be $V/2$; then the steam, upon leaving the blade would have an absolute velocity of zero; that is, all of its kinetic energy would be absorbed by

the blade. While it is necessary in practice that the steam jet be delivered to the wheel at an angle, and there must always be some absolute velocity at exit or *residual* velocity, it is clear that to obtain the maximum work from the jet, the blade velocity must approximate one-half the jet velocity. This means a rim velocity of nearly 1400 ft. per sec., necessitating a wheel of high-grade material and special design, which will be considered in Chap. XXXI.

To illustrate the change in pressure and velocity of the steam in its passage through nozzles and blades, the diagrams of Fig. 24 are given. Velocity and pressure changes are represented by straight lines for convenience and for lack of exact information.

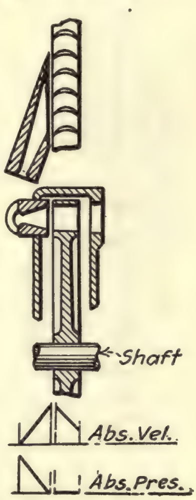

Fig. 24.—Simple impulse turbine.

The only pressure change is in the nozzle and this is accompanied by an increase in velocity, a maximum being reached at exit. There is no pressure change in passage through the blades, but a decrease in absolute velocity as the kinetic energy of the jet is given up to the wheel. The relative velocity of jet and blade is assumed to be constant, friction being neglected. The residual velocity (which is absolute), as the steam leaves the blades, is partly dissipated by friction as it forms eddies in the steam remaining in the casing, and helps to clear the casing of steam sufficiently to maintain a constant pressure.

9. Compound Impulse Turbines. *Velocity-stage.*—It was shown in Par. 8 that a very high rim velocity is necessary for good economy in a simple impulse turbine. This necessitates a rotative speed so high that for most turbine applications, gear transmission becomes necessary. To obviate this, or to make possible direct-connection, especially in large powers compounding is resorted to.

If two or more wheels carrying blades are placed upon a shaft, and between each row of blades, fixed guides are located to direct the steam leaving one wheel into the blades of the following wheel, the velocity of the jet is only in part taken up by the first wheel, part by the second wheel and so on, until upon emerging from the last wheel, the residual velocity may be the same as that of a simple impulse turbine. Then the rim speed is proportional to the number of wheels, inversely, which is also the number of stages. Thus, if a simple turbine runs 12,000 r.p.m., a 3-velocity-stage turbine with the same sized wheels, and the same steam and exhaust pressures will run 4000 r.p.m. Due to greater difference

between jet and rim velocity, and a greater number of blades receiving the impulse of the steam, the total turning force is greater, but it acts at less velocity; the power, which is proportional to the product of force and velocity, is the same as for the single wheel running at the higher speed, neglecting differences in frictional resistance and other losses.

A diagram of a 2-stage turbine is shown in Fig. 25.

As with the simple turbine, the only pressure drop is in the nozzles, accompanied by velocity increase. There is partial velocity decrease in each wheel, with no change of velocity through the guides.

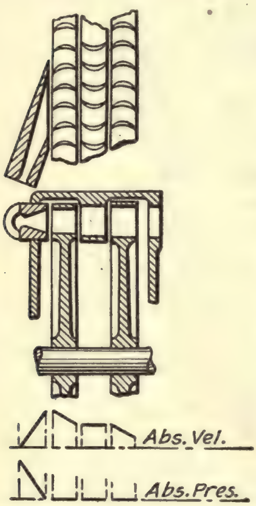

FIG. 25.—Velocity-stage impulse turbine.

It must be remembered that these velocity changes in the moving blades are absolute, the velocity relative to the blade surface being assumed constant. Modifications are made in practice to allow for friction, and different forms of blade and guide sections are sometimes used. These will be considered in Chap. XV.

Two and three stages are most used in velocity-stage turbines, although four and five stages are sometimes seen. The nozzles and guides extend around but a portion of the circumference of the casing, acting upon but a part of each wheel at one time. A set of nozzles or guides, and the row of blades into which they discharge comprise a stage in all types of compound turbines.

10. Compound Impulse Turbines. *Pressure-stage.*—When steam expands from boiler to exhaust pressure, it gives up a certain amount of heat which may be transformed into mechanical energy E. If this is utilized to produce velocity V in steam jets for turbine propulsion, we know by mechanics that the kinetic energy of W lb. of steam equivalent to E is given by

$$E = \frac{WV^2}{2g}.$$

If this energy is divided into n equal parts, each of which is used in a compartment containing a turbine wheel by being consecutively exhausted into the next lower compartment until the pressure in the last equals the exhaust pressure, the equation for the energy delivered to each compartment or stage will be,

$$\frac{E}{n} = \frac{WV^2}{2g}.$$

The velocity of the jets will be,

$$V = \sqrt{\frac{2g}{W}}\sqrt{\frac{E}{n}} = \frac{\text{constant}}{\sqrt{n}}.$$

The velocity is therefore proportional inversely to the square root of the number of stages. Then, assuming the same ratio of jet to rim velocity in all cases, the rim velocity of the pressure-stage turbine with wheels of equal size may be proportional to the square root of the number of stages, inversely. Thus, if a simple turbine runs 12,000 r.p.m., a 16-stage turbine will run

$$\frac{12{,}000}{\sqrt{16}} = 3000 \text{ r.p.m.}$$

Figure 26 contains diagrams of a pressure-stage turbine with two stages.

Each stage is like the single stage of the simple turbine, but with less pressure and velocity change.

In order that the radial length of the blades may not be too small, the nozzles of the first stage occupy but a portion of the circumference; the volume has increased sufficiently after the first two or three stages, so that the entire circumference is needed, the blades and nozzles increasing gradually in radial length to accommodate the increasing volume of steam. As the total steam passage is proportional to the product of the radial length of blades or guides and the circumference of the circle, these lengths may be reduced by increasing the diameters of wheels and diaphragms toward the exhaust end of the turbine, and this is sometimes done.

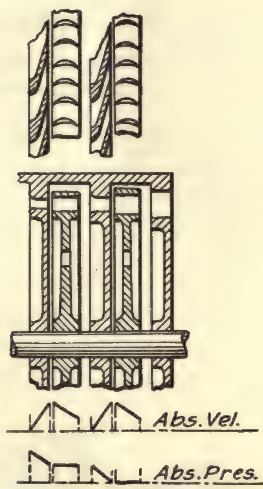

FIG. 26.—Pressure-stage impulse turbine.

11. Reaction Turbines.—Although a single-stage reaction turbine is possible, all practical reaction turbines are compound or multi-stage. There are no nozzles in the ordinary sense in which the term is used, and steam enters around the entire circumference through the usual form of guides. Unlike the turbines previously considered, there is a drop of pressure and increase of volume and relative velocity, throughout all guides and blades, which are proportioned to provide for it. The blades are attached to a drum instead of a disc such as is generally used for impulse turbines, and increase in length toward the exhaust end of the

turbine to accommodate the increasing volume. As there are a large number of stages, the pressure drop is small in each stage, which simplifies matters from the standpoint of construction as will be shown in Chap. XV.

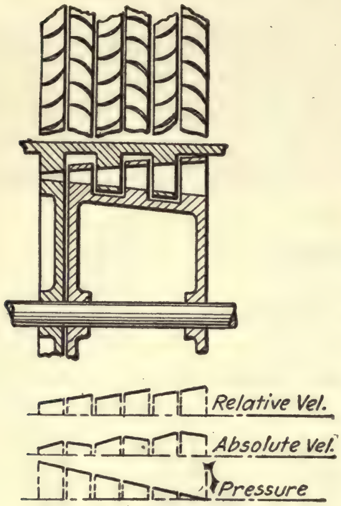

FIG. 27.—Reaction turbine.

Figure 27 shows diagrams for four stages of a reaction turbine. The ends of blades and guides, more usually open in this type, are shown closed, or *shrouded*, according to the practice of at least one builder in this country.

To simplify construction, blade lengths actually increase by groups, two to eighteen rows of blades to each group, which is called a barrel. A glance at the steam tables shows that the volume increases very rapidly as pressure decreases, so that where high vacuum is employed, the blade lengths would be excessive if the drum diameter were constant; to obviate this, the diameter is increased toward the low-pressure end, allowing shorter blades for the required steam passages. Each portion of the drum of a different diameter is called a cylinder and there are usually from two to five barrels on each cylinder. This is shown in Fig. 28. Due to the higher peripheral velocities

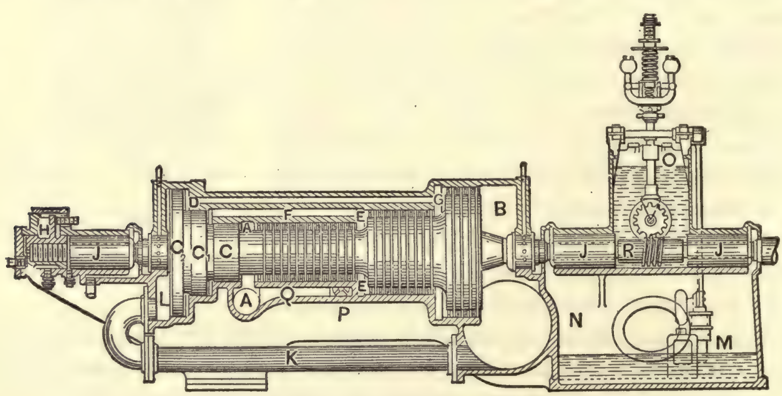

FIG. 28.—Reaction turbine.

of the larger cylinders, the velocity of steam flow increases with the size of the cylinder.

The steam pressure acting on the annular spaces due to increasing the

drum diameter causes an end-thrust on the turbine shaft. This is balanced as shown by discs at the left, of the same diameter as the different cylinders, and connected respectively with them by steam passages.

12. Combinations.—The turbine types set forth in the preceding paragraphs represent either separately or in combination, practically all successful turbines built in the United States. Some slight modifications which may be made as a compromise between practical conditions will be considered in Chap. XV, but in general, these types may be said to include the elements of steam turbine design. These are separated below, and inasmuch as the term compound is now much used in a broader sense to designate unit design, it will be omitted in connection with the elementary turbines.

Elements:
- *A*. Single-stage impulse.
- *B*. Velocity-stage impulse.
- *C*. Pressure-stage impulse.
- *D*. Reaction (multi-stage).

Double-flow Turbine.—The blading may be so arranged that steam is admitted near the center of the shaft and flows toward both ends where it is exhausted. Such a turbine is known as a double-flow turbine, and may be either of the impulse or reaction type or a combination of the two.

Pressure-velocity-stage Turbine.—This consists of a pressure-stage turbine with two or more velocity stages in one or more of the pressure stages. The Curtis turbine as formerly built by the General Electric Company, and still made in the smaller units, is perhaps the best known example. There are from two to five pressure stages with one or two velocity stages per pressure stage. Assuming a simple impulse turbine as in Pars. 9 and 10, running 12,000 r.p.m., a combination of the calculations of those paragraphs would give for a 4-stage turbine with two wheels per stage,

$$\frac{12{,}000}{2 \times \sqrt{4}} = 3000 \text{ r.p.m.}$$

The Curtis marine turbine, built by the Fore River Shipbuilding Corporation, has a varying number of velocity stages; such a turbine is illustrated and described by Prof. Peabody in his book, The Steam Turbine, in which the first pressure stage has four rows of moving blades, the second to the sixth stages three rows, and each of the ten stages following, two rows. The first stage of some turbines have two velocity stages and the rest one stage; these are sometimes known as the Curtis-Rateau type.

Repeated-flow turbines are velocity-stage turbines using a single row of moving blades. The steam from the nozzles having given up part of its

velocity to the blades, is redirected to the wheel. This method is usually confined to small turbines.

Impulse-and-reaction Turbine.—The first stage of this turbine contains an impulse wheel, or in some designs, there may be several impulse pressure stages with a varying number of velocity stages per stage. The exhaust from these then enters a reaction turbine, generally on the same shaft, where the remainder of the work is done. These turbines are sometimes known as the disc-and-drum type, the impulse wheels being disc wheels, while the reaction blading is on a drum, according to usual reaction-turbine practice. The reaction portion of the turbine is sometimes of the double-flow design.

Due to full peripheral admission in the reaction turbine, the blading at the high-pressure end is very short, and due to the low velocity, and expansion during the passage of steam through the blading, the tip leakage is comparatively large. The leakage from this cause is much less in impulse blading, which may also have partial admission and greater length, therefore the impulse principle is best adapted for the high-pressure end of the turbine, while for low pressures the reaction turbine is satisfactory, and considered by some to be more efficient.

Low-pressure Turbines.—These may be of any type, designed to receive low-pressure steam, such as the exhaust from reciprocating engines. Extensions to reciprocating engine plants have been made in this way, increasing both output and economy, but new plants will probably not be designed in this way: in fact, a large reciprocating plant was recently replaced by turbine power when the demand for increased power was made.

Mixed-pressure Turbines.—These may be turbines of any type, single or combined, arranged to receive lower-pressure steam at one of the lower stages. They may operate as low-pressure turbines, as high-pressure turbines, or with a combination of high and low pressures.

Bleeder turbines are so designed that steam may be withdrawn for heating or some industrial process from one of the lower stages. The stages below that from which the steam is taken are designed to operate with a decreased weight of steam.

Compound Turbine.—While all but the single-stage impulse turbines are compound in a sense, the present classification refers to the division of the turbine into two or more elements. The compound reaction marine turbine is sometimes composed of three elements, one high-pressure and two low-pressure. These are on separate shafts, forming a cross-compound turbine. Several large units of large power have recently been built of the cross-compound type with one high-pressure and one low-pressure element. A large tandem-compound turbine with high-pressure and

low-pressure elements in separate casings but on the same shaft, was recently installed. It is of the Rateau or pressure-stage type. The high-pressure element has ten pressure stages, there being two velocity stages in the first stage. The low-pressure element is a double-flow Rateau turbine with two pressure stages on either side.

Condensing and noncondensing turbines are both used. As further explained in Chap. XV, the influence of the condenser on capacity and economy is much greater in the turbine than in the reciprocating engine, so that, except for some special cases and for small powers, turbines are usually condensing.

Some of the advantages of the different combinations will be mentioned in Chap. XV. Other arrangements possessing merit may probably be made, and as with the steam engine, the adoption of any one design seems rather remote. The use of the different elemental types permits considerable flexibility of design, which is probably desirable.

Governing.—Steam turbines are governed by throttling or by admitting steam to a varying number of nozzles according to the load. The control of steam to the first stage is sufficient for the usual range of load: when large overloads are imposed, the governor admits high-pressure steam to some lower stage. In addition to the regular governor, practically all turbines are furnished with an emergency governor which, when a certain speed above normal is reached, automatically closes the throttle.

The speed of steam turbines, except when reducing gears are used, is dependent upon the machinery driven, which usually necessitates a multi-stage turbine of either the pressure-stage-impulse, or the reaction type.

References

Westinghouse 45,000 kw. Cross-Compound Turbine. *Power*, April 18, 1916.
Exhaust Steam Turbines. *Power*, June 6, 1916.

CHAPTER V

THE INTERNAL-COMBUSTION ENGINE

13. On account of its superior relative thermal efficiency, especially in the smaller units, the internal-combustion engine has been attractive since its first really practical introduction by Dr. Otto in 1876. The obstacles to reliable operation have gradually been overcome by persistent designers and experimenters until today, there seems to be but few power problems where it may not be used as an alternative to the steam engine. For automobile, launch, small yacht and aerial propulsion it seems to hold an almost undisputed field, and no other form of motor is suitable for submarine service. Internal-combustion reversing engines of considerable power have been successfully applied to marine propulsion. Large gas engines have been popular in Europe and have been used to some extent in this country, notably in connection with blast furnaces where the waste gas is used as fuel, thus effecting great economy. In small and medium powers, the internal-combustion engine is adapted to nearly all classes of service.

Auxiliary apparatus is not essential to the thermal cycle of an internal-combustion engine. With steam, isothermal expansion occurs in the boiler and piping as well as in the engine cylinder, and for the condensing steam engine, isothermal compression occurs partly in the piping and condenser; but with the internal-combustion engine, the cycle is complete within the engine, and aside from the manufacture or preparation of the fuel, which is not a part of the cycle, no heat transfer, expansion or compression occurs outside the engine cylinder. The mechanism concerned with the supply and ignition of fuel will be treated under valve gears in Chap. XX, but no auxiliaries analogous to boiler, condenser, etc. will be considered.

14. Classification and Cycles.—Many of the classifications given in Chap. III for the steam engine apply to the internal-combustion engine, and as it is obvious when they do not, no repetition is deemed necessary.

All practical internal-combustion engines operate with either four or two-strokes per cycle and are known respectively by the abbreviated terms four-cycle engines and two-cycle engines. Practically all types are, or

may be, built upon either of these cycles. The thermal cycle is really the same in each case, the difference being in the mode of receiving the fresh charge and exhausting the products of combustion.

Internal-combustion engines are also classified by the manner of burning the fuel. If the fuel is burned suddenly with an explosion, the piston movement is so slight during combustion that the volume is practically constant; the cycle is therefore known as the *constant-volume* cycle. If combustion is slower, so that the pressure remains nearly constant as the piston moves away from the end of the stroke until combustion is complete, the cycle is the *constant-pressure* cycle.

If an engine uses gaseous fuel it is known as a gas engine—although this term is often applied to all internal-combustion engines—if liquid fuel is used, it is known as an oil engine. Engines are also commonly known by the names of the particular gas or oil they use, as producer-gas engine, gasoline engine, kerosene engine or alcohol engine.

The following classification will cover all types treated in this book:

$$\text{4-cycle or 2-cycle}\begin{cases}\text{Constant-volume}\begin{cases}\text{Gas}\\ \text{Oil}\begin{cases}\text{Light-oil.}\\ \text{Heavy-oil.}\end{cases}\end{cases}\\ \text{Constant-pressure—Diesel.}\end{cases}$$

The 4-stroke, constant-volume cycle was the one employed by Dr. Otto and is commonly known as the Otto cycle, while the 4-stroke, constant-pressure cycle was the practical cycle of Dr. Diesel and is known as the Diesel cycle; but it is not uncommon for the 2-stroke cycle for these two methods of combustion to be also thus designated. The Otto and Diesel cycles are fully described in Chap. VI, Pars. 25 to 30, but a general description of the 4-stroke and 2-stroke cycles will now be given.

Four-cycle Engine.—In Fig. 29 are diagrams showing the position of the inlet and exhaust valves for the four strokes of the cycle. Above are shown conventional indicator diagrams for the Otto and Diesel cycles. There are really six steps in the cycle as follows:

1. *Suction.*—Starting from the position at the head end of the cylinder shown by the dotted lines, with the inlet valve open and exhaust valve closed as shown in Fig. 29–A, the piston moves to the right. A mixture of fuel and air, or with some engines, air alone, enters the inlet valve, which is either held open against the pressure of a light spring by suction, or against a heavier spring by a cam-operated mechanism. This stroke is indicated by the line 1–2 on the indicator diagrams. At the end of the stroke the inlet valve closes.

2. *Compression.*—With both valves closed, the piston now moves to the left as shown in Fig. 29–B, compressing the charge. This is shown on the indicator diagrams by line 2–3.

3. *Ignition* now takes place along line 3–4 of the indicator diagrams. This is assumed instantaneous for the constant-volume engine and is caused by an electric spark, or by the heat of compression to be explained

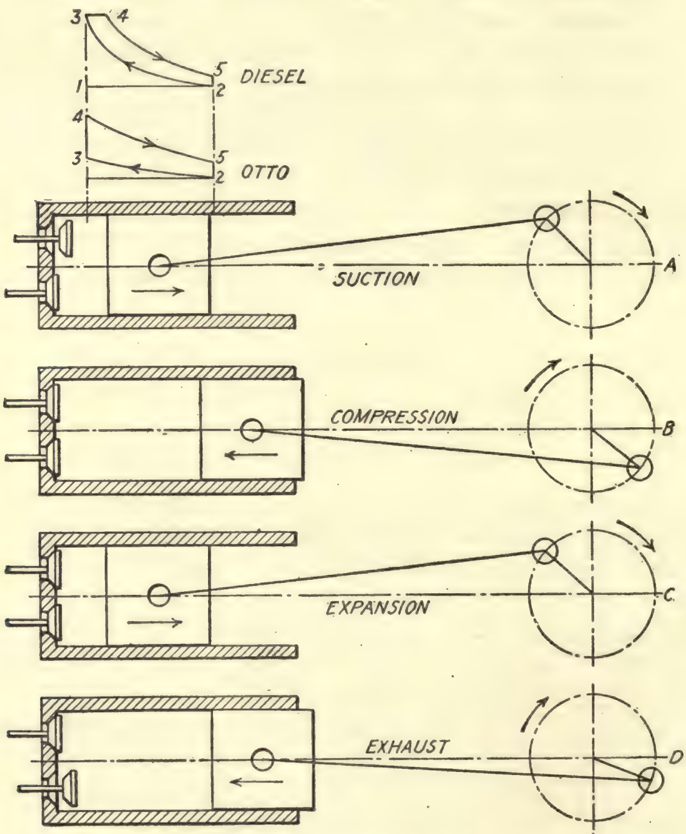

Fig. 29.—Four-stroke cycle.

presently. If air alone enters by the inlet valve during the suction stroke, as in the heavy-oil engines, oil is admitted to the cylinder during the same stroke, or is now sprayed into the end of the cylinder, against a hot plate or into a hot bulb if the constant -volume cycle is used. With either hot bulb or hot plate, combustion is assumed instantaneous. With the Diesel cycle, oil is sprayed into the heated air and burns more slowly

at nearly constant pressure, while a small portion of the stroke is accomplished by the piston.

4. *Expansion.*—With both valves closed as in Fig. 29–C, the piston moves to the right as the pressure falls, drawing line 4–5 of the indicator diagrams.

5. *Exhaust.*—The exhaust valve, always mechanically operated, opens, and the pressure drops along line 5–2.

6. *Exhaust Stroke.*—With the exhaust valve open as in Fig.29–D, the piston moves to the left, drawing line 2–1 and expelling the products of combustion. This completes the cycle.

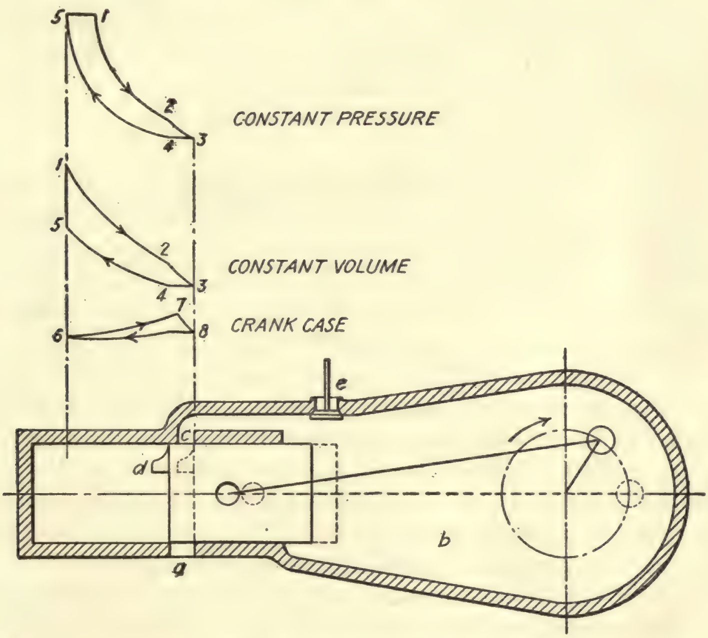

Fig. 30.—Two-stroke cycle.

In practice, the valve openings and ignition do not occur exactly at the ends of the stroke; neither is combustion instantaneous in constant-volume engines. This is treated in Chap. XX. Actual indicator diagrams also differ somewhat from the conventional. An actual constant-volume diagram is shown in Fig. 34 and a constant-pressure diagram is shown in Fig. 38.

Two-cycle Engine.—In this engine the suction and exhaust strokes are eliminated. There are no inlet and exhaust valves in the cylinder, and charging and exhaust are accomplished by means of ports in the cylinder

wall uncovered at the proper time by the piston, which performs the function of a valve. Figure 30 is a diagram of a 2-cycle engine, with indicator diagrams for the constant-volume and constant-pressure cycles.

When combustion is complete at 1 on the indicator diagrams, the piston moves to the right and the pressure drops during expansion, drawing the curve 1–2. At the point 2, shown by the full-line piston position, the exhaust port (*a*) begins to open. As the piston moved to the right it compressed the fresh charge of air and fuel (or air alone for Diesel and other heavy-oil engines) in the crank case (*b*). As the exhaust port (*a*) is opened, the burnt gases rush out and the pressure in the cylinder drops. A little further movement of the piston to the right opens the inlet port (*c*) connected with the crank case, and the fresh charge under light pressure rushes into the cylinder, is deflected by the projection (*d*) on the piston so that it may not pass across the cylinder and out of the exhaust port (*a*). This deflected stream aids in the expulsion of the burnt charge.

During the recharging period the piston moves to its extreme right position, completely uncovering both ports as shown by the dotted lines, and completing line 2–3, which is arbitrarily shown as a straight line for convenience of illustration. The pressure in crank case and cylinder at this point is presumably about the same, and equal to that of the atmosphere, and there is no noticeable change until the piston on its return stroke closes the exhaust port (*a*), when compression begins and the line 4–5 is drawn on the indicator diagrams. During this stroke the fresh charge is drawn into the crank case through the check valve (*e*), due to the partial vacuum formed by the movement of the piston.

Ignition now occurs, and with certain heavy-oil engines the fuel is sprayed into the cylinder exactly as with a 4-cycle engine, except that it occurs every revolution instead of every two revolutions.

To follow the crank-case cycle more in detail, assume the piston to be in the extreme position to the right at the point 8 of the crank case indicator diagram. As the piston moves to the left it increases in velocity, then decreases as it nears the end of the stroke; the vacuum varies with the velocity as shown by the line 8–6 drawn during the stroke as the charge is drawn in through check valve (*e*). The crank case is now full of the charge at approximately atmospheric pressure, which is prevented from leaking away by the check valve (*e*), and air-tight joints at all parts of the casing. The piston now moves to the right, compressing the charge. As the volume of the crank case is much greater than the volume displaced by the piston, the compression pressure is not very high, being from 5 to 8 lb. per sq. in. When point 7 of the compression line is reached,

the piston opens the inlet port; the charge rushes in as previously described and the pressure drops from 7 to 8 as the piston reaches the end of the stroke, completing the crank-case cycle.

The work done in the crank case is negative work and must be deducted from the work done in the cylinder as shown by an indicator diagram taken from it.

A study of the valve gear of the 4-cycle engine given in Chap. XX shows that it will run in but one direction unless some special reversing mechanism is employed, while it is obvious from Fig. 31 that the 2-cycle engine will run in whichever direction it is started.

Fig. 31 shows a modification of the 2-cycle engine known as the 3-port engine, in which the check valve of Fig. 30, admitting the charge to the crank case, is replaced by the port (*g*), which is opened by the piston as it nears the end of the stroke. The suction is increased up to the time

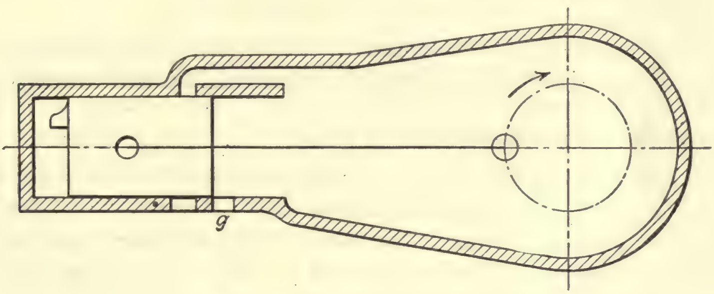

FIG. 31.—Three-port engine.

of the port opening, probably increasing the negative work, and the charging of the crank case must occur during the period of port opening, which is less than for the 2-port engine.

The diagrams given are for single-acting engines in which the piston performs the function of a crosshead. A crosshead must be used with a double-acting engine, and is sometimes used with single-acting engines. When used with single-acting 2-cycle engines, the crank end of the cylinder is enclosed and utilized instead of the crank case for compression of the charge, leakage around the piston rod being prevented by a stuffing box. Double-acting 2-cycle engines of large power, using gaseous fuel, are provided with separate charging pumps for gas and air, operated from the engine crank shaft.

Cylinder Cooling.—This is a necessity with the internal-combustion engine, due to the extremely high temperatures developed in the cylinder. Without cooling, lubrication would be impossible and the metal would be

weakened. For this purpose, water, or in some small engines air, is circulated around the cylinder wall, withdrawing a portion of the heat generated by combustion. This will be further mentioned in Par. 21.

Compression pressure varies with the fuel and method of operation, and will be considered in Chap. XIV.

With this general description of the essentials of internal-combustion engines, some differentiating features will be mentioned concerning the ultimate subdivisions given early in this paragraph, viz., gas, light-oil, heavy-oil and Diesel engines.

15. Gas engines use fuels which are in the form of a fixed gas. They may be either 4-cycle, 2-cycle, single-acting or double-acting. They are always constant-volume engines, no successful constant-pressure gas engine having yet been developed. They are built for a wide range of power. The fuels used are natural gas, the different forms of manufactured illuminating gas, producer gas and blast-furnace gas. Ignition is by electric spark.

Natural gas is a good fuel but is available in but few localities.

Illuminating gas is too expensive for general use and is only used for small powers.

Producer gas is formed by the partial combustion of fuel. A large variety of fuels may be used, including many waste products, but anthracite coal is perhaps the most used and most satisfactory for power purposes. Producer gas is much cheaper than illuminating gas due to less expensive methods of manufacture, and is a very satisfactory fuel.

Blast-furnace gas, as the name implies, is a product of the blast furnace. It is burned with difficulty under steam boilers, but under high compression pressure makes a satisfactory gas-engine fuel.

16. Light-oil engines are either 4-cycle or 2-cycle, but are practically always single-acting engines. They are constant-volume engines, although light oils may be used with constant-pressure engines. They are comparatively small but have a wide range of application. The fuels commonly used are gasoline, kerosene, distillates—petroleum products between gasoline and kerosene—and alcohol. An engine built for gasoline may burn any of these other fuels, but better results are obtained by some modification; that is, the compression pressure should be lower for kerosene and higher for alcohol, and special devices for handling the fuel are sometimes employed. Air is mixed in a carbureter, the fuel being introduced in the form of a fine spray. Both air and fuel are drawn through the carbureter by the partial vacuum formed during the suction stroke of the 4-cycle engine, or the pump stroke of the 2-cycle engine. Ignition is by electric spark.

17. Heavy-oil engines of the constant-volume type may use any fuel from kerosene to the heavy low-grade oils; usually the heavier oils are used, and seem to be more satisfactory even if the question of economy is not considered. These engines are practically always single-acting. Ignition is due to the heat of compression, no electric spark being required.

The *hot-bulb* engine is one of the earlier forms of the heavy-oil engine. The end of the cylinder of one of the older designs is shown in Fig. 32. The bulb-shaped portion of the clearance space is separated from the cylinder end by a small passage. In this bulb are a series of projecting plates or points. To start the engine the bulb is brought to a dull-red heat by means of a torch and blower, but is thereafter kept hot by the combustion of the fuel. This engine is a 4-cycle engine, and during the suction stroke, while pure air is being drawn into the cylinder, oil is sprayed into the bulb where it is vaporized by the heat stored in the metal walls and projecting plates. As only burnt gas was left in the bulb at the end of the previous exhaust stroke, the oil vapor will not ignite. Should there be air introduced at this time, the temperature is not equal to that required for ignition. When air is compressed the temperature rises, so, as the piston starts upon the compression stroke the temperature gradually rises; at the same time, the air is being forced back into the hot bulb and mixes with the vaporized oil. The projecting plates become incandescent, and at the proper point in the stroke, determined by experiment, the mixture and temperature are such that ignition occurs. The compression pressure is comparatively low in this type.

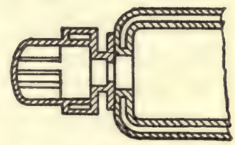

Fig. 32.—Hot bulb.

The newer type of hot-bulb engine and the hot-plate engine are sometimes called medium-compression oil engines. In these, compression is higher, sometimes 300 lb. per sq. in., and the oil is sprayed into the bulb or against the plate at the end of the compression stroke as in the Diesel engine. The temperature attained is not high enough to insure combustion without the aid of the bulb or plate, which is made red-hot by the heat of compression added to the heat of combustion of the previous cycle. Combustion is theoretically at constant-volume, the pressure rising to between 450 and 500 lb. per sq. in., gage. It is probable that combustion continues along the stroke for a short distance. While usually a 4-cycle engine, the 2-cycle principle may be applied to this type.

These engines are often known as semi-Diesel engines, although it is claimed by some that this name applies only to engines in which the fuel is directly injected without the use of air; there seems to be little reason for this distinction.

18. The Diesel engine is the practical representative of the constant-pressure type and has attained the highest thermal efficiency of any heat engine. For the same compression pressure, it will be seen from Chap. VI, Par. 26, that the theoretical constant-volume cycle has a higher efficiency than the constant-pressure cycle, but the absence of fuel during compression permits a much higher compression in the Diesel cycle, offsetting the theoretical advantage of the Otto cycle. The temperature due to this high pressure—450 to 600 lb. gage—is high enough to ignite the fuel as fast as it enters the cylinder without the aid of hot bulb or plate. The fuel is forced in by air pressure much higher than the compression pressure, the design of spray nozzle and timing of discharge being such that combustion is at approximately constant pressure. In most cases, air for fuel injection is provided by a compressor operated from the engine shaft, and stored in high-pressure tanks or bottles; but sometimes the air is led directly from the compressor to the spray nozzle, a definite amount being injected with each charge. In some engines the fuel is injected directly from an oil pump.

While the Diesel engine proper is a 4-cycle engine, the 2-cycle principle is successfully applied.

These engines have been thus far single-acting engines, and burn the same fuels as the heavy-oil constant-volume engines. They may even burn oils harder to ignite, and according to E. W. Roberts in "The Gas Engine" of July, 1916 (Liquid Fuels, Present and Future), they may burn cottonseed oil, olive oil, cocoanut oil and mustard-seed oil, the last named having been used in India twenty years ago.

Diesel engines are usually built for medium powers, although engines developing over 3000 b.h.p. are now in use. While usually a heavy engine due to the high pressure, the capacity is greater for a given cylinder diameter, and it is not unlikely that the 2-cycle Diesel engine will be developed for automobile propulsion and even for the airplane.

19. Governing.—The regulation of small stationary engines using gas or light oils is often accomplished by the "hit-and-miss" governor; that is, when less power is required, one or more explosions are missed. This may be accomplished in different ways, but when the automatic inlet valve is used, the exhaust valve is held open during the cycles when the explosion is to be missed. This relieves the suction, and the inlet valve is not drawn open. This method is simple, and as adjustment may be made to give the best fuel mixture and compression at all times, the engine always operates with the best thermal efficiency if the load is not so light as to unduly cool the cylinder.

For large engines using these fuels, governing by throttling is now

the most common method. This is sometimes applied to the fuel only, especially with gas engines. This changes the proportion of air and gas and is known as *quality* governing. More usually the mixture is throttled as it passes into the cylinder, the ratio of gas to air being kept as nearly constant as possible: this is the *quantity* method. The governing mechanism is more simple for quantity governing, and while the theoretical thermal cycle efficiency is greater for the quality method due to the constant best compression pressure, the better combustion of a constant mixture more than offsets this, according to several leading authorities.

A combination of the quality and quantity methods is sometimes used. As lean mixtures (with a small percentage of gas) do not ignite easily, and burn slowly, a minimum satisfactory ratio of gas to air is handled by the quality method; when more power is required, more gas is admitted and when less power than that given by this minimum mixture, the entire charge is throttled. Although the governing mechanism is somewhat more complicated this method gives good results.

Variable-speed engines such as those used for automobiles, govern by changing the mixture, by throttling and by timing the ignition, all by hand control.

In the heavy-oil engine, governing is effected by changing the amount of oil pumped to the spray nozzle. The pump handles more oil than is required for the maximum load, and that not required is by-passed by means of the governor mechanism.

Governors and their connections will be treated in Chaps. XIX and XX.

20. Starting.—Small engines are commonly started by hand after properly adjusting the fuel valve and ignition apparatus. Electric starters are used for automobile engines, the current for which is furnished by a storage battery which is charged while the engine is running.

Large engines are usually started by compressed air admitted to the cylinder by separate cams. When the engine is up to speed, fuel is turned on and the regular cycle begins. With the Diesel and semi-Diesel engines, air for starting is commonly furnished by the compressor which furnishes the air for fuel injection. It is taken from the lower stage of the compressor and stored in a separate tank, the pressure in which is automatically governed.

21. Cylinder cooling apparatus and exhaust mufflers may be considered as auxiliaries, but are so essential to the operation of internal-combustion engines that they will be briefly mentioned. With the exception of a few small engines, the cylinder is cooled by the circulation

of water. Water cooling may be accomplished by gravity or by forced circulation. A gravity system is shown in diagram in Fig. 33. As the water is heated in the cylinder it rises, flowing to the upper part of the tank where it is cooled by exposure to the air. The water falls to the bottom of the tank as it cools and enters the engine cylinder by the pipe (*b*). As the force causing the flow in a gravity system is very small and the slightest stoppage of the pipe may cause it to cease circulating, a pump is usually placed in the system, making the flow more positive. Where water is scarce, the power great, or large water storage not practicable, various devices are used for cooling the water. In the automobile, a radiator placed at the front of the car, is used, through which air is forced by a fan. For large powers the cooling tower or cooling pond is used, as for the cooling of circulating water for condensers.

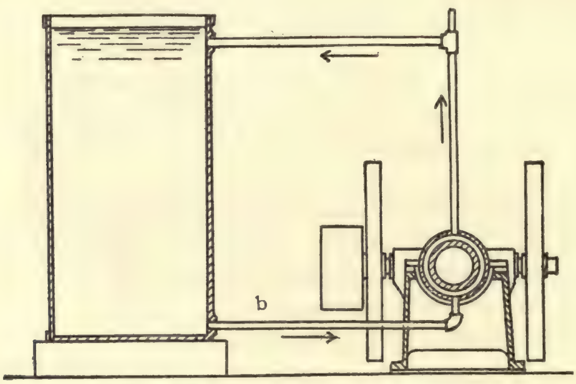

Fig. 33.—Gravity cooling system.

The exhaust of an internal-combustion engine is noisy and disagreeable and is rightly considered a nuisance. To reduce this noise the muffler is used. The muffler is an enlargement of the exhaust pipe containing perforated plates, the object being to reduce the velocity of the gas. Cooling is sometimes resorted to, decreasing the volume of gas passed. There are a great variety of designs, but the principle is the same in all.

The noise due to intake of the charge is often objectionable in engines of large capacity and this is also muffled.

The remaining paragraphs of this chapter will be devoted to illustrations and descriptions of a number of engines built by leading manufacturers. Details of these and of certain auxiliaries will be treated in later chapters.

ILLUSTRATIONS AND DATA FROM PRACTICE

22. The Bruce-Macbeth Co. of Cleveland, Ohio, manufacture gas engines for natural and producer gas. They are of the 4-cycle, vertical, multi-cylinder type and are suitable for a wide range of service. They are provided with dual jump-spark ignition, receiving current from a magneto located on the cam shaft.

Fig. 34 is an indicator diagram from a 4-cylinder, 12¼ by 14 in. Bruce-Macbeth engine, and Fig. 35 is a gas-consumption curve for the same engine using natural gas, the rated capacity being 150 b.h.p.

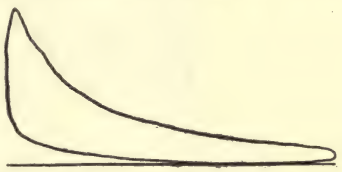

Fig. 34.—Indicator diagram from Bruce-Macbeth engine.

Table 1 gives general information concerning Bruce-Macbeth engines, including net price, which quantity may be subject to fluctuation.

The De La Vergne Type "FH" Crude-oil Engine is built by the De La Vergne Machine Co., New York. These engines are of the horizontal, 4-cycle, hot-bulb, semi-Diesel type, liquid fuel being injected at the end of the compression stroke by means of compressed air. Ignition is due to heat deposited in the walls of the vaporizer chamber or hot bulb by the

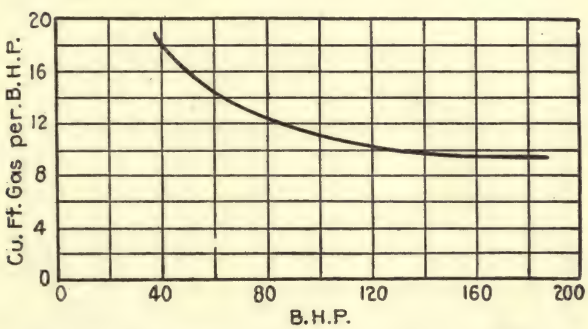

Fig. 35.—Gas consumption of a Bruce-Macbeth engine.

previous explosion. The compression pressure is about 280 lb. per sq. in. gage, and causes a temperature which, while not sufficient to ignite the charge, aids combustion. The explosion pressure is about 480 lb. gage. These engines are of extremely rugged construction and of excellent design throughout. One of them gave a continuous service of 800 hours, as stated in the *Trans.* A. S.M.E. referred to at the end of this chapter.

TABLE 1.—THE BRUCE MACBETH ENGINE COMPANY, FOUR-CYLINDER VERTICAL GAS ENGINE

Natural b.h.p.	Producer b.h.p.	Net price	R.p.m.	Bore in inches	Stroke in inches	Dia. of wheel in inches	Bearings			Crank		Piston		Connecting rod length	Dia. of valves		Weight		Space required		
							Dia. outer	Dia. center	Total length	Dia. inches	Length inches	Dia. inches	Length inches		Inlet	Exhaust	Engine with fly-wheel	Fly-wheel	Length	Width	Height
100	75	3,300	300	10	11	52	5	5	46½	5	3½	2¼	4½	30½	3	3	20,000	3,500	9′–9″	4′–4″	7–8″
150	125	4,500	275	12¼	14	64	6	6	60	6	4½	3	5⅜	35	4	4	29,000	5,000	10′–11″	5′–4″	8–10″
250	190	8,000	250	14	16	78	7¼	7¼	93	7½	5½	3½	6	40	4½	4½	51,000	8,500	15′–7″	6′–6″	10′–5″
350	300	11,800	220	18	18	90	8½	8½	97	9	7	4½	8	45	5¾	5¾	83,000	12,500	18′–2″	7′–9″	12′–5″
Two-cylinder Vertical Gas Engine																					
40	30	1,425	300	9	11	52	4	4	34	5	3½	2¼	4½	30½	3	3	11,000	3,500	6′–5″	4′–4″	7′–4″
40	30	1,500	300	9	11	52	4	4	34	5	3½	2¼	4½	30½	3	3	Valve	in cages	6′–5″	4′–4″	7′–4″
50	35	1,575	300	10	11	52	4	4	34	5	3½	2¼	4½	30½	3	3	12,000	3,500	6′–5″	4′–4″	7′–7″
60	50	1,950	275	11	13	60	5	5½	41	6	4½	3	5⅜	32½	4	4	15,500	4,500	7′–6″	5′–0″	8′–3″
70	60	2,250	275	12¼	13	60	5	5½	41	6	4½	3	5⅜	32½	4	4	17,000	4,500	7′–6″	5′–0″	8′–3″

Indicator diagrams from a Type "FH" engine is shown in Fig. 36.

The fuel consumption of a 20 by 34½ in. De La Vergne Type "FH" engine is given in Fig. 37. This engine is rated at 140 b.h.p. at 165 r.p.m.

The McIntosh and Seymour Diesel Type Oil Engines are built by the

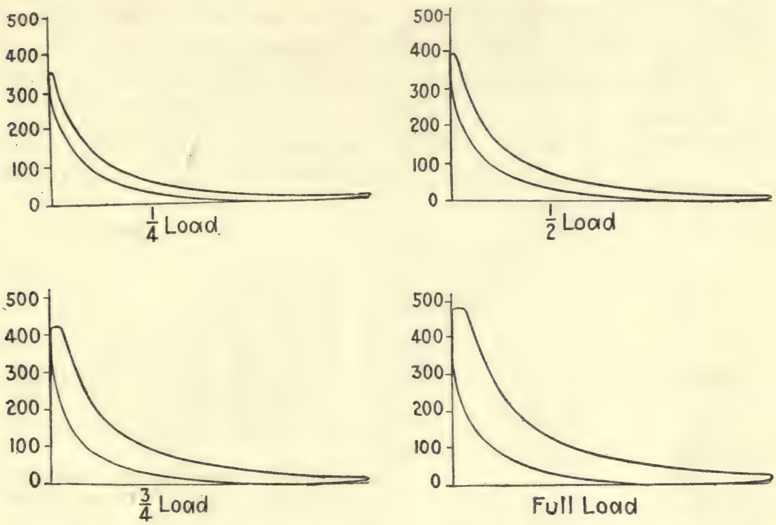

FIG. 36.—Indicator diagrams from a De La Vergne type "FH" engine.

McIntosh and Seymour Corporation, Auburn, N. Y. They are typical vertical, 4-cycle Diesel engines having the fuel injected under air pressure. They are built in two styles, Type *A* having four cylinders and individual frames for each cylinder, bolted to a common base; and Type *B* having from one to six cylinders bolted to a single box frame. Both types have a multi-stage air compresser for fuel-injection and starting air, driven directly by a crank on the end of the main shaft.

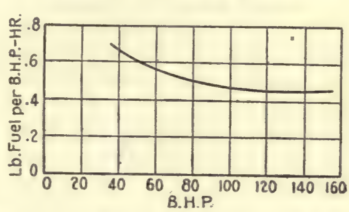

FIG. 37.—Fuel consumption of a De La Vergne type "FH" engine.

The speed of the Type *A* engines ranges from 135 to 170 r.p.m.; and of the Type *B*, from 120 to 280 r.p.m.

A test was published of a Type *A* engine having a cylinder diameter of 18⅞ in. and a stroke of 28⅜ in., and designed to develop 500 b.h.p. at 164 r.p.m. Fig. 38 shows an indicator diagram taken from this engine, and Fig. 39 a curve of fuel consumption.

The Busch-Sulzer Bros. Diesel engine is a typical vertical Diesel engine

and is built by the Busch-Sulzer Bros. Diesel Engine Co. of St. Louis. One working cylinder and the two-stage air compressor are shown in section in Fig. 40, and an end section through the cylinder in Fig. 41.

These drawings are to scale and a good idea of general proportions may be obtained from them.

Continental Motors are built by the Continental Motors Corporation, Detroit, Mich. This company, which makes motors only, has several types, suitable for automobiles and trucks. They are all water-cooled 4-cycle engines.

Model 7W motor has a bore of 3¼ in. and a stroke of 4½ in. Fig. 42 gives a capacity curve at different speeds, showing that the maximum of 42 b.h.p. is at 2140 r.p.m.

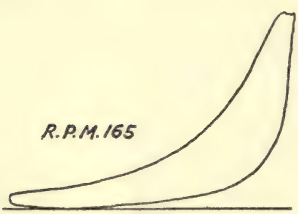

Fig. 38.—Indicator diagram from a McIntosh & Seymour Diesel engine.

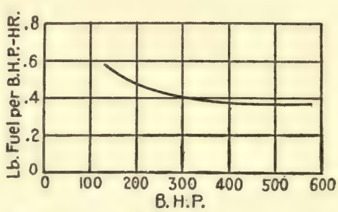

Fig. 39.—Fuel consumption of McIntosh & Seymour Diesel engine.

Some of the most interesting data for this engine are given below:

Ratio of clearance to total cylinder volume	0.225
Firing order	1–5–3–6–2–4
Total weight	575.0 lb.
Flywheel, 16 in. diameter	62.0 lb.
Compression pressure-gage	61.5 lb.

Crank shaft: 3 bearing—2¼ in. diam. Bushings bronze, babbitt lined. Front bearing 2⁹⁄₃₂ by 2⁹⁄₁₆ in.. Center bearing 2¼ by 3 in. Rear bearing 2⁷⁄₃₂ by 3⁵⁄₃₂ in.

Cam shaft: 3 bearing—1 in. diam. Front bearings 1⅞ by 2¹¹⁄₃₂ in. Center bearing 1¹³⁄₁₆ by 2⅜ in. Rear bearing 1¹¹⁄₁₆ by 2½ in.

Connecting rod 8½ (3.78 cranks) long. Crank-end bearing 1²⁹⁄₃₂ by 2 in. Piston pin bearing ⅞ by 1½ in.

Piston 3½ in. long. Three rings, ³⁄₁₆ in. wide.

Model E7 is a four-cylinder truck engine with two cylinders cast en bloc and an aluminum crank case. The cylinders are 4½ in. diameter with a 5½-in. stroke.

Fig. 43 is a capacity curve for a Model E7 engine.

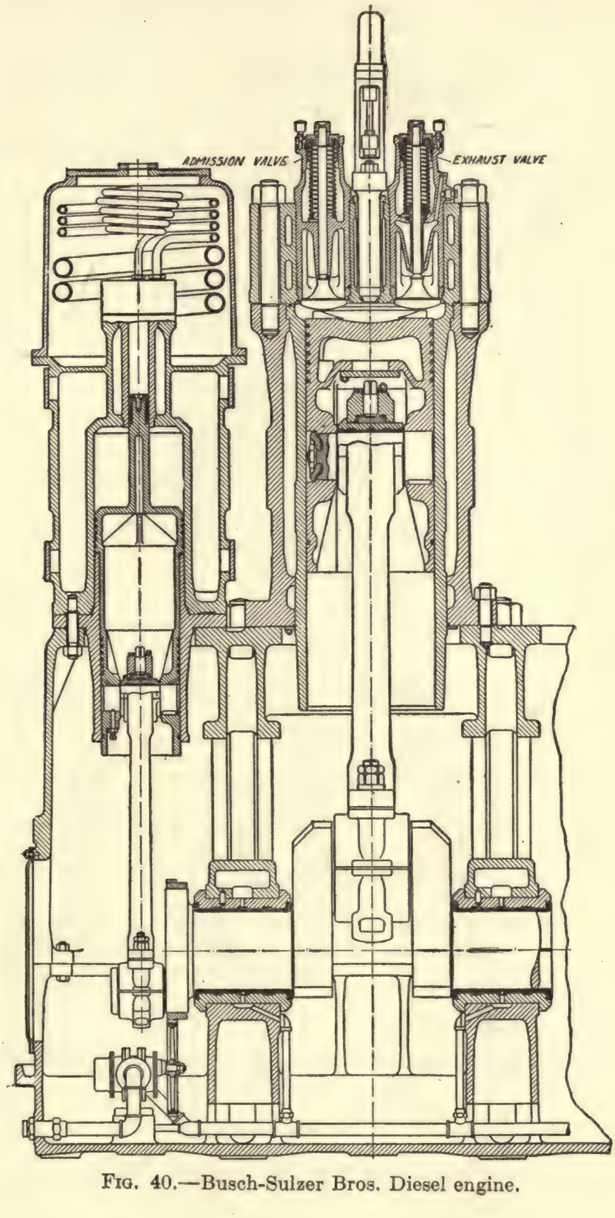

FIG. 40.—Busch-Sulzer Bros. Diesel engine.

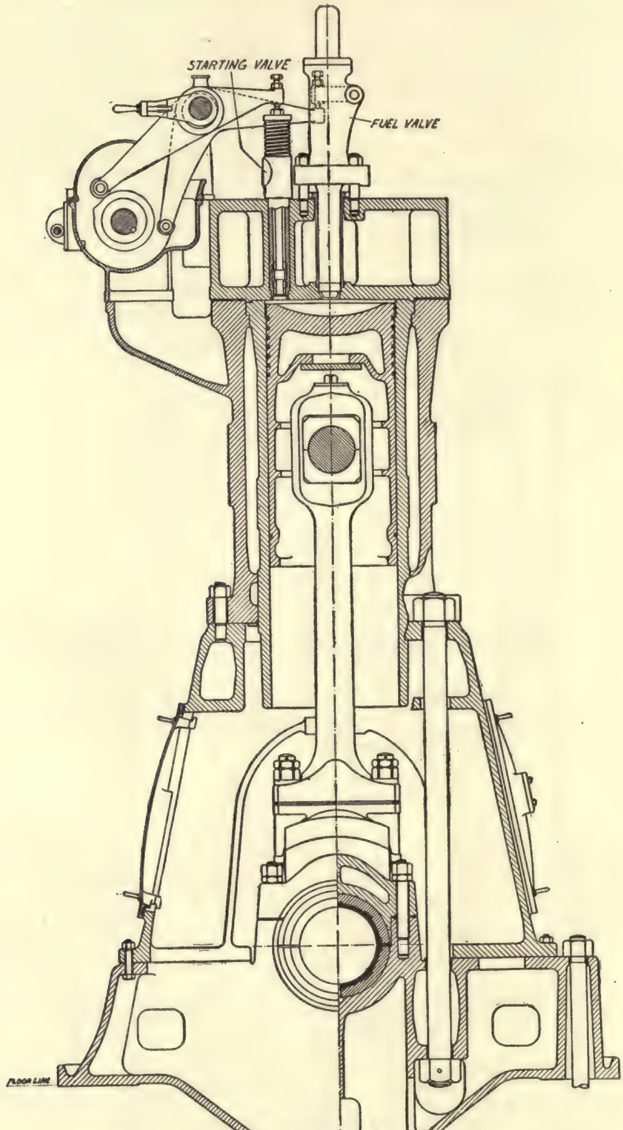

FIG. 41.—Busch-Sulzer Bros. Diesel Engine.

The maximum power is 44 b.h.p. at 1320 r.p.m.
Some of the data for Model E7 is as follows:

Ratio of clearance volume to total cylinder volume	0.239
Compression pressure-gage	55 lb.
Firing order	1–3–4–2
Total weight	660 lb.
Flywheel, 17¼ in. diam., weight	105 lb.

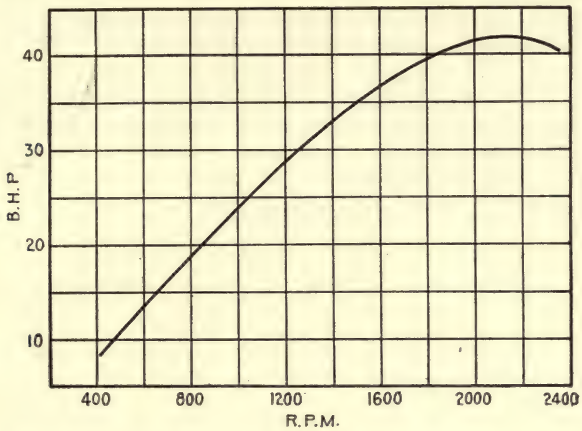

Fig. 42.—Power curve for model 7W Continental engine.

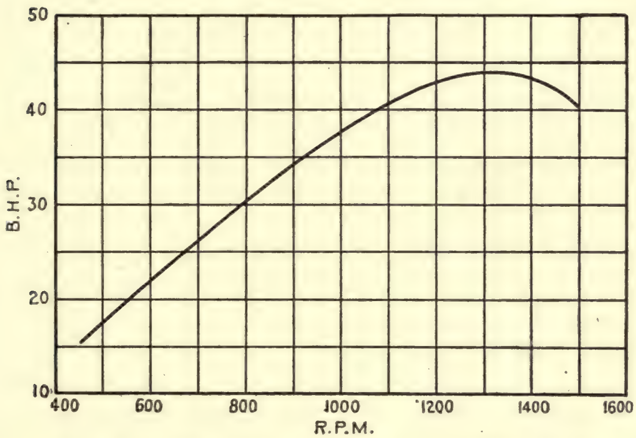

Fig. 43.—Power curve for model E7 Continental engine.

Crank shaft—3 bearing—2¼ in. diam. Bushings bronze, babbitted. Front bearing 2³⁄₁₆ by 3 in. Center bearing 2⁷⁄₃₂ by 3¾ in. Rear bearing 2¼ by 4¹⁵⁄₁₆ in.

Cam shaft—3 bearing—1⅛ in. diam. Front bearing 2¼ by 2⁵⁄₁₆ in. Center bearing 2¼ by 2¼ in. Rear bearing 2¼ by 1¼ in.

Connecting rod 11 in. (4 cranks) long. Crank end bearing $2\frac{1}{4}$ by 3 in. Piston end bearing $1\frac{7}{16}$ by $2\frac{1}{4}$ in.

Piston, $5\frac{5}{8}$ long. Three rings, $\frac{3}{16}$ wide.

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PART II—THERMODYNAMICS

CHAPTER VI

GENERAL POWER FORMULAS AND GASES

23. Power of Heat Engines.—Heat-engine cycles are usually complete on one side of the piston, and if the pressures acting on that side as the piston travels back and forth are plotted, a diagram is produced which is a measure of the work done during the cycle. This diagram is traced for actual engines by an instrument called an *indicator*, and all such diagrams, either actual or for theoretical cycles such as the Carnot cycle, are called *indicator diagrams*, or perhaps more commonly, *indicator cards*.

If the cylinder is double-acting, two cycles, on opposite sides of the piston, are accomplished at the same time, although corresponding strokes are not simultaneous.

Should the cycle depend, for the performance of different functions, upon the two sides of the piston, or upon a main and auxiliary piston (as in 2-cycle gas engines), indicator diagrams must be obtained from these separately and the net work computed. However, in such a case the real work of the cycle is done on one side of the piston and it is this side which will be considered as the *working cylinder-end*.

In practice the indicator diagram is mostly used to determine the *mean effective pressure* (m.e.p.), which, when engine dimensions, speed and the cycle on which the engine operates are known, may be used for the determination of power, or conversely, the determination of cylinder dimensions necessary for a given power.

In all practical engines, all strokes are equal, so let v_s denote the volume swept through by the piston during a complete single stroke, in cu. ft. Let p_1, p_2, p_3 and p_4 be mean pressures in lb. per sq. ft. acting during four consecutive strokes of a 4-stroke-cycle engine, on one side of the piston. On all strokes toward the cylinder-end considered, the working substance resists the movement of the piston and the work is negative.

The pressure in the working cylinder-end may be considered as absolute. The pressure on the opposite side of the piston may be ignored as it does not affect the *indicated work* of the cycle.

The work of the cycle, in ft. lb., for one cylinder-end is:

$$W = p_1v_s - p_2v_s + p_3v_s - p_4v_s = (p_1 - p_2 + p_3 - p_4)v_s = 144P_Mv_s \quad (1)$$

where P_M is the m.e.p. in lb. per sq. in., of the whole cycle, referred to one stroke. Any other cycle would give the same result.

Then:

$$P_M = \frac{W}{144v_s} \quad (2)$$

This m.e.p., exerted throughout one stroke of the piston, would do the work of the cycle for the working substance on one side of the piston.

The indicator diagram is drawn with the length representing the length of stroke; then the area divided by the length gives the m.e.p. when dimensions are in terms of pressure and volume, and in (2), $W/144$ indicates that pressure is in lb. per sq. in.

The work in terms of heat units is:

$$W = \frac{Q_1 - Q_2}{A}$$

where Q_1 and Q_2 are heat quantities received and rejected per cycle, respectively, and A is the reciprocal of Joule's equivalent ($= 1/778$). Substituting this in (2) gives:

$$P_M = \frac{Q_1 - Q_2}{144Av_s} = 5.4\frac{Q_1 - Q_2}{v_s} = 5.4\frac{eQ_1}{v_s}$$

$$= 5.4\begin{bmatrix}\text{B.t.u. converted into work per} \\ \text{cu. ft. of piston displacement}\end{bmatrix} \quad (3)$$

Formulas (2) and (3) are general expressions of m.e.p. for all heat engine cycles, theoretical or practical.

Power is work done in a unit of time; then as 33,000 ft. lb. per minute is the unit of horsepower, the work done per minute divided by 33,000 gives the horsepower. Power determined by means of an indicator diagram, which is the power developed in the engine cylinder, is called *indicated horsepower* (i.h.p.).

Let N_C = the number of cycles per minute; then:

$$i.h.p. = \frac{WN_C}{33{,}000} = \frac{144P_M v_S N_C}{33{,}000} \quad (4)$$

With the usual slider-crank mechanism employed for reciprocating engines there are two strokes per revolution; then the number of strokes per min. is $2N$, where N is the number of revolutions of the crank per minute (r.p.m.). If m denotes the number of strokes per cycle, then the number of cycles per min. per cylinder-end is:

$$N_C = \frac{2N}{m}$$

Then for one working cylinder-end (4) becomes:

$$i.h.p. = \frac{2 \times 144}{33{,}000} \cdot \frac{P_M v_S N}{m} \tag{5}$$

The maximum difference in pressure on the two sides of a piston during a cycle is called the *maximum unbalanced pressure*, and it is this which determines the required strength and consequent weight of the working parts of the engine. If this pressure is high relative to the m.e.p., the engine will be heavy for the work done, and the friction may more than offset the advantage of an economical heat cycle if this is carried too far, as will be presently shown.

Brake Horsepower (b.h.p.) is the power delivered at the engine wheel, or net power, as measured by some form of friction brake or dynamometer. It is the difference between the i.h.p. and the *friction horsepower* (f.h.p.).

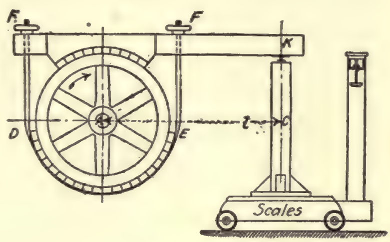


Fig. 44.

Figure 44 is a diagram of a simple form of friction brake. The band consists of blocks of wood held together by band iron, with adjustment at (F). The engine wheel rim sliding inside this band is resisted by friction. The product of this resistance in lb. and the distance traveled by the rim in ft. per min. is the work in ft. lb. per min. absorbed by the brake. This divided by 33,000 gives the b.h.p.

It is usually inconvenient to measure the resistance right at the wheel rim, so a brake arm of length l is made long enough to rest upon a pedestal on the scale platform. If P is the effective force in lb. after deducting the weight of pedestal and unbalanced weight of brake arm, l is the length of the arm in ft. and N the r.p.m., then:

$$b.h.p. = \frac{2\pi lNP}{33{,}000} = \frac{lNP}{5252} \tag{6}$$

The b.h.p. measured in this way may be applied by belting to machinery, by direct-connecting to electric generators or other machinery, or through gears, and is probably not an exact measure of the net work, as bearing friction must be affected differently in the various cases of application.

For electrical machinery the output is measured at the switchboard in kilowatts, which may be reduced to *electrical horsepower* (e.h.p.); or,

$$e.h.p. = 1.34 \times kw.$$

This is the net power after deducting the friction of engine and generator. If the friction of the generator is known the b.h.p. may be found.

In pumping engines *pump horsepower* is the indicated work done in the pump cylinder, the difference between this and the i.h.p. being the f.h.p.

Mechanical efficiency is given by the ratio:

$$e_M = \frac{i.h.p. - f.h.p.}{i.h.p.} = \frac{b.h.p.}{i.h.p.} \tag{7}$$

The friction of an engine does not vary greatly for varying loads, so that if the i.h.p. is small relative to the size of the engine, e_M is small. Then e_M decreases as i.h.p. (which is directly proportional to P_M at constant speed) decreases, the limit being zero—when the i.h.p. is just sufficient to run the engine (see Chap. X).

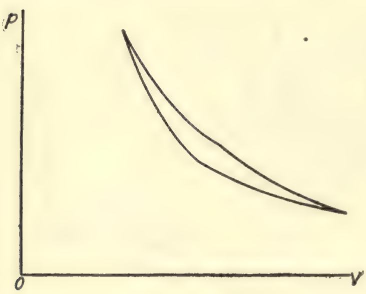


FIG. 45.

A diagram of the Carnot cycle for air is plotted to scale in Fig. 45. The m.e.p. of such a diagram is very small when compared with the maximum pressure, and even though Carnot's cycle were practically possible, an engine employing it would probably be able to do little more than overcome its own friction if gas were the working substance. This would not be true of steam, the m.e.p. in one case being about four times that for air when the same pressure and temperature limits were assumed. Neglecting friction, if a cylinder 20 in. in diameter were used on a steam engine, the air engine with the same mean piston speed, developing the same power would require a cylinder about 40 in. in diameter.

It is obvious therefore that the utility of Carnot's cycle for gas is limited to what it contributes to the science of thermodynamics.

24. Practical Cycles for Engines Using Gas.—In practical efficient engines using gas (air and fuel gas or vapor) as a working substance, the combustion of fuel takes place in the engine cylinder. This necessitates a fresh supply of air for every cycle, with a consequent exhausting of the burnt gases. This may be accomplished with a 2-stroke cycle by the use of auxiliary cylinders; or suction and exhaust strokes may be added, making a 4-stroke cycle.

The constant-volume and constant-pressure cycles are the two cycles now employed for *internal-combustion engines*, and while the theoretical efficiencies are greatly in excess of those actually attainable, due to the impracticability of utilizing such high temperature, the practical

results are satisfactory when compared with the use of steam as a working substance. Both of these cycles were originally intended for 4-stroke cycles, but have been modified to operate with two strokes. In their theoretical consideration this makes no difference, and heat transfer to and from a given weight of gas may be assumed.

25. The Constant-volume Cycle.—Figure 46 is the indicator diagram for the theoretical, 4-stroke, constant-volume cycle, commonly called the Otto cycle. Let it be assumed that the clearance space v_1 contains 1 lb. of air at atmospheric pressure. In practice this is usually a mixture of air and burnt gas, mostly the latter unless some special device is employed for clearing out all the products of combustion, which is known as *scavenging*.

While there are but four piston strokes per cycle, there are six distinct steps as follows:

1. Starting from the position shown by the full lines in Fig. 46, with the inlet valve open and exhaust valve closed as in Fig. 29–A, Chap. V, the piston moves through the distance v_s representing the volume of stroke, to the position given by the dotted lines (Fig. 46). This is the suction stroke, during which the charge of fuel and air (necessary for its combustion) is taken into the cylinder. In some engines using the heavy oils with the constant-volume cycle, only air is admitted to the cylinder proper during the suction stroke, the fuel being admitted into a hot bulb forming part of the combustion chamber, or sprayed against a hot plate or into a hot bulb at the end of the compression stroke. With medium compression (280 to 300 lb. per sq. in. gage), the latter are known as semi-Diesel engines.

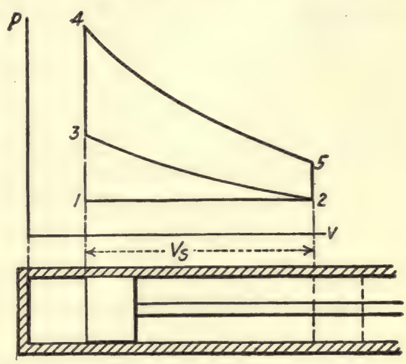


FIG. 46.—Constant-volume cycle.

The suction stroke is simply a mechanical step and plays no direct part in the thermodynamics of the cycle. It affects the power by using time that might have been employed for work. The pressure during the suction stroke is less than that of the atmosphere and any loss resulting is charged to engine friction. This pressure difference will be neglected in the discussion of the theoretical cycle.

2. The charge is next compressed adiabatically along line 2–3 during the compression stroke while both inlet and exhaust valves are closed, as in Fig. 29–B, Chap. V.

3. Ignition is now effected, usually by an electric spark except in oil engines using the hot bulb or plate. The addition of heat at constant volume along line 3–4 is assumed, with rise in temperature and pressure. The heat added is:

$$Q_1 = c_v(T_4 - T_3) \tag{8}$$

in which T is absolute temperature in degrees F., and c_v the specific heat at constant volume.

4. Adiabatic expansion occurs during the working stroke along line 4–5, both valves remaining closed as in Fig. 29–C, Chap. V.

5. In practical engines the exhaust valve opens at or near the end of the expansion stroke, and the drop in pressure along line 5–2 is accompanied by expansion, the temperature T_2 being indefinite. In the theoretical cycle it is assumed that the valves remain closed and that heat is withdrawn along line 5–2, the resulting pressure and temperature being the same as at the end of the suction stroke. This completes the thermal cycle. The heat rejected during the step is:

$$Q_2 = c_v(T_5 - T_2) \tag{9}$$

6. The exhaust stroke 2–1 is one of the two preparatory steps for the next thermal cycle, removing the burnt gas. The exhaust valve is open during this stroke as in Fig. 29–D, Chap. V, and the pressure in actual engines is slightly above atmospheric. The loss is charged to engine friction as with the suction stroke, and the pressure difference is neglected for the theoretical cycle. This completes the practical cycle.

The efficiency of the constant-volume cycle is:

$$e = \frac{Q_1 - Q_2}{Q_1} = \frac{c_v(T_4 - T_3) - c_v(T_5 - T_2)}{c_v(T_4 - T_3)} = 1 - \frac{T_5 - T_2}{T_4 - T_3} \tag{10}$$

Taking exponent $n = k$ for adiabatic changes (in which $k = c_p/c_v$, c_p being specific heat at constant pressure):

$$\frac{T_5}{T_4} = \left(\frac{v_1}{v_2}\right)^{k-1} \text{and} \; \frac{T_2}{T_3} = \left(\frac{v_1}{v_2}\right)^{k-1}$$

or,

$$\frac{T_5}{T_4} = \frac{T_2}{T_3} = \frac{T_5 - T_2}{T_4 - T_3}.$$

Then letting $r = v_2/v_1$ and remembering that:

$$\frac{v_2}{v_1} = \left(\frac{p_3}{p_2}\right)^{\frac{1}{k}}$$

(10) may be written:

$$e = 1 - \frac{T_2}{T_3} = 1 - \frac{1}{r^{k-1}} \tag{11}$$

It is apparent from (11) that efficiency is improved by increasing the compression pressure, which is true in practice with certain limitations (see Chap. IX).

The expansion and compression lines of actual indicator diagrams are not adiabatic, but are approximated by the equation:

$$pv^n = \text{constant};$$

where n is some value usually less than k. However, with any value other than $k\ (=c_p/c_v)$, Equation (11) does not apply; because the values of n which are found approximately from actual diagrams are greater or less than k, indicating that heat is added or subtracted as the working substance changes volume. These quantities of heat must be added to or subtracted from Q_1 and Q_2 in (10). This results in a clumsy formula with assumed values of n at best, and it is better to use (11) for comparison with actual efficiencies. Practical values of n must be used in determining the clearance volume of actual engine cylinders.

Fig. 47.

Constant-volume heat transfer involves instantaneous combustion and exhaust; this is impossible in practice, causing the explosion line to lean. This, together with the fact that the valves do not open and close exactly at the ends of the stroke, causes rounded corners on the diagram.

The theoretical maximum temperature is not permissible due to the burning and warping of the metal, and the impossibility of lubrication, so approximately 30 per cent. of the heat of combustion must be removed by the circulation of water around the cylinder. This lowers the pressure and reduces the area of the diagram. An actual indicator diagram from an engine operating on the 4-stroke Otto cycle is shown in Fig. 47.

A critical examination of the Otto cycle is given in Güldner's Internal-combustion Engines.

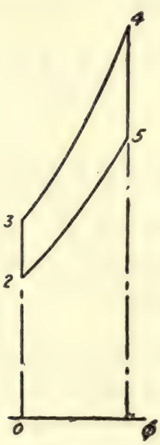


Fig. 48.

The Tϕ (temperature-entropy) diagram for the Otto cycle is shown in Fig. 48, the numerals corresponding to the same points on the pv diagram. The reception of heat at constant volume is along the line 3–4 and adiabatic expansion along 4–5. Rejection of heat at constant volume is along line 5–2 and adiabatic compression along 2–3.

From general thermodynamics, line 3–4 is drawn by means of the equation:

$$\Delta\phi = c_v \log_e \frac{T_X}{T_3}$$

and line 2–5 from:

$$\Delta\phi = c_v \log_e \frac{T_X}{T_2}$$

where T_X is any temperature above T_3 or T_2.

26. The Constant-pressure Cycle.—The Diesel cycle is the best known and most widely used of this class. The theoretical difference between this cycle and the Otto cycle is the combustion of fuel at constant pressure instead of at constant volume. Diesel's original purpose was to approach the Carnot cycle by applying isothermal combustion, but practical results with the later engines employing constant-pressure combustion show improvement over the former method, and the cycle is now generally known as the constant-pressure cycle.

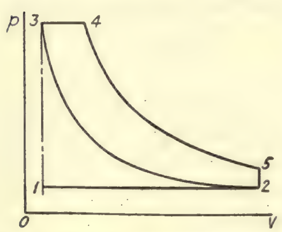


FIG. 49.—Constant-pressure cycle.

The theoretical indicator diagram for the Diesel 4-stroke cycle is shown in Fig. 49. The suction and exhaust strokes, like those of the Otto cycle, are purely mechanical, and the operation of the inlet and exhaust valves are the same as for the Otto cycle.

Some form of liquid fuel, like crude petroleum, has thus far been the only fuel used in the Diesel engine.

The six steps of the cycle are:

1. The suction stroke, line 1–2, during which fresh air only is drawn into the cylinder.

2. The compression stroke, along line 2–3, compresses the air adiabatically to a pressure of about 500 lb. per sq. in. gage, and a temperature high enough to positively ignite the fuel, although none has been supplied up to this time.

3. The fuel valve is now opened (slightly before the end of the stroke), and as the piston starts upon its stroke, oil is injected in the form of spray at a rate which allows the pressure to remain practically constant as the fuel burns along line 3–4. Ignition is effected by the heat of compression and no electric spark is required. The amount of fuel injected is controlled by the governor and depends upon the load on the engine.

The air used for fuel injection is furnished at a pressure from 50 to 100 per cent. in excess of the engine compression pressure, by either an independent compressor or by a compressor connected to the engine.

Sometimes the fuel is pumped directly into the cylinder without the use of compressed air.

The heat received is:

$$Q_1 = c_p(T_4 - T_3) \tag{12}$$

4. The remainder of the working stroke is occupied with adiabatic expansion along line 4–5.

5. The rejection of heat is assumed at constant volume along line 5–2, completing the thermal cycle.

The heat rejected is:

$$Q_2 = c_v(T_5 - T_2) \tag{13}$$

6. The exhaust stroke removes the burned charge from the cylinder, completing the practical cycle.

The efficiency of the Diesel cycle is:

$$e = \frac{Q_1 - Q_2}{Q_1} = \frac{c_p(T_4 - T_3) - c_v(T_5 - T_2)}{c_p(T_4 - T_3)} = 1 - \frac{c_v(T_5 - T_2)}{c_p(T_4 - T_3)} = 1 - \frac{T_5 - T_2}{k(T_4 - T_3)} \tag{14}$$

This may be written:

$$e = 1 - \frac{T_2\left(\frac{T_5}{T_2} - 1\right)}{kT_3\left(\frac{T_4}{T_3} - 1\right)} = 1 - \frac{\frac{T_5}{T_2} - 1}{k\frac{T_3}{T_2}\left(\frac{T_4}{T_3} - 1\right)} \tag{15}$$

But:

$$\frac{T_5}{T_4} = \left(\frac{v_4}{v_2}\right)^{k-1} \text{and} \frac{T_3}{T_2} = \left(\frac{v_2}{v_1}\right)^{k-1}$$

And from Charles' law,

$$\frac{T_4}{T_3} = \frac{v_4}{v_1}$$

Then:

$$\frac{T_5}{T_2} = \frac{T_5}{T_4} \cdot \frac{T_4}{T_3} \cdot \frac{T_3}{T_2} = \left(\frac{v_4}{v_2}\right)^{k-1}\left(\frac{v_4}{v_1}\right)\left(\frac{v_2}{v_1}\right)^{k-1} = \left(\frac{v_4}{v_1}\right)^k$$

Substituting in (15) and letting $v_2/v_1 = r$, as for the constant-volume cycle, and $v_4/v_1 = \epsilon$, (15) becomes:

$$e = 1 - \frac{\epsilon^k - 1}{kr^{k-1}(\epsilon - 1)} \tag{16}$$

This may be written:

$$e = 1 - \frac{1}{r^{k-1}}\left[\frac{\epsilon^k - 1}{k(\epsilon - 1)}\right] \tag{17}$$

If the quantity in brackets is omitted (17) is the same as (11), and as this quantity is always greater than unity it is obvious that for the

same value of r, the efficiency of the constant-volume cycle is greater than that of the Diesel. The greater value of r in the Diesel cycle, allowable because of the absence of fuel during compression, accounts for the superior economy usually attained by the Diesel engine.

An actual indicator diagram from a Diesel engine is shown in Fig. 50.

Theoretical and actual efficiencies may be obtained for both the constant-volume and constant-pressure cycles in terms of heat quantities based upon fuel supplied, in which case:

$$e = \frac{2545}{\text{B.t.u. per horsepower per hr.}} \tag{18}$$

The $T\phi$ diagram for the Diesel cycle is shown in Fig. 51. It is similar to Fig. 48 except that the reception of heat is at constant pressure along line 3–4, the equation being:

$$\Delta\phi = c_p \log_E \frac{T_X}{T_3}.$$

Fig. 50.

The rejection of heat along line 5–2 is at constant volume.

As T_2/T_3 is not equal to T_5/T_4, the relation of the efficiency formula to the diagram is not apparent, as it is with the constant-volume cycle.

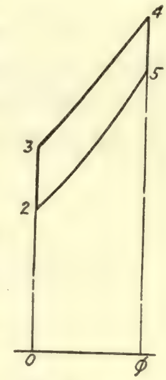


Fig. 51.

27. Volumetric Efficiency.—The volumetric efficiency of an internal-combustion engine is the ratio of the volume of the charge (fresh air and fuel) v_c at some standard pressure and temperature, to the volume of stroke v_s. This is equivalent to the ratio of the actual charge weight w_c to the weight, at some standard pressure and temperature, of a volume of the same substance equal to v_s, the volume of stroke; let this weight be w_s. The weight w_c, at actual pressure and temperature will occupy the volume qv_s, where q is a factor depending upon the relative volume of fresh charge to total cylinder contents. In theoretical computations it is usually assumed that q is unity. With poor valve design and setting q may be less than unity, and with exceptionally good valve design and setting it may be greater. With complete scavenging, when all the burnt gas is expelled from the cylinder:

$$q = \frac{v_2}{v_2 - v_1} = \frac{r}{r - 1}.$$

The general equation for gas is:

$$pv = wRT$$

in which R is a constant (= 53.35 for air).

Letting the subscript 2 refer to actual cylinder conditions, and absence of subscripts to a standard condition, we may then write:

$$w_c = \frac{p_2\, qv_S}{RT_2} \text{ and } w_S = \frac{pv_S}{RT}.$$

Then:

$$e_v = \frac{v_c}{v_S} = \frac{w_c}{w_S} = q\frac{p_2T}{pT_2} \tag{19}$$

The factor q is difficult of practical determination but is of considerable importance in connection with economy and capacity. Aside from affecting the volume of charge it affects the density by its influence upon T_2, and any measure which may be taken to increase it by proper valve setting will increase both economy and capacity.

28. Temperature rise due to combustion must be known in order to determine theoretical efficiencies of internal-combustion-engine cycles.

Let h = the heating value per cu. ft. of gaseous fuel or per lb. of liquid fuel. For gas this must be at some standard pressure and temperature.

a = cu. ft. of air supplied per cu. ft. of gaseous fuel or per lb. of liquid fuel at standard pressure and temperature.

σ = the volume of fuel for which a is supplied.

c = specific heat in general—c_v or c_p.

Referring to Fig. 46, Par. 25, and Fig. 49, Par. 26, letting w_2 be the weight of total cylinder contents in lb., the B.t.u. necessary to raise the temperature from T_3 to T_4 is:

$$w_2c(T_4 - T_3).$$

The heat supplied per cu. ft. of mixture at standard pressure and temperature is:

$$\frac{h}{a + \sigma}$$

The volume occupied by the charge at standard pressure and temperature is:

$$e_vv_s$$

where e_v is found by the method of the last paragraph.

Then the total heat supplied per cycle is:

$$Q_1 = e_v\, v_s\, \frac{h}{a + \sigma} \tag{20}$$

Then:

$$w_2c(T_4 - T_3) = e_v\, v_s \frac{h}{a + \sigma}.$$

But,

$$w_2 = \frac{p_2v_2}{RT_2} \text{ and } v_s = v_2 - v_1.$$

Substituting these in (20), the equation may be written:

$$T_4 - T_3 = e_v \frac{h}{a + \sigma} \cdot \frac{RT_2}{cp_2} \cdot \frac{r - 1}{r} \tag{21}$$

$T_4 - T_3$ may be written:

$$T_3 \left[\frac{T_4}{T_3} - 1\right].$$

Substituting in (21) gives:

$$\frac{T_4}{T_3} = 1 + e_v \frac{h}{a + \sigma} \cdot \frac{RT_2}{cp_2T_3} \cdot \frac{r - 1}{r}. \tag{22}$$

From thermodynamics:

$$T_3 = T_2r^{k-1},\ r = \left(\frac{p_3}{p_2}\right)^{\frac{1}{k}} \text{ and } r^k = \frac{p_3}{p_2}.$$

Substituting in (22) gives:

$$\frac{T_4}{T_3} = 1 + e_v \frac{h}{a + \sigma} \cdot \frac{R}{cp_2} \cdot \frac{r - 1}{r^k}$$

$$= 1 - e_v \frac{h}{a + \sigma} \cdot \frac{R}{cp_3} (r - 1) \tag{23}$$

In theoretical cycles it is common to assume the pressure and temperature the same as the standard at which a and h are measured, and that the volume of charge equals the volume of stroke; but as this is in no wise a necessary assumption, e_v will be retained in the equations. Its value is given in (19). The value of a for theoretical cycles is usually, but not necessarily, assumed as just sufficient to support perfect combustion.

The quantity σ is obviously unity for gaseous fuel. For oil engines it is the volume of 1 lb. of oil vapor, and as this is about 3 per cent. of the volume of air supplied, it is usually ignored.

Standard pressure is taken as 14.7 lb. per sq. in. absolute. Standard temperature is usually 32 degrees F., but sometimes 62 degrees. The A.S.M.E. has adopted 60 degrees F.

Taking numerical values and making substitution, assuming the mixture to have the same characteristics as air, special equations may be written.

For the constant-volume cycle using gas, $c = c_v$, and:

$$\frac{p_4}{p_3} = \frac{T_4}{T_3} = 1 + \frac{316}{p_3} \cdot e_v \cdot \frac{h}{a + 1} (r - 1) \tag{24}$$

For constant-volume cycle for oil:

$$\frac{p_4}{p_3} = \frac{T_4}{T_3} = 1 + \frac{316}{p_3} \cdot e_v \cdot \frac{h}{a}(r - 1) \tag{25}$$

For the constant-pressure cycle (Diesel):

$$\epsilon = \frac{v_4}{v_1} = 1 + \frac{225}{p_3} \cdot e_v \cdot \frac{h}{a}(r - 1) \tag{26}$$

For values of h and a see Par. 75, Chap. XIV.

29. The mean effective pressure of any heat engine is given by Equation (3), Par. 23, and is:

$$P_M = 5.4\frac{eQ_1}{v_s} \tag{27}$$

where Q_1 is the heat furnished per cycle and e the thermal efficiency.

Substituting the value of Q_1 from (20) in (27) gives:

$$P_M = 5.4ee_v \cdot \frac{h}{a + \sigma} \tag{28}$$

which is the m.e.p. in lb. per sq. in. for any internal-combustion engine.

The quantities a and σ may be as in the preceding paragraph.

Then for all *gas engines*, h is heating value per cu. ft., and:

$$P_M = 5.4ee_v \cdot \frac{h}{a + 1} \tag{29}$$

For all *oil engines*, h is heating value per lb., and:

$$P_M = 5.4ee_v \cdot \frac{h}{a} \tag{30}$$

For theoretical cycles e and a may be theoretical values; or if actual practical values are used, P_M will be the actual m.e.p. For values of h and a see Par. 75, Chap. XIV.

30. Conventional indicator diagrams, drawn to scale to give a certain m.e.p. are sometimes convenient in the absence of actual diagrams, for calculations concerning strength or crank effort. The m.e.p. may be determined from the preceding paragraph, using practical values as nearly as possible. Then a value of n may be chosen for the exponent of the curve equation which will give values approximating actual expansion and compression curves. The fundamental equations used relate to pressure and volume only.

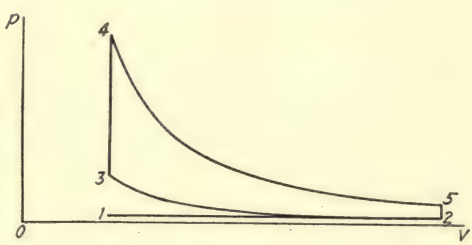


FIG. 52.

Constant-volume Cycle.—As in the previous discussion, all pressures except P_M are in lb. per sq. ft., and subscripts correspond to points on Fig. 52.

$$r = \frac{v_2}{v_1} = \left(\frac{p_3}{p_2}\right)^{\frac{1}{n}} \tag{31}$$

$$r^{n-1} = \left(\frac{p_3}{p_2}\right)^{\frac{n-1}{n}} \tag{32}$$

$$\text{clearance} = \frac{v_1}{v_2 - v_1} = \frac{1}{r-1} \tag{33}$$

The work per cycle in ft. lb. is:

$$\begin{aligned} W &= \frac{p_4 v_1}{n-1}\left(1 - \frac{1}{r^{n-1}}\right) - \frac{p_3 v_1}{n-1}\left(1 - \frac{1}{r^{n-1}}\right) \\ &= (p_4 - p_3) \cdot \frac{v_1}{n-1}\left(1 - \frac{1}{r^{n-1}}\right) \end{aligned} \tag{34}$$

$$144 P_M = \frac{W}{v_2 - v_1} = \frac{(p_4 - p_3)\left(1 - \frac{1}{r^{n-1}}\right)}{(n-1)(r-1)} \tag{35}$$

If P_M is known the pressure rise in lb. per sq. in. is:

$$\frac{p_4 - p_3}{144} = \frac{P_M(n-1)(r-1)}{1 - \frac{1}{r^{n-1}}} \tag{36}$$

FIG. 53.

Constant-pressure Cycle (*Diesel*).—The values of r and r^{n-1} and the clearance are given by (31), (32) and (33) as for the constant-volume cycle.

A convenient ratio is:

$$\epsilon = \frac{v_4}{v_1} \tag{37}$$

$$\text{Ratio of expansion} = \frac{v_2}{v_4} = \frac{r}{\epsilon} \tag{38}$$

$$\text{Cut-off} = \frac{v_4 - v_1}{v_2 - v_1} = \frac{\epsilon - 1}{r - 1} \tag{39}$$

The work per cycle in ft. lb. is:

$$W = p_3(v_4 - v_1) + \frac{p_3 v_4}{n-1}\left[1 - \left(\frac{v_4}{v_2}\right)^{n-1}\right] - \frac{p_3 v_1}{n-1}\left(1 - \frac{1}{r^{n-1}}\right) \tag{40}$$

$$144 P_M = \frac{W}{v_2 - v_1} = \frac{p_3}{r-1}\left[\epsilon - 1 + \frac{\epsilon - 1 - \frac{\epsilon^{n-1}}{r^{n-1}}}{n-1}\right] \tag{41}$$

If P_M is known, ϵ may be found by trial and error.

CHAPTER VII

STEAM

A knowledge of the fundamentals of thermodynamics is desirable, but not essential for a practical understanding of the formulas expressing the properties of steam. For equations involving entropy, such knowledge is necessary and is presupposed. Introductory to Chap. XV, a brief review of a few of the principal equations will be given in this chapter. It will also give a review of these principles for general use.

31. Formation of Steam under Constant Pressure.—When steam is "raised" in a boiler which is initially partly filled with comparatively cold water, heat is added, and after a certain temperature is reached the water boils and steam is given off. The pressure rises gradually until the desired pressure is reached. If steam is now drawn from the boiler for the operation of a steam engine, more steam is formed and the pressure is maintained as nearly constant as possible. Then we may say that all the steam which is formed after the engine begins to work is formed under constant pressure; that is, every pound of water which enters the boiler at a certain temperature is heated, and converted into steam at a constant pressure.

Steam is formed in this way for most of its practical applications, and the heat quantities in the steam tables are based on this assumption. The study of the properties of steam is also simplified by assuming its formation at constant pressure.

Let a vertical cylinder contain 1 lb. of pure water upon which rests a frictionless piston, exerting a constant pressure p upon the water. With the water in the cylinder at initial temperature t_0, assuming no leakage of water, steam or heat, the three stages of steam formation are as follows:

1. Heat is added and the temperature gradually rises from t_0 to some temperature t at which steam begins to form. This temperature depends upon the pressure exerted by the piston, and for every value of p there is a certain value of t, at which, if more heat be added, water will be converted into steam.

2. Heat is still added, and the water at the temperature corresponding to the pressure p, begins to form into steam at the same temperature and

under the constant pressure p. The movement of the piston due to the expansion of the water as it is heated from t_0 to t during stage (1) is so slight as to be negligible; but now the piston rises noticeably due to increase of volume as steam is formed, and continues to rise until all the water is converted into steam. The steam during this stage is *saturated steam*, being in contact with, and at the same temperature of the water from which it is formed. At the end of the stage, when all the water is converted into steam it is called *dry saturated steam* and its volume is denoted by s.

3. As more heat is added to this dry saturated steam the temperature rises to some temperature t_s and there is an increase of volume if the pressure remains constant. The steam is now *superheated.* Each degree rise in temperature above the temperature of saturation is called a degree of superheat.

Had the volume remained constant as the heat was added, the pressure would have risen. This is called superheating at constant volume; but, as with the formation of saturated steam, constant pressure has the most practical application, especially in power plant operation, and is assumed in this book.

When saturated, the characteristics of steam differ considerably from those of a gas, but approach them more nearly as superheating is increased.

Superheated steam at a given pressure may have any practical temperature higher than that due to saturation at that pressure.

32. The relation of pressure and temperature in saturated steam has been determined by direct experiment, the results being given in steam tables. A pressure-temperature curve shows that the pressure rises with the temperature at a rate which increases rapidly with increase of temperature.

For water, as is the case when feedwater is being pumped into a boiler, the pressure may be greater than that corresponding to its temperature in the steam tables, but never less. Should it be made less, some of the heat in the water would be given up to evaporate a portion of the water, evaporation continuing until the temperature was reduced to correspond to the pressure. *The heat of the liquid always corresponds to the temperature and is dependent upon it, regardless of the pressure.*

33. Supply of Heat in the Formation of Steam at Constant Pressure.—The arbitrary zero of the steam tables is at 32 degrees F., and all heat quantities are measured from this zero. The heat required to raise the temperature of the water from 32 degrees F. to that of the boiling temperature t is called the *heat of the liquid*, and is expressed by:

$$h = c(t - 32) \tag{1}$$

in which c is the specific heat, which is nearly unity at ordinary temperatures.

Most saturated steam is not entirely dry, the fraction x of a given weight being steam, the remainder water.

The heat taken in as water is changed to steam at constant pressure is called the *latent heat*, and is denoted by L. The *total heat* of *dry saturated steam* is given by the formula:

$$H = h + L \tag{2}$$

Let C denote the heat content of steam in any condition from water to superheated steam. Then *wet steam* is expressed by:

$$C = h + xL \tag{3}$$

The heat content of *superheated steam* is:

$$\begin{aligned} C &= h + L + c_P(t_S - t) \\ &= H + c_P(t_S - t) \end{aligned} \tag{4}$$

in which c_P is the specific heat at constant pressure.

34. Adiabatic expansion is never realized in practice, but is approached in the flow of steam through nozzles. In such expansion from a pressure indicated by subscript 1 to a lower pressure denoted by 2, this would give:

$$\phi_1 = \phi_2 \tag{5}$$

in which ϕ is the entropy of the steam. The entropy of wet steam is:

$$\phi = \theta + \frac{xL}{T} \tag{6}$$

where T is the absolute temperature in degrees F. and θ is the entropy of water above 32 degrees F. This may be found in the steam tables, as may also the entropy of evaporation, $\frac{L}{T}\cdot$ If x is known for one pressure, it may be found for the other by combining (5) and (6).

The entropy of superheated steam is:

$$\phi = \theta + \frac{L}{T} + c_P \log_E \frac{T_S}{T} \tag{7}$$

in which c_P is the mean specific heat between T_S, the temperature of the steam, and T, the absolute temperature due to the pressure.

35. The Rankine Cycle.—If C_1 and C_2 are heat contents at the same entropy, it may be shown that for any initial and final condition of the steam the efficiency of the Rankine cycle with complete expansion is given by:

$$e = \frac{C_1 - C_2}{C_1 - h_2} \tag{8}$$

The mean effective pressure for the cycle in lb. per sq. in. is:

$$P_M = 5.4\frac{C_1 - C_2}{v_C - \sigma} \tag{9}$$

where v_C is the volume after expansion, in cu. ft., of 1 lb. of steam, and σ is the volume of 1 lb. of water (= 0.017, nearly).

For the Rankine cycle with incomplete expansion:

$$e = \frac{C_1 - C_C + \dfrac{(P_C - P_2)(v_C - \sigma)}{5.4}}{C_1 - h_2} \tag{10}$$

where C_C is the heat content at the same entropy at terminal pressure, P_2 the back pressure in lb. per sq. in.

The mean effective pressure is:

$$P_M = 5.4\frac{C_1 - C_C}{v_C - \sigma} + P_C - P_2 \tag{11}$$

36. Flow of Steam.—It may be shown that the frictionless flow of steam in ft. per sec., when the initial velocity is zero, is expressed by:

$$V = \sqrt{\frac{2g}{A}}\sqrt{C_1 - C_2} \tag{12}$$

If p_2 is less than about $0.58p_1$, it is found by experiment that the jet will not retain its form upon leaving the nozzle, and a diverging nozzle must be used.

37. Effect of Friction on Steam Flow.—If the fraction y of the heat drop from initial pressure p_1 to throat or exit pressure (either of which may be taken as p_2) is required to overcome the surface friction of the nozzle, the velocity will be:

$$V_N = \sqrt{\frac{2g}{A}}\sqrt{(1 - y)(C_1 - C_2)} \tag{13}$$

The heat quantity $y(C_1 - C_2)$ is returned to the steam, increasing the heat content, dryness factor and entropy at pressure p_2. If x_N is the new dryness factor (x_2 being that due to adiabatic expansion to p_2),

$$h_2 + x_N L_2 = h_2 + x_2 L_2 + y(C_1 - C_2)$$

or,

$$x_N = x_2 + \frac{y(C_1 - C_2)}{L_2} \tag{14}$$

and,

$$\phi_2 = \theta_2 + \frac{x_N L_2}{T_2} \tag{15}$$

The specific volume is:

$$v_2 = \sigma + x_N u_2 \tag{16}$$

Where u is the change of volume from water to steam.

The area of opening required in sq. in.

$$144a = \frac{144w(\sigma + x_N u_2)}{V_N} \tag{17}$$

With high initial superheat, steam at the throat of a nozzle may still retain superheat. Then (14) would be replaced by:

$$H_2 + c_P(T_N - T_2) = H_2 + c_P(T_S - T_2) + y(C_1 - C_2)$$

From which:

$$T_N = T_S + \frac{y(C_1 - C_2)}{c_P} \tag{18}$$

Then:

$$\phi = \theta + \frac{L_2}{T_2} + c_P \log_E \frac{T_N}{T_2} \tag{19}$$

T_N is the resulting absolute temperature, T_S the temperature due to adiabatic expansion, and, due to the small difference, c_P has been assumed to be the same on both sides of the original equation. Such problems are easily solved with entropy table or chart.

A barely possible case would be the change from slightly wet steam to superheated steam; then:

$$h_2 + L_2 + c_P(T_N - T_2) = h_2 + x_2 L_2 + y(C_1 - C_2)$$

and

$$T_N = T_2 + \frac{y(C_1 - C_2) - L_2(1 - x_2)}{c_P} \tag{20}$$

The entropy is given by (19).

The energy lost by friction and returned to the steam as heat may also be expressed in terms of velocity; letting $q = V_N/V$, the ratio of actual to theoretical velocity:

$$\text{Heat returned} = \frac{A}{2g}(V^2 - V_N{}^2) = \frac{V^2(1 - q^2)}{50{,}000} \tag{21}$$

This may replace $y(C_1 - C_2)$ in (14), (18), and (20), making general equations for any passages when velocity decrease due to friction is known or assumed.

CHAPTER VIII

ECONOMY

Notation.

P = pressure in lb. per sq. in.

v = volume of steam in cu. ft. per lb.

σ = volume of water in cu. ft. per lb.

C = heat content in B.t.u. per lb. above 32 degrees F., of steam of any condition.

h = heat in the water in B.t.u. per lb. above 32 degrees F.

w = actual steam consumption per horsepower per hour, usually, but not necessarily in terms of i.h.p.

w_R = theoretical steam consumption for the Rankine cycle per horsepower-hour.

e = actual thermal efficiency.

e_B = thermal efficiency at brake, or economic efficiency = $e_M e$.

e_M = mechanical efficiency (see Chap. X).

F = heat factor, or ratio of actual to theoretical efficiency.

38. The Steam Plant.—Formulas generally used for expressing the thermal efficiency of the steam engine or turbine are:

$$e = \frac{42.42}{\text{B.t.u. per h.p. per min.}} \tag{1}$$

$$e = \frac{2545}{\text{B.t.u. per h.p. per hr.}} \tag{2}$$

Both forms are used by different writers, the second being that given in the Rules for Conducting Steam Engine Tests by the A.S.M.E., with the exception that the constant given is 2546.5. As the slide rule is usually accurate enough for such work the simpler and more usual constant will be used in this book.

Though (1) and (2) are simple expressions, they permit of several interpretations. If the object of a test is to determine the economy of the complete power plant, the heat supplied to all the auxiliaries must be added to that supplied to the engine or turbine. The total heat supplied for power purposes is the difference between the heat content of all steam used by engines and auxiliaries, the pressure and quality being measured near their respective throttle valves; and the heat content of all feed

water measured near the boilers, provided that steam for heating or industrial processes is not taken from the same boilers. In this case, equivalent water at raw water temperature should be deducted from the total feed water.

The determinat on of heat quantities and the apparatus employed are fully discussed in treatises on mechanical laboratory methods and in Rules for Conducting Steam Engine Tests, Report of Power Test Committee, *Trans.* A.S.M.E., vol. 37. When all heat quantities are obtained, the efficiency may be obtained by (2).

39. The Prime Mover.—For the engine or turbine designer the economy of the engine, independent of its auxiliaries, is perhaps of the most importance. The steam consumed by the engine alone is then considered, and as the condensed exhaust steam is available for feed-water whether so applied or not, it is assumed that the temperature of the feed-water before heating is that corresponding to the back pressure. Then the heat delivered to the engine per lb. of steam is:

$$C_1 - h_2$$

and if w is the steam consumption (also called water rate) in lb. per horsepower-hour, the B.t.u. per horsepower-hour for the engine will be:

$$w(C_1 - h_2).$$

Should there be steam jackets or a reheating receiver, the heat supplied to these should be added, as they directly affect the cylinder efficiency. As the condensed steam from jacket and receiver is available for feed-water at the pressure of supply, which is usually that of the initial cylinder pressure, $h_2 = h_1$. If w_1 is the weight of steam supplied to jacket or receiver, or both, the heat will be:

$$w_1(C_1 - h_1).$$

Then the total B.t.u. per horsepower-hour will be:

$$w(C_1 - h_2) + w_1(C_1 - h_1).$$

The efficiency will be:

$$e = \frac{2545}{w(C_1 - h_2) + w_1(C_1 - h_1)} \tag{3}$$

As steam jackets are not very commonly used, and their use is decreasing with the use of superheated steam, they may ordinarily be neglected. Then (3) becomes:

$$e = \frac{2545}{w(C_1 - h_2)} \tag{4}$$

The water rate of Rankine's cycle for complete expansion is:

$$w_r = \frac{2545}{C_1 - C_2} \tag{5}$$

And for incomplete expansion:

$$w_r = \frac{2545}{C_1 - C_c + \dfrac{(P_c - P_2)(v_c - \sigma)}{5.4}} \tag{6}$$

The subscript c refers to terminal pressure.

Formula (5) is useful in steam-turbine design.

40. I.H.P. or B.H.P.—For an intelligent conception of thermal efficiency and economy the term horsepower must be defined. This is commonly taken as i.h.p. or as b.h.p. Then (2) may be written:

$$e = \frac{2545}{\text{B.t.u. per i.h.p.-hr.}} \tag{7}$$

or:

$$e = \frac{2545}{\text{B.t.u. per b.h.p.-hr.}} \tag{8}$$

Formula (7) is commonly applied to steam engines, but (8), sometimes called the thermal efficiency at brake, or economic efficiency, is a more correct measure of economical performance for any heat engine. For engines of the same type, when the mechanical efficiency is about the same for the usual or rated load, (7) gives a fair measure for comparison, but for widely varying types, or where the ratio of maximum to mean pressure differs greatly, the only true measure of economy must include the friction of the engine.

There is no indicated horsepower for the steam turbine, although by assuming mechanical efficiency the so-called *turbine horsepower* is sometimes calculated.

For any type of prime mover driving electrical machinery the term electrical horsepower (e.h.p. = 1.34 × kilowatts) is used. This allows for the combined friction of engine and generator. Economy may then be expressed in B.t.u. per e.h.p.-hr.

The most common method of expressing economy of steam engines is in lb. of water per i.h.p.-, or per kw.-hr. This may be used for the comparison of simple noncondensing engines using saturated steam, but is generally unsatisfactory and is being replaced by the heat-unit method. To show the discrepancy of basing economy upon water rate alone, assume engine A working with 140 lb. absolute pressure and exhausting to atmosphere with a steam consumption of 20 lb. per i.h.p.-hr.; while engine B, with a steam pressure of 165 lb. absolute and 2 lb. absolute back pressure uses 12 lb. Assuming initially dry saturated steam in both cases, the B.t.u. per h.p.-hr. for A is:

$$20 \times (1188 - 180) = 20{,}160$$

and for B:

$$12 \times (1193 - 94) = 13{,}188.$$

The ratio of steam consumption of A to that of B is:

$$\frac{20}{12} = 1.66$$

and of heat consumption:

$$\frac{20{,}160}{13{,}188} = 1.53$$

which is nearly 14 per cent. less.

41. The Heat Factor.—It is obvious from the $T\phi$ diagram, that if the thermal efficiency of the Carnot cycle were to equal unity the temperature of exhaust would be zero absolute. There are natural limits which make this physically impossible, and even with the best of our heat engines cause the efficiency to appear pitiably small. As the greater part of this seeming deficiency is not attributable to faulty design or construction, it is fair to take as a standard of comparison a cycle which is as near perfection as these natural limits will allow. This standard is usually the Rankine cycle with complete expansion. The discarding of adiabatic compression is perhaps not necessitated by natural physical conditions, but there is sufficient practical difficulty attending its attempted application to warrant its rejection.

The Rankine cycle with incomplete expansion is sometimes taken as a standard when the terminal pressure of the actual engine is known. This seems like a reasonable practice when it is remembered that both capacity and economy demand a certain amount of terminal drop; moreover, it is the true measure of cylinder efficiency.

Denoting the efficiency of the Rankine cycle by e_R, the ratio of efficiencies, variously known as the Rankine cycle ratio, efficiency ratio, and heat factor (the last term being used in this book), is:

$$F = \frac{e}{e_R} = \frac{w_R}{w} \tag{9}$$

In applying (9) it is customary to take e as the indicated thermal efficiency, and in transferring an indicator diagram to the $T\phi$ diagram (see Berry's Temperature-entropy Diagrams) it is necessarily so assumed, but there is no reason in practical work why e should not be based upon net work performed, or b.h.p., for steam engines as well as for internal-combustion engines. Practical values of F range from 0.5 to 0.75.

Cycle design consists in selecting values of initial pressure, superheat, vacuum and ratio of expansion which will give the highest cycle efficiency while remaining within commercially practical limits. In

designing the engine, proper values of piston speed, rotative speed and cylinder dimensions must be selected and all details so carefully proportioned that both the indicated and brake efficiencies of the engine shall approach as nearly as possible the efficiency of the theoretical cycle. A high cycle efficiency combined with a high heat factor insures an economical engine, as the product of the two equals the thermal efficiency, which after all, though uncomplimentary, is the absolute measure of economy when based upon the brake horsepower. This statement will no doubt be challenged, as it disregards interest on investment, depreciation and other fixed charges. Exception must of course be made for temporary installations, and when the fuel is all refuse, as from wood-working machinery, there being often a surplus to dispose of. In this case the engine should be of the simplest type. However, in general, true economy must always tend toward the conservation of those natural resources which are in greatest danger of depletion; from this viewpoint, thermal efficiency will be the criterion of economy.

In the efficiency of the Rankine cycle the feed-water temperature is assumed as that due to exhaust pressure. The cycle for a noncondensing plant in which the feed-water temperature is lower than that of the exhaust steam, should only be applied as the ideal cycle when the boiler plant is included, as the temperature of the feed water has no influence on the economy of the engine. Then, neglecting the steam jacket or reheating receiver, the heat factor is:

For complete expansion:

$$F = \frac{e}{e_R} = \frac{2545}{w(C_1 - C_2)} \tag{10}$$

For incomplete expansion:

$$F = \frac{e}{e_R} = \frac{2545}{w\left[C_1 - C_2 + \frac{(P_C - P_2)(v_C - \sigma)}{5.4}\right]} \tag{11}$$

The heat factor for complete expansion is especially useful in steam-turbine design, the steam consumption being reduced to turbine horsepower.

From what has preceded it is clear that in reporting economy or efficiency it must be plainly stated whether or not the auxiliaries are included, and whether it is based upon i.h.p. or b.h.p. In comparing engines operating under different conditions, as condensing with noncondensing, or those using superheat with those using saturated steam, the heat consumption of auxiliaries required for any given mode of operation should properly be included. This may be found in the same manner as for the main engine.

Should a portion of the exhaust or receiver steam be utilized for heating or industrial purposes, its total heat content may be deducted from that charged to engine and auxiliaries on the ground that this steam must otherwise have been furnished by the boiler. Should all of the exhaust steam be so utilized the thermal efficiency would then equal the heat factor.

On the other hand it may be argued that the manufacturing plant is utilizing waste heat from the engine. At any rate, such an arrangement fosters economy and is often a deciding factor in favor of the steam engine.

Should all of the exhaust steam be required, a simple type of engine with small ratio of expansion could be employed. This would deliver a greater weight of dryer steam and possess numerous practical advantages if large overload capacity were not required. The heat factor based upon the Rankine cycle with incomplete expansion would probably be greater, and this is the measure of economy for an engine working under these conditions. Cylinder condensation and radiation losses should be reduced to a minimum, and the maximum amount of heat should pass out with the exhaust. Such an engine would be wasteful only upon condition that the exhaust steam were wasted. In some cases even the use of superheated steam would be well-advised, should this provide the maximum heat content with a minimum of fuel consumption.

Theoretically, the heat consumption of auxiliaries is an insignificant fraction of that required by the main engines, but may practically exert considerable influence upon plant economy. The use of uneconomical auxiliaries is never justified except in refuse-burning plants, even though all of their exhaust steam is required for heating feed water.

With the expection of that which refers specifically to the steam engine, the preceding paragraphs of this chapter apply as well to the steam turbine, remembering that expansion should always be complete in the turbine.

42. Internal Combustion Engines.—A common method of expressing the economy of gas- and oil engines is in cu. ft. of gas or lb. of oil per i.h.p.- or per b.h.p.-hr. For producer-gas engines lb. of coal per h.p.-hr. is usually given in stating engine performance. This practice is useful only in comparing engines using the same fuel, but if the heating value of the fuel is known the B.t.u. per h.p.-hr. may be found. If the fuel is gas the heating value of the gas must be based upon the temperature of the gas as it passes through the meter, or both volume and heating value be reduced to some standard temperature.

In the A.S.M.E. Code the higher heating value of the fuel is used as a

basis for computing the heat charged to the engine, while the Vereines Deutscher Ingenieure employs the lower heating value. In view of this difference of opinion, even among American engineers, all test reports should state whether the high or low value is used. The use of the lower value may be justified upon similar ground that the heat factor of the steam engine is based upon the Rankine rather than the Carnot cycle, in that the lowering of the exhaust temperature to a point where the latent heat of the water vapor is available is a physical impossibility. On the other hand, as opinions are at variance concerning the proper deduction to be made in determining the lower heating value, greater uniformity may be obtained by the use of the higher value until there is a more general agreement in the determination of the lower value, which, after all, must be more or less arbitrary.

The heating value being determined, the heat consumption will be:

$$\text{B.t.u. per h.p.-hr.} = \frac{\text{Lb. fuel per hr.} \times \text{heating value per lb.}}{\text{horsepower}} \qquad (12)$$

or:

$$\text{B.t.u. per h.p.-hr.} = \frac{\text{Cu. ft. fuel per hr.} \times \text{heating value per cu. ft.}}{\text{horsepower}} \qquad (13)$$

Then, as with steam engines and turbines, the thermal efficiency is:

$$e = \frac{2545}{\text{B.t.u. per h.p.-hr.}} \qquad (14)$$

Either indicated- or brake horsepower may be taken in (12) to (14), b.h.p. being the more correct, as explained already. Both values are sometimes given in test reports. In this connection it must be remembered that indicator diagrams are not obtainable from engines with very high speed, and that a brake test of very large engines is not always feasible.

43. Efficiency Ratio of Internal-combustion Engines.—Actual efficiency, either indicated- or brake- may be compared with the theoretical cycle efficiency. This is given by (11) and (17), Chap. VI, for constant-volume and constant-pressure cycles respectively. In these formulas it is usual to assume air as the working substance, with the corresponding value of the ratio of specific heats. The ratio for gas mixture theoretically required for complete combustion is slightly different. The ratio of actual to theoretical efficiency may be called the heat factor as with steam cycles. It is not used for design as with steam turbines nor is it commonly given in reporting tests as with steam engines and turbines. This may in part be due to lack of uniformity in assuming conditions for the theoretical cycle, a difficulty not experienced with the ideal steam cycle.

44. Comparative Economy.—Any extensive collection of test data, or

an elaborate discussion of the relative economy of the different heat engines is not within the scope of this book. While a great many tests have been recorded which include about every form of heat engine over a wide range of power, the results for any given class vary considerably among themselves, and the reduction of all conditions to an absolute standard by which the different classes may be compared with perfect fairness is nearly impossible. Best efficiencies differ to a considerable extent from the average in all types, and tables may easily be compiled exhibiting the excellence of a favorite type to the disparagement of its rival.

It may be said for the steam engine and the steam turbine, that under practically the same working conditions for machines of the same grade and power, their maximum thermal efficiencies are the same, and the selection of one or the other must be governed by other conditions for which one may be better adapted than the other.

The internal-combustion engine undoubtedly has a higher maximum thermal efficiency than steam machines. There is less difference in the efficiencies of large and small powers than in steam engines and turbines, giving the advantage to the internal-combustion engine for small and intermediate powers.

Some data of heat-engine performance are given in Chaps. III to V. Economy under changing loads is also considered in Chaps. XII to XV.

Fuel economy is not the only factor in the selection of an engine. Interest on investment, rental, depreciation and other items having a purely financial bearing are sometimes opposed to the highest fuel economy and govern the selection, not only of the type, but the grade of engine within that type. This condition will probably continue until there is a more acute public conscience, or a new era of economic conditions forces upon us a more careful consideration of the fuel question.

CHAPTER IX

CYLINDER EFFICIENCY

45. Cylinder efficiency implies losses due in some manner to the cylinder, and suggests practice as opposed to theory. But all practice has its theory even though it be unrecognized; or, due to its complexity we are unable to follow it through its various ramifications. All theory is not so simple, or so easily checked by experiment as that dealing with the effect of heat upon gases and vapors; and while the passage of heat through metal and other substances has been theorized and experimented upon, the effect of the metal walls of a cylinder upon a vapor or gas which is going through a certain cycle within it involving changes of pressure and temperature; and with the duration of this cycle varying from two seconds to less than one-tenth of a second; is indeed difficult to analyze.

Considering the conductivity of the working substance and of the metal of the cylinder, and the fact that the cylinder is jacketed with some heat-resisting material, steam or water; also the impossibility of instantaneous temperature measurements, and the general un-get-at-able-ness of the whole thing for reliable observations; about the best we can do is to cover the entire effect with the heat factor, diagram factor, or the thermal efficiency found from tests.

It seems worth while, however, by the application of simple laws to various phases of engine construction and operation, to study the question of cylinder losses with a view to reducing them.

46. The Steam Engine Cylinder.—A comparison of the actual with the theoretical steam consumption of a steam engine shows that much more steam enters the cylinder than is required to do the work, therefore some of it must be condensed in its passage through the cylinder.

Initial Condensation.—Scientific investigations, notably those of Hirn, of the behavior of saturated steam in an engine cylinder, have established the fact that nearly all of the condensation occurs as the steam enters the cylinder, the heat being withdrawn by the walls which are at a temperature much below that of the steam. This is known as *initial condensation,* and continues up to the point of cut-off.

Condensation and Re-evaporation.—This term really includes initial condensation, but it will be discussed here in its relation to the remainder

of the cycle, beginning at cut-off. Condensation continues, partly due to the cooling effect of the cylinder walls, and partly to the conversion of heat into work which would occur if expansion were adiabatic. This double condensation causes the expansion curve to fall below the adiabatic for a ways as may be seen in Fig. 54.

The transfer of heat since the beginning of the stroke has raised the temperature of the cylinder walls, so that the temperature of the steam as it expands eventually reaches a point below that of the walls. As heat must always flow from a higher to a lower temperature, the steam now begins to receive heat from the cylinder walls. This begins to check further condensation and in some cases will re-evaporate condensed steam to such an extent that the steam is dryer at the end of expansion than at cut-off. This has the effect of raising the expansion line above the adiabatic as shown in Fig. 54.

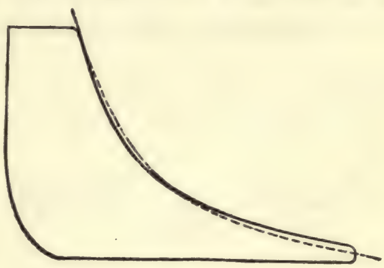


Fig. 54.

As the exhaust valve opens, the drop in pressure due to free expansion causes further drying. The cylinder walls continue to give up heat to the steam during the exhaust stroke until the closure of the exhaust valve.

Compression and Clearance.—The cylinder walls at this end of the cylinder are now at their minimum temperature, and the steam enclosed in the clearance space is usually assumed to be dry. Some experiments, however, indicate that clearance steam is not free from moisture at the beginning of compression; this is probably true of small engines, with which most of the compression experiments were made. Completing the stroke compresses the steam, raising its pressure and temperature.

Rankine's cycle with clearance shows that if compression raises the pressure in a nonconducting cylinder, so that it equals the initial pressure, and if expansion is complete, clearance does not effect the steam consumption; but expansion is not complete in practice, and clearance influences the ratio of expansion considerably, the effect varying with changing cut-off. The effect of clearance and compression is also dependent upon other factors.

The whole effect of condensation and re-evaporation so far has been to cool the cylinder walls, so that the incoming steam for the next cycle will give up part of its heat by condensation. Re-evaporation during the latter part of the stroke increases the work done slightly, but is equivalent

to heat added at a temperature less than the maximum and is therefore inefficient. Assuming no compression, and that the exhaust valve closes at the end of the stroke enclosing dry steam in the clearance space, initial condensation will be due to the comparatively cool walls at the next admission of steam.

It is probable that during engine operation, the inside surface of the cylinder walls never reaches a temperature as high as that of the incoming steam, or as low as that of the exhaust steam, so that with a small amount of compression the temperature of the steam in the clearance space will not be as high as the cylinder-wall temperature; there will then be no further loss of heat, and the clearance steam may be slightly superheated. This condition, with the saving of the clearance steam, is conducive to economy.

FIG. 55.

On the other hand, a higher compression may raise the temperature of the steam above that of the walls, and the surface of the walls being large in proportion to the weight of steam in contact with them will cause the steam to condense, lowering the pressure as shown in Fig. 55.

It is well known that the presence of water will cause steam to condense much more rapidly than contact with metal of the same temperature; therefore if compression causes condensation, initial condensation for the next cycle will be increased.

There has been much discussion about compression and some noteworthy experiments, the range of which, however, has not been comprehensive enough to formulate any rules governing the selection of compression pressures. Mr. Robert R. Fisher in *Power*, July 29th, 1913, gives the results of tests on a 10 by 30 in. simple, noncondensing Corliss engine, which showed that up to about 45 per cent. of the absolute initial pressure, compression was beneficial to economy. The effect of speed will be considered presently.

It has been shown by certain recent experiments that the percentage of clearance volume has little effect on economy. The large clearance, usually considered so wasteful, holds a larger weight of steam proportionate to the cooling surface of the walls in a well-designed cylinder, tending to reduce initial condensation. An earlier compression is also necessary to get a given cushion effect; this encloses a greater weight of steam which better resists condensation as the compression temperature rises above that of the walls. The influence of clearance in reducing relative cooling surface is more noticeable at short cut-off, as may be seen

in Formula 1, Par. 47. The effect of clearance and compression and their relation to cut-off in connection with theoretical steam consumption is discussed in Chap. XII, Par. 62.

The cycle just completed in a rather roundabout way was for one end of a cylinder and applies to a single-acting engine. Most steam engines are double-acting, the inlet and expansion stroke for one end being simultaneous with the exhaust stroke for the other end, thus complicating things. From a consideration of the foregoing, it is obvious that the quantitative prediction of results by any mathematical theory however complex and apparently complete, is futile beyond any peradventure.

Hirns analysis, and later the temperature-entropy diagram have been used to study the effect of cylinder walls on saturated steam, and these may be found in works on thermodynamics and the temperature-entropy diagram.

47. Factors Affecting Cylinder Condensation.—These include practically everything that must be considered in the design of an engine of a given power, and with few exceptions enter into the design of all steam engines. These exceptions involve special construction or some special application of heat with the express purpose of improving economy.

Factors affecting cylinder condensation	Common to all engines	Speed.
		Size of cylinder.
		Design of cylinder.
		Range of pressure and temperature.
		Ratio of expansion.
	Special	Steam jacket.
		Compounding.
		Superheated steam.

Speed.—Tests of a given engine run at different speeds have shown that better economy is obtained at the higher speeds. At high speed there is less time for heat interchange between the cylinder walls and the steam, and the minimum temperature is probably never so low.

While speed is a factor which must usually be considered in the design of the engine, the economy of existing engines may sometimes be improved by increasing the speed if the design of the engine will permit of the resulting increase of stress, and the necessary reduction of the m.e.p. to maintain the same power does not necessitate too short a cut-off, the effect of which will be explained presently. If the engine is belted to a jack shaft, the pulley on this shaft must be increased in diameter in the same ratio that the engine speed is increased, to retain the same speed of jack shaft. Should the increased speed cause a greater velocity of the

fly-wheel rim than is permissible—about 1 mile per minute—a smaller fly-wheel must be used, retaining the original jack-shaft pulley. The diameter of the new wheel must be such that the product of the diameter and r.p.m. will be the same as before the speed change.

Speed change is sometimes limited by the area of ports or steam passages, which if too small may cause excessive wire-drawing and offset any gain due to increase of speed.

It is maintained by some engine builders that piston speed has a greater influence on economy than rotative speed, and it may be that the recent practice of higher piston speed is not all attributable to the demand for increased capacity. This will be further mentioned under cylinder design.

Size of Cylinder.—For cylinders of similar form the volume varies directly as the cube of the diameter, while the surface varies as the square of the diameter; this may be seen in Table 2. Therefore, there is less cooling surface per unit volume of steam in large cylinders than in small, and consequently less relative condensation in a given time. This in part accounts for the superior economy usually realized in large engines even though the rotative speed is low. The piston speed, however, is usually fairly high, which goes to support the suggestion in the preceding paragraph; there have also been exceptional records made by large engines when the piston speed was unusually low.

For a given power requirement there is apt to be a conflict between high piston speed and the large unit idea, especially when the nature of the service fixes the number of units. A compromise will usually be effected, governed mostly by financial considerations.

Design of Cylinder.—This includes the clearance volume, ratio of stroke to diameter of cylinder, and the jacket or covering. With the exception of special designs, such as the uniflow engine, it is probably safe to state that the clearance volume should be kept as small as possible. This depends upon the clearance distance between piston and cylinder heads, the type of valves used and the general arrangement of steam chest and ports, which must all be carefully worked out on the drawing board.

The ratio of stroke to cylinder diameter is probably more often determined from general considerations of design and construction than from the standpoint of economy. A large ratio, or relatively long stroke gives a smaller relative amount of cooling surface for the same cut-off and cylinder diameter, as may be seen from Table 2, and permits of a greater piston speed when the rotative speed is limited as in large pumping engines, or is fixed by direct connection to electrical machinery. It

also reduces the percentage of clearance volume for the same piston speed and diameter of cylinder, the design of the ports being the same for a given piston speed.

For engines of the same power and rotative speed a long stroke makes possible a greater piston speed as just stated, but a smaller cylinder, the latter requiring engine parts of smaller cross-sectional area; and although the long stroke necessitates a longer engine, the smaller diameter of pins and bearings, and the lighter parts, will reduce the friction and increase the mechanical efficiency.

The relation between power, cylinder diameter, rotative and piston speed, may be readily seen in the equations of Chap. XII, from which a better understanding may be obtained of the preceding discussion. From these equations it is apparent that for an engine of a given power, assuming the m.e.p. to be the same in any case, a high piston speed either means a high rotative speed or a large ratio of stroke to cylinder diameter; or that both rotative speed and ratio may be greater than with a lower piston speed. Although the relative importance of these two factors may not be definitely stated, they are both conducive to economy and the advantage of higher piston speed may be thus explained.

As the ratio of surface to volume of steam undoubtedly greatly influences condensation, a clearer understanding of how this ratio is affected by cylinder diameter, length of stroke and cut-off will be of advantage, so a simple formula will be derived. Let:

R = the ratio of inside cylinder surface in one end of a cylinder up to cut-off, to the volume enclosed.

D = the diameter of the cylinder in in.

q = the ratio of length of stroke to diameter of cylinder.

l = ratio of the portion of stroke up to cut-off to the entire stroke.

k = ratio of clearance at one end to volume of stroke, neglecting counterbore and ports.

$$\text{Surface} = 2\frac{\pi D^2}{4} + \pi D(l + k)qD$$

$$\text{Volume} = \frac{\pi D^2}{4}(l + k)qD$$

$$R = \frac{\text{surface}}{\text{volume}} = \frac{2[2q(l + k) + 1]}{q(l + k)D} \qquad (1)$$

Assuming a clearance of 4 per cent. Table 2 has been computed from (1), giving values of R for values of D, l and q over range enough to show the effect of each.

The influence of piston speed upon the design of engines with a limited or fixed rotative speed is not apparent, as the greater piston speed neces-

TABLE 2

D in.	Values of R					
	$q=1$			$q=2$		
	$l=\frac{1}{8}$	$l=\frac{1}{4}$	$l=\frac{1}{2}$	$l=\frac{1}{8}$	$l=\frac{1}{4}$	$l=\frac{1}{2}$
10	1.610	1.090	0.772	1.010	0.746	0.586
20	0.805	0.544	0.386	0.505	0.373	0.293
30	0.536	0.363	0.257	0.336	0.248	0.195
40	0.402	0.272	0.193	0.252	0.186	0.146

sitates a smaller cylinder and a greater ratio of stroke to diameter, which have opposite effects upon the value of R. To better compare these effects examples will be given of two engines of different power assumed to have the same m.e.p. and cut-off, but each having several different piston speeds.

The rotative speed will be fixed in each case and suited to the power of the engine. Cylinder diameters were calculated from Formula (25), Chap. XII, from which the following notation was taken.

P = m.e.p.
N = r.p.m.
S = piston speed in ft. per min.
H = horsepower.
L = length of stroke in in.
l and q are used as in Formula (1), from which R is calculated.

Engine No. 1. $P = 70$, $l + k = \frac{1}{4}$, $H = 200$ and $N = 200$.

$S = 600$, $L = 18''$, $D = 14\frac{1}{4}''$, $q = 1.265$, $R = 0.725$
$S = 800$, $L = 24''$, $D = 12\frac{1}{4}''$, $q = 1.960$, $R = 0.660$
$S = 1000$, $L = 30''$, $D = 11''$, $q = 2.730$, $R = 0.630$
$S = 1200$, $L = 36''$, $D = 10''$, $q = 3.600$, $R = 0.622$

Engine No. 2. $P = 70$, $l + k = \frac{1}{4}$, $H = 1000$ and $N = 100$.

$S = 600$, $L = 36''$, $D = 31\frac{3}{4}''$, $q = 1.135$, $R = 0.348$
$S = 800$, $L = 48''$, $D = 27\frac{1}{2}''$, $q = 1.745$, $R = 0.313$
$S = 1000$, $L = 60''$, $D = 24\frac{1}{2}''$, $q = 2.450$, $R = 0.297$
$S = 1200$, $L = 72''$, $D = 22\frac{1}{2}''$, $q = 3.200$, $R = 0.289$

It is obvious that the gain due to long stroke more than offsets the loss due to smaller cylinder as the piston speed becomes greater, giving a decrease in the ratio R. The gain per hundred feet of piston speed decreases with the increase of piston speed, suggesting a practical limit for a given rotative speed, especially if it leads to objectionable construction.

A comparison of the values of R for engines 1 and 2 does not mean that the relative condensation per lb. of steam used is indicated thereby, because this is counteracted in part by the higher rotative speed of No. 1. R is but one of the factors influencing condensation, and it may be that its effect is proportional to some power of this factor less than unity.

The cross-sectional area of the engine parts is less for the smaller cylinder diameter as previously stated, assuming the maximum unbalanced pressure to be the same in all cases; and while this is offset by increased lengths of such parts as are affected by the stroke, making the weight of these parts practically the same in either case, such parts as the piston, crosshead, connecting rod ends, crank pin and shaft, are only affected by the cylinder diameter, resulting in a lighter engine as stated.

Most engine cylinders are of cast iron, which is a good conductor of heat, therefore to reduce heat loss by conduction, radiation and convection, the cylinder must be covered with some heat-resisting material, called a nonconductor. Wood was formerly used, but as higher pressures and temperatures were used the wood was replaced by asbestos or magnesia in the form of plaster, which is more easily applied and more satisfactory. This is covered with a jacket, or lagging, usually of sheet steel, which fulfils the purpose of a finish and also forms a dead air space which aids in retaining the heat. Steam jackets will be considered presently.

Range of Pressure and Temperature.—If the difference between initial and back pressure is great, there will be a correspondingly great temperature difference between the cylinder walls and the steam at the moment of admission. As this is responsible for initial condensation, it follows that the greater this range of pressure the greater will be the condensation. This will cause loss of thermal efficiency unless offset by some opposing factor. For example, increasing the steam pressure in a certain cylinder increases the condensation, but with a proper cut-off the gain in work done due to the increased pressure may be such that the work per given weight of steam is more than with the lower pressure. Likewise the increase of pressure range by lowered back pressure due to connecting a condenser may produce a similar result; in this case the economy is practically always better.

Ratio of Expansion.—The ratio of expansion depends upon the clearance and the cut-off. For an engine already built the clearance is fixed, and any change in the ratio of expansion is determined by the cut-off. For the Rankine cycle of maximum economy expansion is complete, the

terminal pressure equalling the back pressure. Lowered efficiency results if the ratio is decreased or increased, the latter resulting in a loop in the diagram, giving negative work as in Fig. 56.

Complete expansion would also give the highest efficiency in actual engines if it were not for cylinder condensation and friction. Neglecting the latter for the present, it is obvious from Table 2 that if the rate of condensation of a given weight of steam depends upon the surface to which it is exposed—other things being equal—the relative condensation is greater at short cut-off. Thus, a cut-off must be chosen practically always greater than will give complete expansion, and such that the sum of the loss due to incomplete expansion and that due to condensation is a minimum. This cannot be definitely predetermined, and for a given engine may only be found by test; but it is usually from 1/5 to 1/3, depending upon the various factors already discussed and to follow.

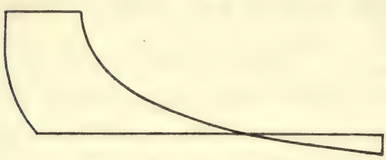


Fig. 56.

Obviously this most economical cut-off must be such as to give the highest thermal efficiency at brake, thus providing for friction. It is also desirable that it correspond to the load to be carried by the engine most of the time, or to the rated horsepower, and this is usually aimed at in steam-engine design. Then by change of cut-off the engine can adapt itself to changes of load from zero brake horsepower to 50 per cent. overload, and in some cases as high as 100 per cent., the economy decreasing as the load varies either side of the rated load, assuming that this was correctly chosen. A typical steam consumption diagram is shown in Fig. 57. The effect of initial condensation shows in a marked degree at the smaller powers.

Steam Jacket.—A form of cylinder construction in which steam surrounds the cylinder barrel and heads is sometimes used to improve economy. The steam is usually supplied from the steam main which supplies the engine, the condensed steam being piped to a trap. The object of this steam jacket is to keep the cylinder walls at a higher temperature throughout the cycle and thus reduce initial condensation. The jacket steam replaces part of the heat carried out with the exhaust due to

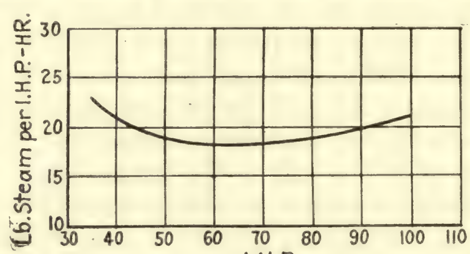


Fig. 57.

re-evaporation, and supplies that carried away by radiation, which would otherwise be taken from the working steam.

While the consumption of steam in the cylinder is reduced by the application of the steam jacket, the jacket consumption partly offsets this result, so that the increase in economy is much less than would otherwise be possible. In some instances there is no gain whatever and sometimes an actual loss, so that the steam supply to the jackets is discontinued.

It is obvious that engines in which the design or working conditions tend to increase condensation are more greatly benefited by steam jackets, the gain in some instances being as high as 30 per cent. Thus, engines at light load with short cut-off show considerable gain, with less, or no gain at a more economical cut-off, and sometimes a loss at overloads.

Comparatively few engines are equipped with steam jackets, their most usual application being to large pumping engines and marine engines of large power. Their diminishing use is probably partly due to the increasing use of superheated steam.

Compounding.—The general principle of compound engine operation is explained in Par. 3, Chap. III. After expansion is effected in the high-pressure cylinder, the steam is exhausted to a receiver, from whence it is admitted to a low-pressure cylinder and again expanded, exhausting finally to the atmosphere or a condenser. In triple- or quadruple-expansion engines this operation is continued through one or two more cylinders. The advantage of compounding may be explained with two-stage expansion and the following discussion will be so confined.

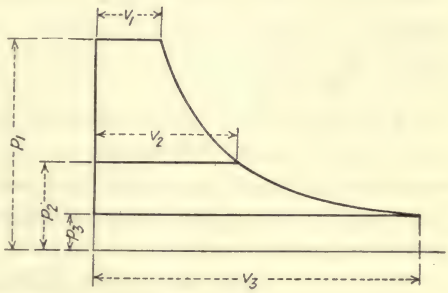


Fig. 58.

Assuming a compound engine to operate on the Rankine cycle with complete expansion, and that for the sake of simplicity the receiver is indefinitely large, so that the transfer of steam to and from the receiver does not change the pressure, the pressure-volume diagram is shown in Fig. 58. Pursuant with current practice let the receiver pressure divide the diagram into two equal parts, equally dividing the work between the two cylinders. This may be done by dividing the difference between the heat content at p_1 and p_3 at the same entropy, by two, subtracting it from the heat content at p_1 and finding from an entropy chart or table the pressure corresponding to the resulting heat content.

For example, assume adiabatic expansion between 151 lb. per sq. in. absolute to 14.7 lb. absolute, at 1.52 entropy. From table or chart, Table 3 may be constructed.

TABLE 3

Pressure	Temp., ° F.	Heat content	Specific volume
$p_1 = 151.0$	359	1153.3	$v_1 = 2.859$
$p_2 = 50.8$	282	1072.2	$v_2 = 7.465$
$p_3 = 14.7$	212	991.1	$v_3 = 22.39$

Thus for a simple engine operating between the same pressures, the temperature range from admission to exhaust is 147 degrees F., while for the compound it is 77 and 70 degrees in the high- and low-pressure cylinders respectively; or approximately one-half of what it would be in the single cylinder, thus reducing the initial condensation.

Referring to Fig. 58, the ratio of expansion for a simple engine would be v_3/v_1, or 7.84. For the compound engine it is v_2/v_1, or 2.61 in the high-pressure cylinder, and v_3/v_2, or 3 in the low-pressure. For the simple diagram assumed, the cut-offs are the reciprocals of these numbers. Thus, while the total ratio of expansion is the same in both simple and compound engine, and theoretically the most economical, in the actual engine the longer cut-offs in the cylinders of the compound would result in less initial condensation than the shorter cut-off in the simple engine.

For the Rankine cycle, assuming as it does a nonconducting cylinder, there would be no advantage in compounding from a thermodynamic standpoint; but it serves to illustrate the effect of compounding upon temperature range and ratio of expansion, two very important factors influencing condensation.

Clearance, compression, the receiver, and the fact that the expansion is not adiabatic, alter the values of Table 3 to some extent when applied to actual engines; these will be considered in Chap. XIII.

Cylinder Ratio and Terminal Drop.—These quantities are interdependent when a nearly equal division of work is desired, and the question of their proper values to insure the best economy has been considerably discussed. A knowledge of the principles of Chap. XIII is necessary for an intelligent consideration of this subject, and a previous study of the same is assumed.

Then it is apparent that within reasonable limits, the power of a compound engine working with a given ratio of expansion is mainly dependent upon the size of the low-pressure cylinder, the high-pressure cylinder, or the ratio of the volume of stroke of the low-pressure to that

of the high-pressure cylinder, having little influence. That this ratio does affect the terminal pressure in the high-pressure cylinder is plainly seen in Fig. 59, in which V_2 is the volume of stroke of the low-pressure and V_1 of the high-pressure cylinder. Volume V_1 is for a low cylinder ratio, or value of V_2/V_1, while V_1', shown by the dotted lines, is for a high cylinder ratio. Terminal drop p is for the low ratio and p' the high ratio.

As elsewhere explained, a certain amount of terminal drop is necessary for practical, economic engine operation, and this is provided for in the low-pressure cylinder by the assumption of a total ratio of expansion which is expected to give the maximum economy at the rated load.

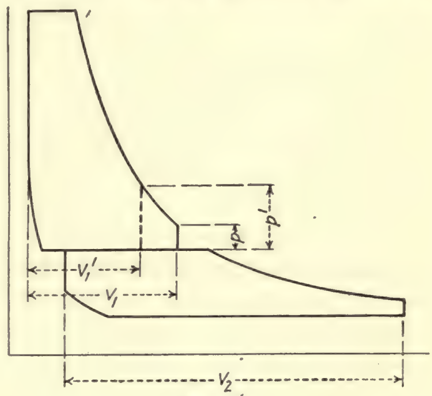


FIG. 59.

Although engines with high cylinder ratios have been built for a good many years, the lower ratios are in the majority, a very common value of the ratio being four. Though there have been high-economy records with both low and high ratios, advocates of the high ratio claim superior economy when working conditions are the same in both cases.

In Fig. 59 it will be noticed that the total area is some less for the high ratio. On the other hand the cut-off, especially in the high-pressure cylinder, is longer. With a high terminal drop the steam will be dryer as it enters the receiver than with low drop, if the quality is the same at terminal pressure.

For the purpose of comparison, let three engines be considered, each to have a rated indicated horsepower of 500, neglecting the diagram factor; a piston speed of 1000 ft. per min., and the same stroke and r.p.m. Also let the initial and back pressures be 140 and 2 lb. absolute, clearance 4 per cent., compression 0.8 stroke in the low-pressure cylinder, and in the high-pressure cylinder such that the two compression curves lie on the same curve. Assume the curves of expansion and compression to be hyperbolas and that the total ratio of expansion is 30 in all cases.

The cylinder ratios are 4, 5.55 and 7.56; cylinder diameters were computed by the formulas of Chap. XIII as was also the theoretical water rate. The temperature range was taken from steam tables for the

pressures used in each case. The quality is given to show the condition the steam may be in at terminal pressure in the high-pressure cylinder to become dry due to free expansion by the time it reaches the receiver pressure. These values are given in Table 4. The cylinder ratios for engines 2 and 3 given in the table are not those used in the calculation, but are the result of rounding up the calculated diameter to even inches. The ratio R was found by Formula (1).

TABLE 4

Cyl. ratio	Cyl. diam.		Cut-off		Temp. range		Ratio R		Quality	Water rate	Relative effect	
	H.p., in.	L.p., in.	H.p.	L.p.	H.p.	L.p.	H.p.	L.p.			H.p.	L.p.
4.00	16	32	0.0985	0.231	135	105	0.653	0.331	0.999	7.76	1.000	1.000
5.55	14.0	33	0.1417	0.233	136	104	0.591	0.329	0.994	7.78	1.024	1.025
7.56	12.0	33	0.1980	0.247	138	102	0.567	0.316	0.986	7.97	1.063	1.067

By a study of Table 4 and the preceding paragraphs of this chapter it is evident that the economy is affected inversely as the ratio R, the temperature range, the quality and the theoretical water rate. Assuming the product of a constant and the reciprocals of these quantities as unity for each cylinder of engine 1, their relative combined influence may be found and is given in the last two columns of Table 4. This indicates an average gain in economy of the highest over the lowest ratio of 6.5 per cent.

The influence of the receiver and piping upon temperature range and condensation have been neglected, it being assumed the same in each case.

Lack of comprehensive experimental data, and the complex nature of the problem make impossible the construction of a formula including these tabulated quantities which may be used to predict economy, but it seems reasonable to believe that they give the direction in which improved economy may be looked for.

Superheated Steam.—When superheated steam enters the cylinder of a steam engine, heat is transferred to the walls as with saturated steam, but instead of immediate condensation upon contact, which still further augments condensation, the superheat must first be withdrawn. Meanwhile, the cylinder walls have increased in temperature, so that, should the steam reach the saturation point before cut-off occurs, the tendency to condensation is greatly reduced. If superheating is carried far enough condensation will not begin until after cut-off, and even be delayed until expansion is partly completed. Engineers differ as to the degree of superheat which may be used to advantage, some maintaining that

economy increases with the amount of superheat, while a number conservatively place the limit at about 100 degrees F., having consideration of the damage to castings and the difficulty of lubrication attending the use of exceedingly high temperatures. Even a small amount of superheat is of great advantage, for though some condensation begins at the walls before the point of cut-off is reached, superheated steam is a much poorer conductor of heat than saturated steam, so that the bulk of the steam is but little influenced.

The effect of superheat on economy is largely indirect, the actual gain often being twice that given by the Rankine cycle.

In addition to the reduction of initial condensation, increased specific volume in part accounts for steam economy, a cubic foot of steam at 140 lb. absolute pressure and 100 degrees superheat weighing but 86 per cent. of saturated steam at the same pressure. This is in part offset by the fact that the expansion curve falls below that of saturated steam, necessitating a larger volume at cut-off to do the same work.

Superheat probably affects several of the factors previously discussed, as it offsets the influence of initial condensation; however, the features of design best adapted to the economical use of saturated steam will probably produce the best results with superheated steam. It seems probable, however, that due to the reduction of condensation at short cut-off, the most economical cut-off will be shorter, making it possible to carry a desired overload without the usual long cut-off and loss of expansive work. On the other hand, as just stated, the expansion curve for superheated steam is said to drop more rapidly than that of saturated steam, necessitating a longer cut-off to obtain the same terminal pressure. This effect is perhaps more noticeable with high degrees of superheat; in fact, the author has failed to notice much difference in this respect in any superheated-steam diagrams that have come to his notice, these having been mostly for a moderate degree of superheat.

The range of pressure may be increased with superheat, making the condensing simple engine more desirable than with saturated steam. The tendency has been, however, where high pressure has been employed with saturated steam, as in locomotives, to reduce the pressure when superheat is employed.

While there are engines specially built for superheated steam, it is advantageously applied to existing engines with various types of valves. This is especially true of locomotives, the addition of the superheater increasing capacity as well as economy.

In the Journal of the A.S.M.E., Jan., 1916, Mr. Robt. Cramer points out the theoretical advantage of using steam of high pressure with mod-

erate superheat, over lower pressure with greater superheat. He makes his comparison between pressures of 200 and 600 lb. per square inch, limiting the temperature in both cases to 600 degrees F. This allows superheat of 218 and 113 degrees respectively. A common pressure for compound engines and steam turbines is 150 lb. gage, and a common range of superheat is from 100 to 150 degrees. Using the nearest values without interpolation from Peabody's entropy table, comparison with a gage pressure of 250 lb. is given in Table 5, assuming a vacuum of 28 in. of mercury and nearly equal maximum temperature. Efficiency is that of the Rankine cycle, given by Formula (8), Chap. VII.

TABLE 5

Absolute pressure	164.800	264.300
Maximum temperature	508.000	503.900
Superheat	142.000	97.900
Entropy	1.650	1.590
Efficiency	0.292	0.328

The higher pressure shows a gain of 12 per cent., and while this may not be practically realized, there probably would be a substantial increase in efficiency, as the superheat is sufficient to greatly reduce if not prevent all condensation. The gain is obviously due to the reception of a larger percentage of heat at high temperature. There is no practical difficulty in the use of 250 lb. steam from the standpoint of either boiler or engine, and it is probable that much higher pressures may be practical. This question has been much discussed since the appearance of Mr. Cramer's paper.

High pressure shows greater theoretical gain than high superheat, which may be readily understood by a study of the entropy diagram; it is probable, however, that where extremely high pressures are used, high superheat will be a desirable, if not a necessary accompaniment, especially in steam turbine operation, as otherwise, condensation will begin too early.

High steam pressures are no novelty, 500 lb. having been used on naval vessels and as high as 1000 lb. in automobiles. A steam car has recently been developed to carry 600 lb. pressure with no superheat. The advisability of carrying such pressures in stationary steam plants is problematical and has a number of noted authorities as its advocates. The question is probably a financial one. If gain in thermal efficiency and capacity offsets interest on investment and insurance, high pressures will be used, the matter of safety having no more bearing than with high-tension electric currents or munition factories.

While no definite rules have been given for obtaining maximum cylinder efficiency, a careful consideration of the foregoing principles, combined with a reasonable amount of practical experience and common sense, should aid the designer in his compromise of conditions to secure good working results.

48. The Internal-combustion Engine Cylinder.—Gas- and oil-engine cycles are discussed in Chaps. V and VI. Due to the high maximum temperature and great temperature range, the theoretical efficiency is high compared to that of the steam engine and turbine. The cumulative effect of the high temperature upon the metal walls of the cylinder is such that the metal is heated to a point where it will not be abl to withstand the pressure if some means are not employed to withdraw part of the heat as fast as it is generated. This is most commonly accomplished by the water jacket, but sometimes air cooling is used with small engines. Heat passes from the working gases through the cylinder walls into the cooling medium, and as the rate of heat flow depends upon temperature difference, more heat is withdrawn at combustion and during the early part of expansion than during the remainder of the cycle, and this lowers the maximum temperature.

Unlike the steam engine, the temperature of the working fluid during exhaust never becomes as low as that of the cylinder walls. The next charge then receives from the walls a portion of this heat of combustion at much less than the maximum temperature. This addition of heat to the charge causes a proportionately higher temperature during the compression stroke, so that for a given compression pressure, the high temperature at the time of ignition causes a higher temperature along the combustion line with its correspondingly greater heat transfer. The heat added to the charge, by increasing its specific volume, reduces its weight; this decreases the capacity of the engine, increasing the relative friction, and the ratio of cooling surface to weight of charge—also promoting heat transfer.

It is commonly stated that the quantity of heat withdrawn from the engine cylinder should be the smallest possible amount consistent with practical operation; but while the theoretical efficiency of an internal-combustion engine is independent of the initial temperature of the charge, it is not probable that actual brake efficiency would be improved by entire absence of cooling, even though lubrication and the strength of the metal were not impaired.

Cylinder dimensions, speed, etc., affect heat transfer to cylinder walls in the same way as in the steam engine, but the problem is an entirely different one. Prevention of heat transfer is the aim in the steam engine,

while in the internal-combustion engine a compromise between thermal efficiency, capacity and lubrication must be effected. It is obvious that the cooling system should be flexible, so that it may be adjusted to give the best operating results. In certain engines the temperature of the cooling water is controlled by a thermostat which may be set at the proper temperature, which is found by experience.

As previously stated, cooling the cylinder has little influence upon economy within reasonable limits. If the heat is not taken out in the cooling water, combustion is suppressed by dissociation as extremely high temperature is reached and the heat is thrown out in the exhaust. As engines increase in size the ratio of cooling surface to volume of cylinder decreases, the natural result being a larger proportion of heat in the exhaust. This is shown by the first three engines in Table 6, taken from Peabody's Thermodynamics. The last engine used blast-furnace gas and was liberally cooled with water.

The amount of cooling water required may be found as follows:

TABLE 6

Engine size, in.	Distribution of heat		
	Work	Jacket	Exhaust
6.75 × 13.70	0.16	0.52	0.32
9.50 × 18.00	0.22	0.44	0.35
26.00 × 36.00	0.28	0.24	0.48
51.20 × 55.13	0.28	0.52	0.20

Let h = heating value per cubic feet of gaseous fuel or per pound of liquid fuel in B.t.u.

w = cubic feet of gaseous fuel or pound of liquid fuel per b.h.p.-hr.

W = pound of cooling water per b.h.p.-hr.

G = gallons of cooling water per b.h.p.-hr.

T_R = Temperature rise in water in degrees F.

q = fraction of total heat supply withdrawn by cooling water.

e_B = thermal efficiency at brake.

Equating the heat withdrawn with the heat taken by the water:

$$qwh = WT_R$$

Also from (8), Chap. VIII:

$$e_B = \frac{2545}{wh} \quad \text{or,} \quad wh = \frac{2545}{e_B}$$

Substituting gives:

$$\frac{2545q}{e_B} = WT_R$$

or:

$$W = \frac{2545q}{e_B T_R} \qquad \text{and} \qquad G = \frac{300q}{e_B T_R}.$$

If $q = 0.35$, $e_B = 0.20$ and $T_R = 70$, then: $W = 61$ and $G = 7.5$.

As a practical suggestion it has been stated that if jacket water is run through the jackets for some time after the engine is shut down, it will prevent the heating of the water remaining in the jacket to the point where salts held in solution will be precipitated, thus forming deposits on the walls, causing overheating. It will also prevent the evaporation of oil from the piston pin.

Lack of agreement among authorities concerning combustion in the cylinders of internal-combustion engines, the properties of gases at high temperatures and the effect of this upon combustion makes the discussion of these subjects seem out of place in a book of this character. Much of value of this nature may be found in Güldner's Internal-combustion Engines, to which the reader is referred.

Güldner suggests a greater ratio of stroke to diameter than has been customary, and greater piston speeds, both of which are being adopted in present-day practice, especially in automobile engines. Their effect upon capacity and weight is perhaps greater than upon economy of operation. High piston speed involves either large valves or high gas velocities. Güldner recommends a gas velocity of 4500 ft. per min. with a maximum of 6000 ft. under the most favorable conditions. With a piston speed of 1000 ft. per min. and a valve lift of ¼ its diameter—which is given as a maximum by Güldner—the valve diameter would be about 0.55 and 0.47 of the cylinder diameter for the two values of gas velocity respectively. This exceeds usual practice at the present time. The piston speed of Güldner's engine given in his book is 730 ft. per min. At maximum valve lift given by him and the valve diameter scaled from his reproduction of a working drawing of the engine, the gas velocity would be 8000 ft. per min. A discussion of this subject with examples from present practice is given in Chap. XX.

As in the steam engine, the tendency is toward higher rotary and piston speeds. A piston speed of 2000 ft. per min. is not uncommon in automobile engines and 3200 ft. has been used in racing engines. Rotary speeds from 1500 to 3000 r.p.m. are also used. Although gas velocities are undoubtedly increased by these high speeds, with a consequent loss, there is still a gain in capacity.

Valve timing plays an important part in both capacity and efficiency and is discussed in Chap. XX. The influence of compression is considered in Par. 76, Chap. XIV.

PART III—FRICTION AND LUBRICATION

CHAPTER X

MECHANICAL EFFICIENCY

Notation.

H_B = brake horsepower (b.h.p.) in general.
H_{BR} = b.h.p. at rated load.
H_I = indicated horsepower (i.h.p.) in general.
H_{IR} = i.h.p. at rated load.
H_F = friction horsepower (f.h.p.).
e = mechanical efficiency at any power.
e_R = the same at rated load.
P = maximum unbalanced pressure in engine cylinder in pounds per square inch.
P_M = mean effective pressure (m.e.p.) in pounds per square inch.
μ = coefficient of friction of engine if $P_M = P$.

49. A general expression for mechanical efficiency of engines is given in Formula (7), Chap. VI. The mechanical efficiency changes with the loading and is obviously zero when all external load is removed and the engine is only overcoming its own friction. The horsepower developed under these conditions is known as the friction horsepower, and the m.e.p., the friction m.e.p.

Friction depends upon many factors, such as the condition of the rubbing surfaces, the system of lubrication and the lubricant used, the relation of work done to the weight of the moving parts; and, other things being equal, it is usually relatively less for a given type of engine as the size is increased.

It is therefore apparent that no fixed rule can be given by means of which friction may be determined with accuracy in all cases, but due to the very conditions named, it is convenient for the purpose of arranging and studying the results of tests, to derive formulas giving relations between indicated, brake and friction horsepowers for different conditions of loading.

To be intelligible, mechanical efficiency must always be stated with reference to some given load; this may be the maximum load or fraction

thereof, or any rated load. The efficiency will then be different for any other load.

The friction horsepower usually increases with increase of load; the increase is often slight, however, and for practical purposes the f.h.p. may be assumed constant. This assumption will be made in the present discussion and the results compared with actual values from practice.

In what follows, H will denote horsepower and e mechanical efficiency. The subscripts B and I denote brake and indicated horsepower respectively. The subscript R refers to rated power; with steam engines this is usually the most economical power, allowing for from 50 to 100 per cent. overload, depending upon the type of valve gear used. For steam turbines and internal-combustion engines, though at or near the load giving best economy, the rated load is more nearly the maximum; in most cases, however, from 10 to 20 per cent. overload may be carried.

When subscript R is not used in conjunction with B and I, any other than the rated power is meant. The f.h.p. is assumed constant for a given engine. For convenience of expression let:

$$\frac{H_B}{H_{BR}} = k \qquad (1) \quad \text{and} \qquad \frac{H_I}{H_{IR}} = q \qquad (2)$$

The expression for mechanical efficiency at any rated load may be written:

$$e_R = \frac{H_{BR}}{H_{IR}} = \frac{H_{BR}}{H_{BR} + H_F} = \frac{1}{1 + \dfrac{H_F}{H_{BR}}} \qquad (3)$$

From (3):

$$\frac{H_F}{H_{BR}} = \frac{1}{e_R} - 1 = \frac{1 - e_R}{e_R} \qquad (4)$$

Efficiency at any load in terms of b.h.p. is:

$$e = \frac{H_B}{H_B + H_F} = \frac{1}{1 + \dfrac{H_F}{H_B}} \qquad (5)$$

From (1), (4) and (5):

$$e = \frac{1}{1 + \dfrac{H_F}{kH_{BR}}} = \frac{1}{1 + \dfrac{1 - e_R}{ke_R}} \qquad (6)$$

Dividing (1) by (2) gives:

$$\frac{k}{q} = \frac{H_{IR}}{H_I} \cdot \frac{H_B}{H_{BR}} = \frac{H_B}{H_I} \cdot \frac{H_{IR}}{H_{BR}} = \frac{e}{e_R}$$

from which $$k = q \cdot \frac{e}{e_R} \qquad (7)$$

To express efficiency in terms of i.h.p., substituting (7) in (6) and solving for e:

$$e = 1 - \frac{1 - e_R}{q} \tag{8}$$

Should e be known at some other than the rated load, e_R may be found from (8); or:

$$e_R = 1 - q(1 - e) \tag{9}$$

Formulas (6) and (8) give the efficiency for any load when it is known at the rated load, when the fraction of the load is given in terms of b.h.p. and i.h.p. respectively.

Results of tests are given in the following tables. Tables 7 to 9 are from Goodman's Mechanics Applied to Engineering, and are from tests on an experimental steam engine with different methods of lubrication.

The data in Tables 10 to 12 are for Westinghouse vertical 4-cycle, 3-cylinder gas engines using natural gas, and were taken from Güldner's Internal-combustion Engines.

Table 7.—Syphon Lubrication

H_I	2.75	9.250	10.230	11.140	12.340	13.950	14.290
H_B	0.00	5.630	7.500	7.660	9.090	11.090	11.250
H_F	2.75	3.630	2.730	3.480	3.250	2.860	3.040
e	0.00	0.608	0.733	0.688	0.738	0.795	0.788

Table 8.—Syphon and Pad Lubrication

H_I	2.48	5.160	6.830	8.300	11.500	13.840	17.020	22.300
H_B	0.00	2.350	3.940	5.610	8.700	10.820	13.890	19.090
H_F	2.48	2.810	2.890	2.690	2.800	3.020	3.130	3.210
e	0.00	0.455	0.578	0.676	0.756	0.783	0.806	0.857

Table 9.—Forced Lubrication

H_I	49.800	102.700	147.600	193.600	217.500
H_B	44.500	97.000	140.600	186.000	209.500
H_F	5.300	5.700	6.500	7.600	8.000
e	0.912	0.945	0.955	0.962	0.964

Table 10.—25- by 30-in. Gas Engine

Nominal load	H_I	H_B	H_F	N	k	e	
						Actual	Calc.
1/3	262.5	207.5	55.0	151.7	0.375	0.791	0.754
2/3	449.3	383.8	65.5	150.0	0.693	0.855	0.850
Rated	621.4	553.1	68.3	149.2	1.000	0.891	0.891
Max.	676.7	605.6	71.1	146.7	0.095	0.895	0.894

TABLE 11.—19- BY 22-IN. GAS ENGINE

Nominal load	H_I	H_B	H_F	N	k	e	
						Actual	Calc.
½	147.2	121.3	25.9	206.3	0.507	0.825	0.780
Rated	273.5	239.0	34.5	203.0	1.000	0.875	0.875
1⅓	365.5	325.3	40.2	198.0	1.360	0.890	0.862

TABLE 12.—13- BY 14-IN. GAS ENGINE

Nominal load	H_I	H_B	H_F	N	k	e	
						Actual	Calc.
0	18.57	0.00	18.57	265.5	0.000	0.000	0.000
¼	53.69	30.81	22.88	264.0	0.273	0.574	0.534
½	85.55	64.37	21.18	264.0	0.570	0.752	0.706
¾	114.69	90.25	24.44	260.0	0.800	0.787	0.770
Rated	139.73	112.86	26.87	258.0	1.000	0.808	0.808
Max.	164.22	143.17	21.05	256.8	1.270	0.873	0.842

TABLE 13.—HORNSBY—AKROYD OIL ENGINE

Nominal load	H_I	H_B	H_F	N	k	e	
						Actual	Calc.
⅓	13.10	9.00	4.10	203.0	0.336	0.687	0.677
⅔	22.40	17.96	4.44	202.4	0.672	0.802	0.807
Rated	32.15	27.74	4.41	202.6	1.000	0.862	0.862

TABLE 14.—GÜLDNER GAS ENGINE

Nominal load	H_I	H_B	H_F	k	e	
					Actual	Calc.
¼	18.85	11.15	7.70	0.317	0.592	0.550
⅓	22.20	13.30	8.90	0.378	0.600	0.594
½	26.40	17.65	8.25	0.502	0.668	0.660
⅔	31.10	21.50	9.60	0.612	0.692	0.704
¾	37.40	25.90	11.50	0.737	0.692	0.741
Rated	44.20	35.10	9.10	1.000	0.795	0.795

Table 13 contains data given by Güldner for a Hornsby-Akroyd oil engine.

Table 14 is for a Güldner gas engine.

From these data it may be seen that for practical use in designing, formulas (6) and (8) may be used. It may also be seen that the value of

e for a given load increases with the size of the engine, both for steam and gas; but it is considerably greater for the steam engine in proportion to the power. It is less for Diesel engines that for gas engines; W. H. Adams, *Trans.* A.S.M.E., vol. 37, p. 460, gives 0.75 for 4-cycle engines and 0.70 for 2-cycle engines. The Hornsby-Akroyd oil engine, Table 13, has a rated load efficiency of 0.862, although developing only 18 b.h.p., but the compression was only about 45 to 50 lb., and the explosion pressure only 180 to 200 lb.

There seems to be some relation between pressure and mechanical efficiency for engines having about the same m.e.p. When the ratio of maximum unbalanced pressure to m.e.p. is high, the engine parts must be heavier, and while theoretically, friction work depends upon mean rather than maximum pressure, high maximum pressures seem to interfere with lubrication; at any rate, the friction m.e.p. increases with the ratio of maximum to mean pressure.

The formula for mechanical efficiency may be written:

$$e = \frac{H_I - H_F}{H_I} = 1 - \frac{H_F}{H_I} \tag{10}$$

An empirical expression, considering the pressure ratio just mentioned, may be written:

$$\frac{H_F}{H_I} = \mu \cdot \frac{P}{P_M} + m - \frac{H_B}{10{,}000} \tag{11}$$

in which P is maximum unbalanced pressure and P_M the m.e.p. The coefficient of friction μ may be assumed as the ratio of friction to useful work if the pressure were uniform throughout the stroke, or if $P = P_M$; its value may be taken from 0.02 to 0.06. H_B may be taken as the b.h.p. of one cylinder end and m a constant depending upon the type of engine.

Assume for average values, the following:

For simple steam engines.............................. $\frac{P}{P_M} = 2$ and $\mu \frac{P}{P_M} = 0.08$.

For compound steam engines.......................... $\frac{P}{P_M} = 2.5$ and $\mu \frac{P}{P_M} = 0.10$.

For gas—or other constant-vol. engines............... $\frac{P}{P_M} = 4$ and $\mu \frac{P}{P_M} = 0.16$.

For Diesel engines.................................... $\frac{P}{P_M} = 6$ and $\mu \frac{P}{P_M} = 0.24$.

Values of m for steam engines and 4-cycle internal-combustion engines are given in Table 15. These are rather arbitrary and from 0.05 to 0.15 may be added for 2-cycle engines, the larger values being for small, high-speed engines.

Formulas (10) and (11) may be used with judgment for obtaining a fair approximation to ordinary values found in practice, but of course wide deviations will be found. A wider study of data would probably result in more accurate constants, or perhaps a better formula.

TABLE 15.—VALUES OF m

Type of engine	m
Steam-small, with unbalanced valve	0.07
Steam-compound	0.02
Steam-locomotive	0.08
2-cylinder, single-acting, internal-combustion	0.02
3-cylinder, single-acting, internal-combustion	0.03
4- and 8-cylinder, single-acting, internal-combustion	0.04
6- and 12-cylinder, single-acting, internal-combustion	0.06
1-cylinder, double-acting, internal-combustion	0.02
2-cylinder, double-acting, twin internal-combustion	0.04
2-cylinder, double-acting, tandem internal-combustion	0.05
4-cylinder, double-acting, twin-tandem internal-combustion	0.06
Small engines with 1 or 2 cylinders, poorly attended	0.04

Reference

Mechanics applied to engineering, Prof. Goodman.

CHAPTER XI

LUBRICATION

Notation.

d = diameter of bearing in inches.

l = length of bearing in inches.

k = ratio of length to diameter = l/d.

D_S = diameter of standard simple engine cylinder, designed for some standard pressure. (See Par. 63, Chap. XII and Par. 72, Chap. XIII.)

a = area of rubbing surface of bearing in square inches; this is the projected area (ld) for cylindrical bearings.

P = total mean load on bearing in pounds.

p = mean pressure in pounds per square inch; for cylindrical bearings, per square inch of projected area.

p_M = maximum pressure in pounds per square inch.

S = shearing stress in turbine shaft in pounds per square inch.

V = velocity of rubbing surface in feet per minute.

N = r.p.m.

M = speed in knots, or nautical miles per hour.

H = horsepower of engine (i.h.p.).

h = heat in B.t.u. per hour developed in bearing.

C, K and m are constants in formulas.

50. General Principles.—Friction, lubrication and the proportioning of bearing surfaces are so intimately related in connection with the design of heat engines that they may be considered together in one chapter. The preceding chapter dealt with the effect of friction upon power, and later chapters will give the general construction of bearings, while in the present chapter a study of some of the factors influencing friction, and means of reducing it will be attempted.

The laws of friction are exceedingly complicated, and while much valuable experimental work has been done, and most lubricating problems are practically handled, no comprehensive mathematical treatment has yet been presented. References to valuable material are given at the end of this chapter.

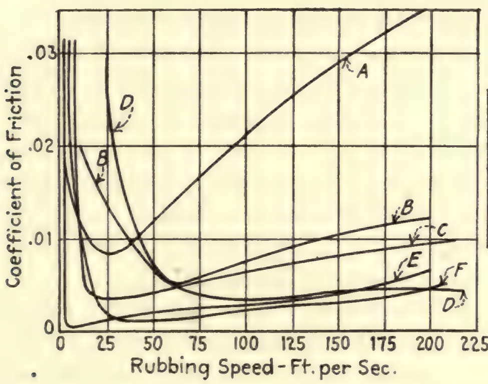
Proper lubrication is effected when a thin film of any lubricant lies between two surfaces in such a way that there is no metallic contact. There is then no appreciable wear. Without lubrication, there is wearing

of the surfaces, and with great unit pressures the surfaces may "sieze," or become welded together, so that if further movement is forced the surfaces become torn and "scored."

There may be all stages between no lubrication and perfect lubrication, with behavior varying accordingly; for this reason it is well to consider the laws of dry as well as of lubricated surfaces. The laws of friction of dry surfaces credited to Morin have been revised in the light of more recent experiments and are given by Goodman in Mechanics Applied to Engineering, in parallel with the laws of lubricated surfaces, and they will be given here.

Dry Surfaces	Lubricated Surfaces
1. The frictional resistance is nearly proportional to the normal pressure between the two surfaces.	1. The frictional resistance is almost independent of the pressure with bath lubrication so long as the oil film is not ruptured, and approaches the behavior of dry surfaces as the lubrication becomes meager.
2. The frictional resistance is nearly independent of the speed for low pressures. For high pressures it tends to decrease as the speed increases.	2. The frictional resistance of a flooded bearing, when the temperature is artificially controlled, increases (except at very low speeds) nearly as the speed, but when the temperature is not controlled the friction does not appear to follow any definite law. It is high at low speeds of rubbing, decreases as the speed increases, reaches a minimum at a speed dependent upon the temperature and the intensity of pressure; at higher speed it appears to increase as the square root of the speed; and finally, at speeds of over 3000 feet per minute, some believe that it remains constant.
3. The frictional resistance is not greatly affected by temperature.	3. The frictional resistance depends largely upon the temperature of the bearing, partly due to the viscosity of the oil, and partly to the fact that the diameter of the bearing increases with a rise of temperature more rapidly than the diameter of the shaft, and thereby relieves the bearing of side pressure.
4. The frictional resistance depends largely upon the nature of the material of which the rubbing surfaces are composed.	4. The frictional resistance with a flooded bearing depends but slightly upon the nature of the material of which the surfaces are composed, but as the lubrication becomes meager, the friction follows much the same laws as in the case of dry surfaces.

Dry Surfaces	Lubricated Surfaces
5. The friction of rest is slightly greater than the friction of motion.	5. The friction of rest is enormously greater than the friction of motion, especially if thin lubricants be used, probably due to their being squeezed out when standing.
6. When the pressures between the surfaces become excessive, seizing occurs.	6. When the pressures between the surfaces become excessive, which is at a much higher pressure than with dry surfaces, the lubricant is squeezed out and seizing occurs. The pressure at which this occurs depends upon the viscosity of the lubricant.
7. The frictional resistance is greatest at first, and rapidly decreases with the time after the two surfaces are brought together, probably due to the polishing of the surfaces.	7. The frictional resistance is least at first, and rapidly increases with the time after the two surfaces are brought together, probably due to the partial squeezing out of the lubricant.
8. The frictional resistance is always greater immediately after reversal of direction of sliding.	8. Same as in the case of dry surfaces.



Curve	Pressure Lb. Sq. In.	Temp. Cent.
A	43	20
B	43	50
C	57	25
D	213	25
E	75	40
F	150	40

Fig. 60.

The way in which friction varies with velocity for lubricated surfaces is shown in Fig. 60 taken from Goodman. A comparison of curves A and B also shows the influence of temperature as stated in Law 3 for lubricated surfaces. A comparison of curves C and D, and of curves E and F also shows the effect of pressure.

It is usually considered that friction and wear are so closely related as to be nearly synonymous terms, but experiments at low pressures show that for dry surfaces the friction is less than for a bearing flooded with oil, but it is certain the wear must be greater. The friction of a lubricated

bearing is due to the shearing of the lubricant, which does not damage the bearing although heat is produced with a rise in temperature.

From Law 3 of lubricated surfaces, a hot bearing, within reasonable limits, is more efficient than a cool one, as the thinning of the lubricant due to temperature reduces the friction; this is also shown by curves *A* and *B*, Fig. 60.

Martin gives an instance of a steam turbine bearing in which the heat generated at 90 degrees F. was double that generated at 165 degrees, showing less friction at the high temperature. He also states that certain steam turbine bearings running at a temperature of 195 degrees F. have given no trouble. High temperature is normal for a properly lubricated high-speed bearing, but the oil must be kept sufficiently viscous to keep the rubbing surfaces apart. The greater the normal viscosity of the lubricant, the higher will the temperature be, due to greater shearing resistance, and the greater the allowable bearing pressure at a given speed.

It is not advisable to run a bearing at its least friction, involving as it does high temperature with a less viscous condition of the lubricant, as an increase in temperature from any cause may still further decrease the viscosity so that the oil film will not be dragged completely around the bearing. To control the temperature, some bearings are constructed so that cold water may be circulated around them, and in steam turbine practice with forced lubrication, the oil is passed through a cooler before being fed to the bearings.

The temperature of steam turbine bearings is not all due to the shearing of the oil, but in part to the transmission of heat of the steam through the shaft, so a bearing temperature of 180 degrees F. is probably perfectly normal for a turbine if there is no foaming of the oil.

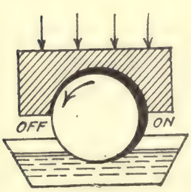


Fig. 61.

A. G. Christie, *Trans.* A.S.M.E., vol. 34, p. 435, states that if the bearings of a steam turbine are flooded with oil at 100 degrees F. so that its temperature upon leaving is about 125 degrees, the life of the oil is much longer than when very hot oil is used, and the wear of the bearings absolutely prevented. He further says that the use of water-cooled bearings is to be discouraged, water being much better employed in an oil cooler.

A light oil, with ample continuous feed, properly filtered and cooled will probably give minimum friction with maximum life of oil and bearings.

With oil-bath lubrication as in Fig. 61, oil is drawn up around the bearing as shown, the layer being thicker on the "on" side than on the "off"

side. This has been shown to be so by actual experiment by Goodman, and that in case of wear, it was greater on the off side.

Experiments by Tower also show that the oil pressure varies, being a maximum nearly midway between the on and off sides, and greater on the off side than on the on side. He also shows that to feed oil at the center where the pressure is greatest causes the oil to flow out rather than in. The results of some of Tower's tests, given by Goodman (Mechanics Applied to Engineering), are given in Table 16, which shows the seizing pressure and coefficient of friction for different methods of supplying lubricants, and for different kinds of bearings. Bath lubrication gives the best results and may be taken as the standard of lubrication. Equivalent methods are those in which the bearing is flooded with oil, or has stream feed, such as some of the forced feed systems.

TABLE 16

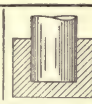
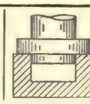
Form of bearing							
Seizing load, lb. per sq. in.	100	150	370	550	600	200	90
Coefficient of friction.......	0.01	0.01	0.006	0.006	0.001	0.01	0.035

TABLE 17

Mode of lubrication	Relative friction	
	Tower	Goodman
Bath..	1.00	1.00
Saturated pad....................................	....	1.32
Ordinary pad.....................................	6.48	2.21
Syphon..	7.06	4.20

Pad lubrication, giving next best results, is accomplished by having the journal in contact with oil-soaked pads. Waste in railway journal boxes is used for this purpose. A comparison between methods of applying lubricant is given in Table 17, from Goodman.

51. Lubricants and Their Application.—As acid is produced by the decomposition of animal and vegetable oils, they are little used for heat engine lubrication, except as the former may be combined in small

proportions with mineral oil to form cylinder oil for saturated steam, promoting better retention of the oil on the surfaces. Mineral oils may then be considered as the lubricants for heat engines, and a few important characteristics will be mentioned. Further information may be found in Gill's Engine Room Chemistry and in treatises on Testing and Experimental Engineering.

Viscosity, or "body," is the degree of fluidity or internal friction of an oil. This varies with the temperature and must be tested at some standard temperature, commonly 70 degrees F. for engine oils and 212 degrees F. for cylinder oils. The least viscous oil which will stay in place and do its work should be used, as it absorbs less power in friction. Viscosimeters for testing the viscosity of oils are commonly constructed upon the principle of the rate of flow through a standard orifice, and the results are given in seconds. The longer it takes for a given quantity of oil to flow through the orifice, the greater the viscosity. When viscosity is given, the viscosimeter should be named.

Density or specific gravity of mineral oils is usually measured with the Baumé hydrometer, and expressed in degrees Baumé (degrees B.). The specific gravity is found by the formula:

$$\text{Specific gravity} = \frac{144.3}{134.3 + \text{deg. B}}.$$

The cold test is the temperature at which the oil will just flow. In stationary power plants this is of little importance, but for exposed work in cold climates it must not be overlooked.

The flash point is the temperature at which oil gives off vapors in sufficient quantity to explode when mixed with air, but the oil will not continue to burn.

The fire test or burning point is the temperature at which vapor enough is given off to burn continuously.

The gumming test is to ascertain the amount of change taking place in an oil when in use. Gumming may seriously interfere with the distribution of oil to bearing surfaces.

Acid Test.—Acid in mineral oils is usually sulphuric, used in the refining process and not entirely washed out. An acid content exceeding 0.3 per cent. is considered harmful.

The friction test, to determine the coefficient of friction is made with a machine specially devised for this purpose. It must be made at standard temperature and with a definite method of applying the lubricant.

There is considerable variation in the characteristics of oils as given by different authorities, but a few values are given in Table 18.

TABLE 18

Kind of oil	Gravity, deg. B	Flash point, deg. F.
High-pressure cylinder oil	26.0 to 28	550 to 600
General cylinder oil	24.0 to 27	530 to 560
Heavy engine oil	25.5 to 28	385 to 410
Gas engine cylinder oil	26.0	410 to 500
Automobile engine oil	29.5	430
Air compressor oil		500 to 530

The cold test is usually 30 or 32 degrees F.

A turbine builder recommends a pure mineral oil of 600 degrees test.

Charles G. Sampson, *Trans.* A.S.M.E., vol. 35, p. 151, says that for a blast furnace gas engine main bearing, crosshead, crank pin and guides, an oil having a specific gravity of 0.888, and a flash point of 435 degrees F. gave good results. For the cylinders, when the piston speed was 600 ft. per min., an oil having a specific gravity of 0.902, and a flash point of 380 degrees F. was satisfactory, but was not at 850 ft. per min.; for this speed a specific gravity of 0.92 and flash point of 502 degrees F. were used. He states that with the slower speed and lighter oil the wear was excessive.

Mathot says that the flash point of gas engine cylinder oil is negligible; that it should have a viscosity similar to bearing oil; in fact the same oil is used for bearings and cylinder in the best practice.

Greases are composed of oils and fats mixed with soap, forming a more or less solid mass. They are used for heavy pressures or in places difficult to lubricate with oil; or, where the movement is comparatively small, as in valve gears. For extra heavy pressures, solid lubricants, such as soapstone, mica or graphite are mixed with the grease.

Systems.—The individual system of lubrication is where each bearing is provided with an oil cup or grease cup, with *restricted feed*, or with an oil hole with *intermittent feed*. The latter is fortunately passing out of date.

The forced feed or continuous system may be *direct*, in which oil under pressure of from 3 to 20 lb. per sq. in. is forced directly to the bearings, returned through a strainer or filter—and sometimes through a cooler—to the pump, where it again starts upon its circuit; or it may be a gravity system, in which the oil is pumped to an elevated tank located upon the wall or on the engine frame, and flows by gravity to the bearings. In this system the filter may be in the basement, on the ground floor, or elevated with the storage tank, with which it is sometimes integral.

The forced feed system may have restricted feed, the supply being throttled by a valve, or the wearing parts may be continuously "flooded" with oil, with stream feed, being a form of oil bath, and this method is gaining favor for heat engine lubrication. The oil pump may be a plunger pump operated from some moving part of the engine as the rocker arm or valve lever, or it may be a centrifugal pump (see Fig. 252, Chap. XIX) operated by some rotating part. With steam turbines, a gear pump is often driven by an extension of the governor spindle. In large units, or in a plant in which there are a number of engines of tur-

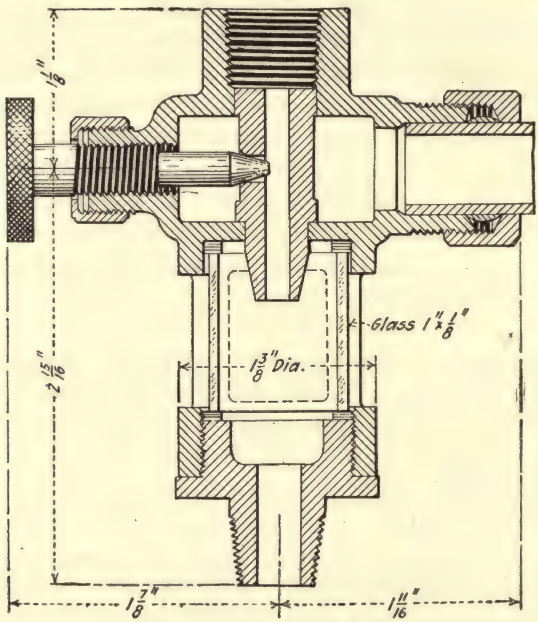


Fig. 62.—Sight-feed valve.

bines, a separate pump driven by a steam cylinder, turbine or motor supplies oil for the entire plant. Manifold oilers, sometimes having a common pump and sometimes a pump for each line which is led from it to a bearing, are often used. Sometimes a separate compartment contains cylinder oil, the manifold oiler thus furnishing complete lubrication.

The splash system of lubrication, used mostly on the smaller engines, consists of an enclosed crank case containing oil at such a level that the crank or counterbalance enters the surface at each revolution, splashing the oil upon the bearing surfaces. In single-acting engines of the trunk-

piston type, this also lubricates the piston, which is lubricated separately in other systems, sometimes with a different grade of oil.

Mathot depreciates the splash system, in that it gives impurities no time to settle; they are thus brought continuously to the wearing surfaces.

A combination of the forced-feed and splash system is used in some instances, notably with automobile engines. The oil is forced into the shaft bearing by a gear pump, from thence to the crank pin through holes in the shaft from which it is thrown. It is claimed that the rapid motion of the cranks (which do not dip in the oil in the bottom of the

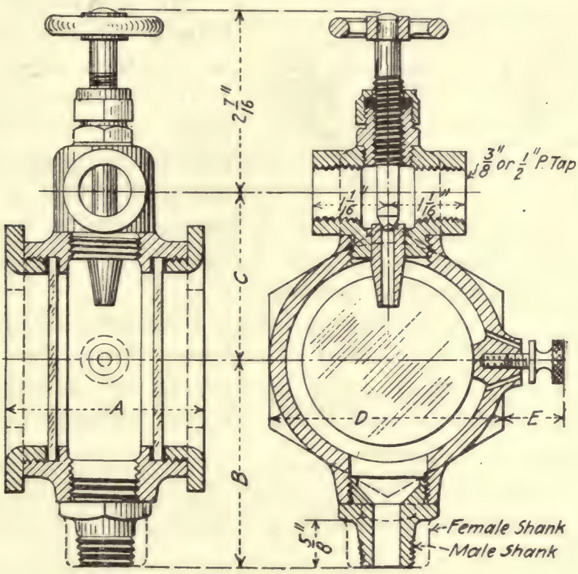


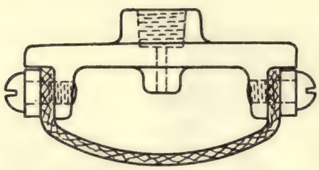
FIG. 63.—"Clear vision" sight-feed oiler.

oil pan) whip the oil into a fine spray, providing ample lubrication for the cylinders.

In feeding oil directly to the bearing, either with an oil cup or with the forced-feed system, a sight feed is generally employed; a sight-feed valve is shown in Fig. 62. The valve may be placed on top of the bearing or it may be a part of an oil cup. The flow of oil in both cases may be regulated by a valve. A special sight-feed oiler is shown in Fig. 63. This is made by the Richardson-Phenix Co.

In some cases the sight-feed valve is one of several in a manifold oiler located on some part of the engine, or turbine frame, with pipes leading to the different bearings; or it may be located near the bearing, but in the return pipe. In either case it shows whether the oil is flowing.

When the individual system is used on engines which are shut down once or twice a day, moving parts such as crosshead pins, crank pins and eccentrics may be supplied with oil cups, but as these cannot be filled while in motion, special devices must be resorted to for continuous operation. For the crosshead, the most common is the wiper and wiper cup, shown in Fig. 64. Oil drops upon the wick from a sight-feed cup or valve and is wiped off into the cup near the end of the stroke. Eccentrics are also oiled in this manner. An alternative in both cases is the telescopic oiler (also called trombone oiler).



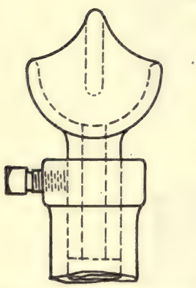


Fig. 64.—Crosshead wiper.

For the crank pin a centrifugal oiler is used. A convenient form, especially for the individual system, is the Nugent oiler. The weight keeps the cup in an upright position at the center line of the shaft. Sometimes a stand, fastened to the floor or engine frame replaces the hanging weight, especially in the forced-feed system.

The telescope oiler is sometimes used for the crank pin in center-crank engines.

Many oiling devices, as well as system layouts, may be seen in the catalogs referred to at the end of this chapter.

Oil guards, enclosing the crank and connecting rod, and sometimes the eccentrics, are often used, and may be seen in engine catalogs.

All of these appliances are also used with the continuous system. In some installations, oil cups are provided with each bearing, which, when filled with oil through the system piping, are shut off by a valve; in case the oiling system is shut down for cleaning or repairs, the cup valves are opened and lubrication provided.

Grease Cups.—Two types of grease cups are used, one a screw-feed cup, the other an automatic cup. In the latter, when the handle is screwed up to the top of the stem, the spring exerts pressure upon the grease, the feed being regulated by a screw which forms a plug cock.

Self-oiling bearings have an oil pocket in the bottom from which oil is carried to the top of the shaft by rings or chalns which are supported on the shaft. A gage glass on the outside of the bearing indicates the amount of oil in the pocket. They are used on the smaller steam turbines and sometimes in connection with a forced-feed system; they are also

used on outer bearings of engines and sometimes for main bearings of large engines. Güldner objects to them for internal-combustion engines as they reduce the wearing surface and weaken the bearing. Bearings are shown in Chaps. XXIX and XXXIII.

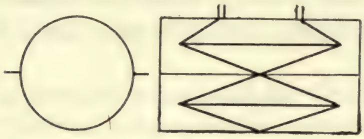


FIG. 65.—Gas engine main bearing grooves.

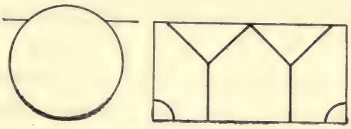


FIG. 66.—Steam engine main bearing grooves.

Clearance.—For proper lubrication there must be some clearance between journal and bearing. No hard-and-fast rule may be made' and standards differ with different builders. Goodman says that for ordinary lubrication the clearance should be 0.001 of the journal diameter,

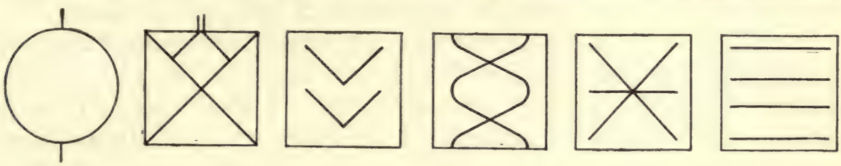


FIG. 67.—Crank- and crosshead box grooves.

and slightly more for flooded bearings. Martin says the clearance for steam turbine bearings should be from 0.006 to 0.008 in.

Oil Grooves.—It is sometimes claimed that oil grooves are unnecessary, and for the uniform conditions under which most bearing tests are made,

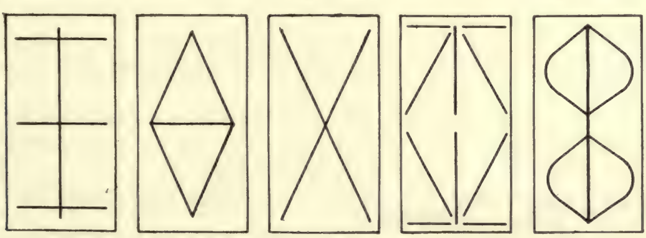


FIG. 68.—Crosshead shoe grooves.

this may be true; but judging from the practice of most builders, it is apparent that there is a general feeling in favor of them. Their form and arrangement have been the subject of considerable discussion, and no attempt of a settlement of the question will be made here; but a number

of forms given by leading authorities and used by successful builders will be given. The grooves are shown as though laid out on a flat surface equal in length to the projection of the circular arc which they represent, and of a width equal to the length of the bearing.

There is probably no doubt but that bearings should be chamfered at the edges, and the corners of chamfer and grooves rounded as shown in Fig. 69.

FIG. 69.—Grooves and chamfers.

Bearing Metal.—It has long been considered that proper lubrication cannot be secured between surfaces of the same material, and while this does not hold for cylinders and valves, at least, it is nevertheless true that in nearly all cases, the journals and bearings of heat engines are of different materials. The journals are practically always of steel, while the bearing boxes are either of some form of bronze, or are lined with a soft metal such as babbitt or some other "anti-friction" metal, the latter method being predominant.

From Law 4 for lubricated surfaces, it makes little difference what the bearing metal is if the bearing is amply lubricated, as the journal is not in contact with the bearing; but bearings are not always perfectly lubricated, so the selection of the material is important, especially for heavy loads or high speeds.

For heavy loads subject to blows, bronze is usually employed, although the crank-pin boxes of most gas engines are lined with babbitt. While all bearings should be properly fitted, this is especially true of bronze, but if carefully fitted and well lubricated, it will probably outwear babbitt.

Goodman gives the advantages and disadvantages of soft white metal for bearings as follows:

Advantages.

(*a*) The friction is much lower than with hard bronzes, cast iron, etc., hence it is less liable to heat.

(*b*) The wear is very small indeed after the bearing has once got well bedded (see disadvantages).

(*c*) It rarely scores the shaft, even if the bearing heats.

(*d*) It absorbs any grit that may get into the bearing, instead of allowing it to churn round and round, and so cause damage.

Disadvantages.

(*a*) Will not stand the hammering action that some shafts are subjected to.

(*b*) The wear is very rapid at first if the shaft is at all rough; the action resembles that of a new file on lead. At first the file cuts rapidly, but it soon clogs, and then ceases to act as a file.

(*c*) It is liable to melt out if the bearing runs hot.

(*d*) If made of unsuitable material it is liable to corrode

The crosshead pins of nearly all steam- and internal-combustion engines, having comparatively small wearing motion, are provided with bronze bearings, as are some small valve-gear pins, but the main and outer bearings, the crank-pin box and the crosshead shoes are babbitted. Sometimes steam engine pistons have rings or strips of babbitt, but this is not the rule. Steam turbine shaft bearings are usually babbitt lined.

The term babbitt is used in a rather general way; there are a number of anti-friction metals on the market which are used for the same purpose.

The design of the various bearings is considered in connection with the different engine details of which they form a part.

52. Bearing Proportions.—The bearing formulas in common use are empirical and are not based upon the laws of friction. The most common expression is:

$$pV = C \tag{1}$$

in which p is the pressure in pounds per square inch of projected area, V the velocity of rubbing surfaces in feet per minute, and C a constant which differs for various bearings. The mean pressure is assumed, which may equal the maximum in some cases. In steam- and gas-engine work it is the resultant of all forces acting upon the bearing surface, which, for main and outer bearings includes weight of shaft, wheel, crank, etc., pull of belt and piston thrust; the latter, including inertia, is explained in Par. 103, Chap. XVI.

The piston thrust is often the chief factor in internal-combustion engines, especially in small and medium powers, and consists of the mean pressure due to gas pressure and inertia of reciprocating parts. This will be further treated in Chap. XVI.

A maximum pressure p_M is also prescribed, beyond which it is best not to go. Strength considerations, the most important from the standpoint of safety, are discussed in the chapters dealing with the details involved.

It is apparent that preliminary assumptions must be made, and the results checked by other limiting conditions.

Let P be the total mean load on the bearing, l the length and d the diameter of the bearing in inches, and N the r.p.m. Then:

$$p = \frac{P}{ld} \tag{2}$$

and:

$$V = \frac{\pi d N}{12} \tag{3}$$

Substituting in (1) gives:

$$l = \frac{\pi PN}{12C} = 0.262 \frac{PN}{C} \tag{4}$$

Table 19 gives values of p_M and C which have been used in practice.

TABLE 19

	Bearing	p_M	C
Steam Engine	Main bearing	125 to 200	50,000
	Crosshead pin	1200 to 1500	......
	Crosshead pin—locomotive	3500 to 4800	......
	Crank pin	1000 to 1200	200,000
	Crankpin—locomotive	1200 to 1700	......
	Crosshead shoe	35 to 75	50,000
	Piston	25	30,000
Int. Comb. Eng.	Main bearing	350 to 400	42,000
	Crosshead—or piston pin	1000 to 1800	.....
	Crank pin	1000 to 1700	90,000
	Crosshead shoe	35 to 45	
	Trunk piston	21	

The values of C for the steam engine were given by James Christie in the American Machinist, Dec. 15, 1898, and have been used as a check by the author since that time. For light loads, high speeds and efficient lubrication, it was stated that the values may be doubled.

The values for internal-combustion engines are those given by Güldner, and coincide with those given by other authorities. In all cases it is assumed that steel journals and bearings lined with babbitt are used. Exception is made for crosshead boxes—which are usually of bronze-and crossheads and pistons.

Where the force acting upon a bearing is rapidly alternating, better lubrication is possible, as time is given for the oil to flow between the surfaces when the pressure is removed. This obtains to some extent with all cylindrical bearings of a reciprocating engine, but especially for the connecting-rod bearings; this accounts for the larger value of p_M and C given for crank and crosshead pins. These bearings also have better air circulation due to their motion.

A little computation will show that under the same conditions of operation regarding temperature, lubrication, etc., Formula (1) gives absurd results for widely varying surface speeds unless C is varied with the speed. A formula credited to the late Edwin Reynolds has been

used for the main bearings of engines, and has also been used by the author for heavy-duty mill bearings; it is:

$$p\sqrt{V} = K \tag{5}$$

the notation being the same as for (1). For extremes of speed the constant K must vary, but a given value covers a wider range of conditions than does C in Formula (1). Table 20 from the author's note book will illustrate.

TABLE 20

Service	K
Main bearing—horizontal engine	$2500\sqrt[3]{\frac{Ds}{30}}$
Main bearing—vertical engine	$3000\sqrt[3]{\frac{Ds}{30}}$
Sugar mill gear drive	2500
Sugar mill roll shaft	5200
Freight car journal	4900

The constants for the engine bearings are modified to give a smaller value for small engines, assuming less efficient lubrication; this, however, will probably not hold today in many cases, and the quantity within the radical may be ignored. Neglecting this, the same value is used for slow turning sugar mill gearing as for the horizontal engine. The sugar mill roll shaft works under a steady hydraulic load of over 300 tons at extremely low speed; the diameter is limited due to the housing bolts, and strength considerations reduces its length. Another extreme is the freight car journal, which is of limited size, under heavy pressure and fairly high speed.

Substituting (2) and (3) in (5) gives:

$$l = \frac{P}{K}\sqrt{\frac{\pi N}{12d}} = 0.512\,\frac{P}{K}\sqrt{\frac{N}{d}} \tag{6}$$

In solving directly for the bearing dimensions we may take $l = kd$; then:

$$d = \sqrt[3]{\frac{\pi P^2 N}{12k^2K^2}} = 0.64\sqrt[3]{N\left(\frac{P}{kK}\right)^2} \tag{7}$$

The ratio k will be considered in connection with the details concerned. In main bearings it ranges usually from 1.5 to 2. The smaller values are most used where a large diameter is required for strength. A very common value for stationary steam engines of moderate size is 2.

Steam Turbine Bearings.—Martin gives the values of C in Formula (1) as from 150,000 to 180,000 for stationary turbines driving electrical ma-

chinery, and 90,000 for marine turbines. A. G. Christie, *Trans.* A.S.M.E., vol. 34, p. 435, gives a value of p from 80 to 100 when V is 3600. This gives C from 288,000 to 360,000, and K from 4800 to 6000.

Martin gives a formula due to Nicholson, which, insofar as pressure and speed are concerned, follows the laws of friction of lubricated bearings qualitatively at least; it is as follows:

$$l = \frac{P}{md^{3/4}N^{1/4}} \tag{8}$$

This formula is given here as indicating the possible trend in the design of turbine bearings. The value of m is given as 40. According to Martin, Dr. Lasche successfully ran a bearing $10\frac{1}{4}$ in. in diameter by $4\frac{3}{8}$ in. long, 33 ft. per second with a pressure of 167 lb. per sq. in. of projected area. With the same total load, taking m as 40, (8) gives a length of 1.96, for which the pressure is 372 lb. To adapt (8) to Dr. Lasche's bearing, m must equal 18.3.

A value on m which would border upon conservatism according to present-day practice in the United States might be used without destroying the general rational form of the equation; this would give k, the ratio of length to diameter between about 1.75 and 3, the smaller ratio being for the higher speeds.

To better make a comparison of (4), (6) and (8) as applied to steam turbine bearings with the value of the constants already mentioned, approximate data from an actual turbine will be taken and the results given in Table 21.

$P = 25{,}000$, $N = 750$, $d = 12$ and $V = 2350$.

TABLE 21

Line	Formula	p	C	K	m	l	k
1	Actual	53.5	126,000	2,600	5.45	39.00	3.25
2	(4)	64.0	150,000	....	.....	32.60	2.72
3	(4)	76.8	180,000	....	.....	27.20	2.27
4	(4)	123.0	288,000	....	.....	17.00	1.42
5	(4)	153.0	360,000	....	.....	13.60	1.14
6	(6)	51.2	......	2,500	.....	40.70	3.40
7	(6)	61.5	......	3,000	.....	34.00	2.83
8	(6)	100.0	......	4,800	.....	21.00	1.75
9	(6)	123.0	......	6,000	.....	17.00	1.42
10	(6)	87.0	......	4,200	.....	24.00	2.00
11	(8)	393.0	......	.....	40.00	5.30	0.44
12	(8)	180.0	......	.....	18.30	11.60	0.97
13	(8)	98.0	......	.....	10.00	21.25	1.77

The diameter of the bearing may be found tentatively thus:

$$d = 75\sqrt[3]{\frac{H}{SN}} \tag{9}$$

where H is the horsepower and S the shearing stress (see Chap. XXXII).

It is interesting to note that in line 6 the value of K giving a length about the same as that of the actual bearing (line 1) is the same as for the horizontal engine and the sugar mill gears—a wide range indeed.

Lines 4, 5, 9, 11 and 12 seem extreme when compared with present-day practice, but 2, 3 and 7 are no doubt conservative. Lines 8, 10 and 13 give less conservative values, but there is little doubt but that friction would be less. Formula (6) would increase the length for higher speeds and decrease it for lower—a familiar practice; while (8) would do the reverse. Formula (6) would therefore reduce unit load with increased speed, while (8) would increase it; there are a number of formulas given by different authorities giving results in a similar direction as (8), but of simpler form (see Leutwiler's Machine Design). Formula (8), when m is 40, is as exceedingly generous when applied to heavily loaded, slow-speed shafts, as it is sparing with the turbine bearing, and gives absurd dimensions. In view of successful practice with such bearings, it is obvious that it is not a general rational bearing formula.

It is apparent that all bearing formulas have their limits, but that (6) will adapt itself to a wider range of conditions with results which are now considered practical, than either (4) or (8), especially if a single constant is to be used, although it does not express the laws of friction of lubricated bearings.

Thrust bearings are used to locate the shafts of steam turbines. The amount of thrust is indefinite and their proportions are rather arbitrary. In ship propulsion, they must take the thrust of the propeller; this is due to the fraction q of the horsepower of the engine or turbine which is transmitted to the propeller. In good practice, $p \gtreqless 75$ for $V = 500$, V being the mean speed of the bearing surface. Then $C = 37{,}500$ and $K = 1677$. Let M be the speed of the ship in knots per hour, H the horsepower of the engine or turbine and a the area of the bearing surface in square inches. Then:

$$\frac{6080\,MP}{60 \times 33{,}000} = qH$$

or:

$$P = \frac{308qH}{M}$$

Then:

$$p = \frac{P}{a} = \frac{309qH}{aM}$$

and:

$$a = \frac{308qH}{pM} \tag{10}$$

By (1):

$$p = \frac{37{,}500}{V}$$

and:

$$a = \frac{qVH}{122M} \tag{11}$$

By (5):

$$p = \frac{1677}{\sqrt{V}}$$

and:

$$a = \frac{qH\sqrt{V}}{5.08M} \tag{12}$$

Tables 19, 20 and 21 may be used as a guide in the design of bearings, but it does not seem wise to make dogmatic statements at this stage of bearing formula development. The length of a bearing is often computed, then modified by judgment or by some old rule of thumb; or it may be designed by rule of thumb and checked by one or more formulas. The author has more confidence in Formula (5) and the derived Formulas (6) and (7) than in any other one formula, with the values of K given in Table 20. For steam turbines K may be taken from 3000 to 5000 according to the intrepidity of the designer.

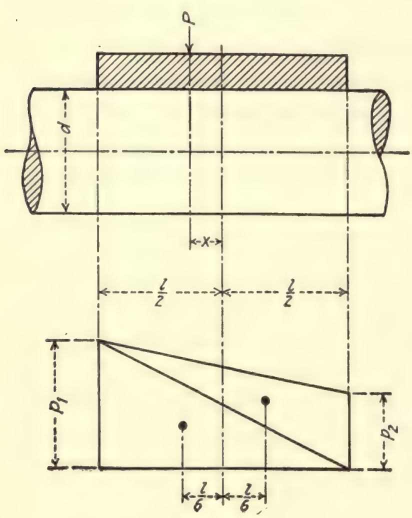


FIG. 70.

Eccentric bearings have sometimes been used to obtain increased bearing surface when space would not allow an increase of length except by eccentric loading. That the desired end—reducing bearing pressure—is often defeated by this arrangement may be shown by the following equations. Fig. 70 is a diagram of a bearing with eccentric load placed

x inches from the center of the bearing. Below is a pressure-distribution diagram which assumes a condition similar to a rigid bearing resting upon an indefinitely large number of springs exactly alike, the deflection of which measures the unit bearing pressure at that point in the bearing length. The actual case is near enough to this to give an idea of the effect of eccentric loading.

The center of area of the load diagram being at x, it remains to find the end ordinates, p_1 and p_2, which will agree with this condition. For this purpose the diagram may be divided into two triangles whose centers of gravity are $l/6$ from the center of the bearing; then equating moments gives:

$$\frac{p_1 dl}{2}\left(\frac{l}{6} - x\right) = \frac{p_2 dl}{2}\left(\frac{l}{6} + x\right) \tag{13}$$

Also:

$$\frac{p_1 + p}{2} \cdot dl = P \tag{14}$$

Rearranging Equations (13) and (14) and adding together gives:

$$p_2 = \frac{P}{dl}\left(1 - \frac{6x}{l}\right) \tag{15}$$

and:

$$p_1 = \frac{P}{dl}\left(1 + \frac{6x}{l}\right) \tag{16}$$

From (15) and (16) it may be seen that if x equals $l/6$, p_2 is zero, while p_1 is twice the mean pressure on the bearing.

53. Forced-feed Systems from Practice.—Few engine and turbine builders manufacture their own lubricating appliances, but these, which are of great variety, are obtained from manufacturers making a specialty of this line. A few catalogues and bulletins are referred to at the end of this chapter. These may be had for the asking, and space will not be used to reproduce the numerous cuts necessary for an adequate description.

Fig. 71 shows a Bowser 7F oil clarifying outfit located in the basement of the engine room, and connected with a gravity tank, forming a continuous oiling system. Oil is forced to the tank by a separate steam pump; it is then led to the bearings, which are also shown fitted with emergency oil cups. The 7F outfit is regarded as a portable outfit for small plants, and is given here to illustrate a simple piping arrangement. The more complete Bowser system will be mentioned later.

Taking a as the area of the wearing surfaces of a thrust bearing, or the

projected area of a cylindrical bearing in square inches, Martin gives for the heat generated by a bearing per hour in B.t.u.:

$$h = a\left(\frac{V}{60}\right)^{1.38} \tag{17}$$

where V is the velocity in feet per minute as before. Then the weight of oil which must be supplied by the pumps in pounds per hour he gives as $\Sigma h/4$, and the cooling surface in the cooler as $\Sigma h/500$ sq. ft. He states that the tanks may have a capacity of $\frac{1}{10}$ the total quantity of oil pumped per hour.

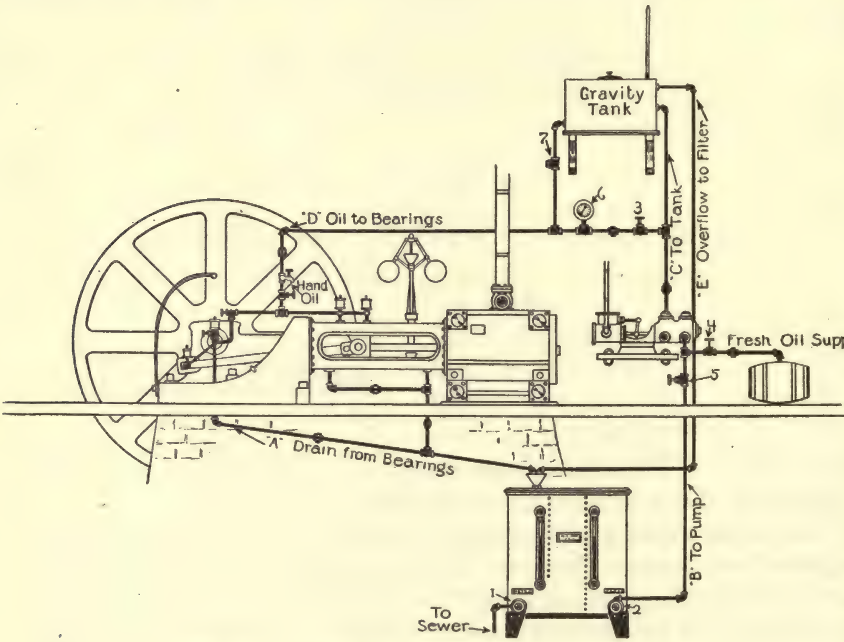


Fig. 71.—Gravity system applied to Corliss engine.

Filtration systems may be classified as follows:

1. *Continuous circulating systems,* in which all the oil used is continuously passed through a filter.

2. *Partial filtration,* in which part of the dirtiest oil is continuously removed from the circulating system, passed through a filter and returned to the system.

3. *Batch filtration,* in which all of the oil is removed and filtered, the machine being supplied with fresh oil while the dirty oil is being purified.

Method (1) may be used on steam and gas engines where large quan-

tities of oil are not needed for cooling as in the case of the steam turbine. For the latter, the filtering surface required would be excessive, and it is unnecessary to pass all the oil through the filter.

Method (2) may be used when excessive quantities of oil are used, as for large steam turbine plants.

Method (3) is used for splash lubrication systems and where ring-oiling bearings are used.

In all clarifying systems, precipitation is used to some extent, but it is a large factor in Method (3).

The systems thus far illustrated in this paragraph have been continuous circulating systems. Another example is the 2F filtration system of the S. F. Bowser Co., Inc., Ft. Wayne, Ind. This system was especially designed for reciprocating-engine plants where basement space is available for the two floor units; viz., the separator and refuse tank, and the filter tank.

A quite generally applicable system is the Bowser 6F filtration system. This is a continuous system with large precipitating capacity. The system was designed to meet several important operating conditions as follows:

1. For power plants where there are no basements and the engines are located on the ground floor.

2. When large volumes of water return with the dirty oil.

3. For general application to large plants requiring an individual filtering and circulating system of considerable capacity, or a central system to which all the units in the power plant are connected.

4. The 6F system can be used with reciprocating steam engines, gas or Diesel engines, and other machinery when a speedy separation of large volumes of impurity is essential. This system is of well nigh universal application and can be specified for any class of lubrication requiring stream feed.

The 6F system is also recommended when it is desired to filter a large number of batches so frequently that it is necessary to pass oil through with a very short time for precipitation. It would thus take the place of the 5F system of batch filtration. A diagram of the 6F system, which may aid in an understanding of its operation is given in Fig. 72.

54. Cylinder Lubrication.—Steam cylinders are lubricated by introducing oil with the steam, sometimes by means of a simple gravity oil cup on the steam chest, but in stationary engines, more often by forcing the oil into the steam pipe by a hydrostatic lubricator. A small pipe from the top of the lubricator connects with the steam pipe about three feet above where the oil enters the steam pipe; the condensed steam in this small pipe gives the head which forces in the oil.

Positive Lubrication.—Hand oil pumps are sometimes used to supplement the hydrostatic lubricator, but these depend upon the engine-man for operation. Probably the most reliable method of lubrication for cylinders, for either steam- or internal-combustion engines is by a positive lubricator operated from some moving part of the engine. The stroke of the pump is adjustable, so that the proper quantity of oil is fed at each stroke of the piston. Such lubrication may feed into the steam pipe or

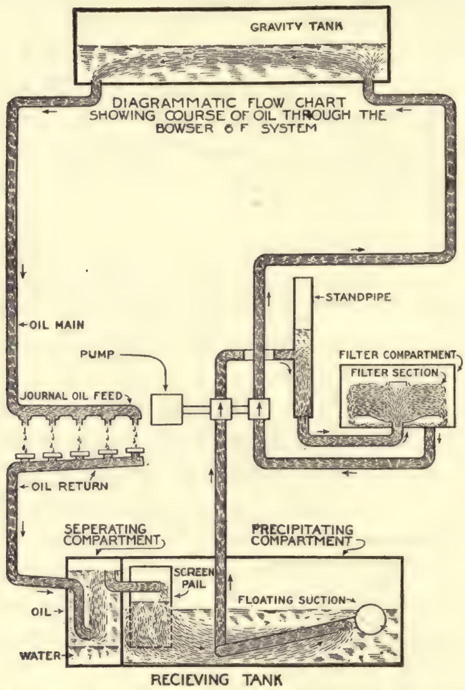


Fig. 72.—Diagram of Bowser 6F. filtration system.

directly into the cylinder, and in some large engines, may have branches going to the valves, especially if superheated steam is used.

In using superheated steam in cylinders previously using saturated steam, the rounding off of the edges of valves and ports has been advised to prevent wiping off the oil.

For high steam pressures and superheated steam, pure mineral oil is best, but it is sometimes difficult to get it to adhere to the surfaces; graphite has been found effective in this case. It is best mixed with oil and may be applied by a graphite lubricator, a description of which may be found in the catalogues cited.

The cylinders of internal-combustion engines are best lubricated by a positive sight-feed oil pump which forces the oil onto the piston between the rings at the crank end of the stroke. The usual place for oil injec-

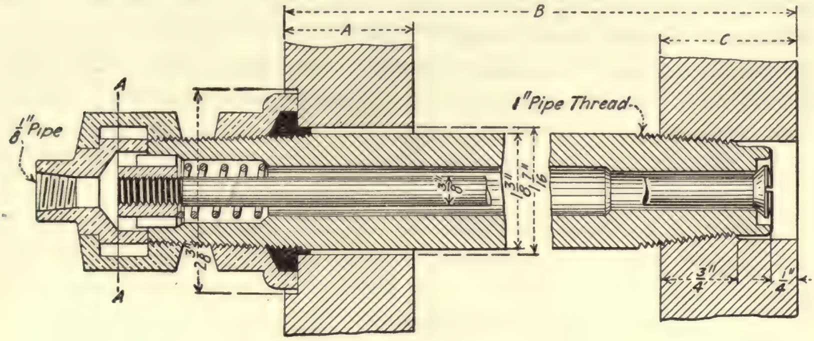


FIG. 73.

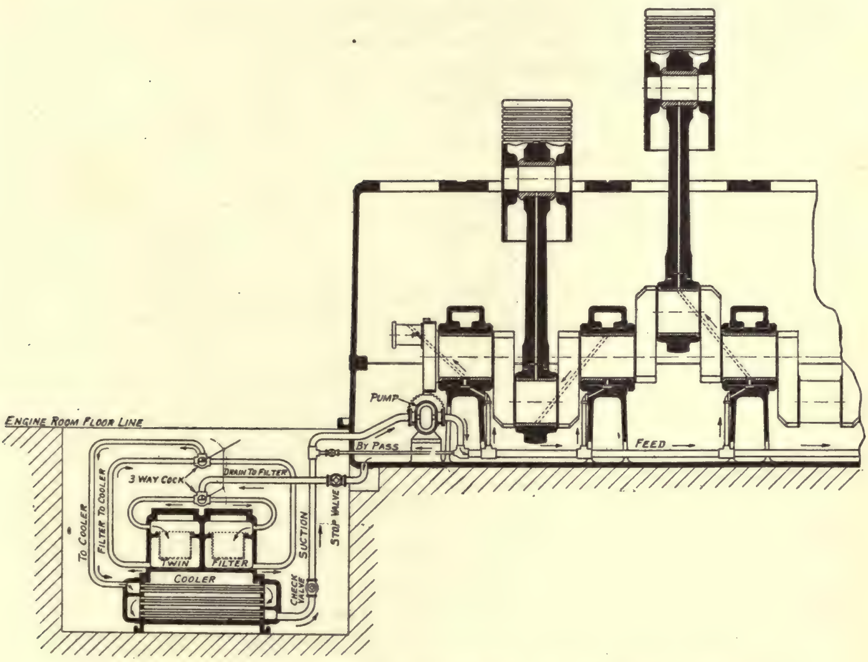


FIG. 74.—Lubricating system of Busch-Sulzer Bros. Diesel engine.

tion is between the first and second rings counted from the head of the piston. Güldner says that oil grooves on the surface of the cylinder are of doubtful value if a forced feed is not used and the forcing of the oil is

not rightly timed. He also says that oil should be supplied at several points on the circumference of the piston in large engines.

A check valve is used in the cylinder wall for forced feed and this must be constructed so as to penetrate the water jacket. Such a valve, furnished by the Richardson-Phenix Co. is shown in Fig. 73.

In many internal-combustion engines, especially those with small cylinders, the cylinders are not directly lubricated; the oiling system of such engines properly comes under Par. 53. In these engines, the bearings have forced feed but the cylinders are lubricated by oil thrown from the cranks.

The system of forced lubrication used on the Busch-Sulzer Bros. Diesel Engine is shown in Fig. 74. The oil flow is indicated by the arrows.

References

Mechanics Applied to Engineering, Goodman.
Laws of Lubrication of Journal Bearings. *Trans.* A.S.M.E., vol. 37, p. 167.
Bulletins of S. F. Bowser and Co., Inc., Fort Wayne, Ind.
Bulletin of The Richardson-Phenix Co., Milwaukee, Wis.
Catalog of Wm. W. Nugent and Co., Chicago, Ill.

PART IV—POWER AND THRUST

CHAPTER XII

THE SIMPLE STEAM ENGINE

Notation.

P = pressure per square inch absolute.

P_B = back pressure in pound per square inch absolute.

P_T = terminal pressure in pounds per square inch absolute.

P_C = compression pressure in pounds per square inch absolute.

P_M = mean effective pressure (m.e.p.) in pounds per square inch.

P_X = maximum total unbalanced pressure exerted by piston.

p = maximum unbalanced pressure per square inch. Also pressure used in general discussion.

p_S = maximum unbalanced pressure per square inch taken as some standard pressure.

V = volume of stroke. Also volume in general.

v = specific volume—cubic feet per pound. Also volume in general.

n = exponent in equation, pv^n = constant.

k = ratio of clearance volume at one end of cylinder to volume stroke.

l = ratio of portion of stroke up to cut-off, to entire stroke; commonly known as cut-off.

x = ratio of portion of stroke up to exhaust closure, to entire stroke; usually called compression.

c = ratio of volume of cushion steam in one end of cylinder at initial pressure, to volume of stroke.

F = ratio of volume of cylinder feed in one end of cylinder at initial pressure, to volume of stroke.

r = ratio of expansion.

r_C = ratio of compression.

C = heat content of one pound of steam (see Chap. VII).

H = indicated horsepower (i.h.p.).

A = effective piston area in square inches acted upon by the steam.

D = cylinder diameter in inches.

D_S = diameter of cylinder when designed for standard unbalanced pressure p_S.
L = stroke of piston in inches.
N = revolutions of engine crank per minute (r.p.m.).
S = mean piston speed in feet per minute.
W = theoretical steam consumption (or water rate) in pounds per i.h.p.-hr. measured from indicator diagram.
w = weight per cubic foot of dry saturated steam.
f = diagram factor—ratio of actual to theoretical m.e.p.
Log_E = hyperbolic, natural or Naperian logarithm; it is the result of integration and used only as a factor.

55. Indicator Diagrams.—The indicator diagram is of great importance in all phases of steam engine design, and due to the fact that the maximum and minimum steam pressures are known with accuracy and the opening and closing of inlet and exhaust valves may be positively effected for any load or speed, a conventional indicator diagram may be safely used for all design problems.

Although the curves employed are not theoretical curves for steam from a thermodynamic standpoint, such a diagram is commonly known as a theoretical diagram to distinguish it from a diagram traced with an indicator, giving actual pressures within the engine cylinder. The difference between the two is treated under diagram factors in a later paragraph.

A conventional diagram is traced through a complete cycle of events in Chap. III, a perusal of which is assumed. An important use of the diagram is the determination of the m.e.p., and to greatly simplify the operation, admission and release are assumed to occur at the extreme ends of the stroke. The discrepancy due to this is usually small except for high-speed engines with single-eccentric gears, running with light loads, and is covered by the diagram factor already alluded to.

The conventional curve most convenient for plotting and calculation has the equation:

$$pv^n = \text{constant}.$$

The exponent n may equal or be greater or less than unity. If n is unity the curve is a rectangular hyperbola, which has been most widely used, and represents with considerable accuracy the expansion curves for saturated steam, and for superheated steam when the superheat is moderate; this may be seen in Fig. 75 which is reproduced from an actual diagram; the hyperbola is plotted from the same point of cut-off. After examining a number of diagrams from engines using 100 degrees superheat, it seems that little is to be gained by using other than the hyperbola for

the conventional curves of superheated steam; the deviation is probably not greater than for some saturated steam diagrams.

The actual point of cut-off on many diagrams is not well defined, as may be seen in Fig. 76. The A. S. M. E. Committee on Standardizing Engine Tests (*Trans.* A. S. M. E., vol. 24, p. 749), in order that this "point may be defined in exact terms for commercial purposes, as used in steam-engine specifications and contracts," recommends that a horizontal line be drawn touching the point of maximum pressure as *pb*, Fig. 2. A hyperbola from *b* to *c*, forming a continuous smooth curve with the expansion line marks the cut-off at *b*. The fraction of cut-off is ab/ag, and is called the *commercial cut-off*. The actual cut-off is some later in the stroke and should be provided for in valve-gear design; the commercial cut-off, however, will be assumed in power calculations.

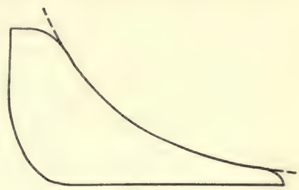


FIG. 75.

To draw the hyperbola through two points of a curve on an actual diagram, a clearance line must be located. Referring to Fig. 77, let x represent the unknown distance to the clearance line, p_1 and p_2 the absolute pressures at the two points and a the horizontal distance between them. Then from the equation of the hyperbola:

$$p_1x = p_2(a + x)$$

from which:

$$x = \frac{p_2a}{p_1 - p_2}.$$

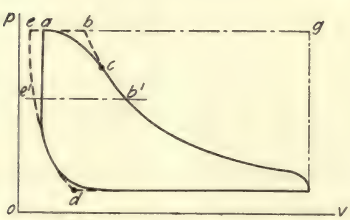


FIG. 76.

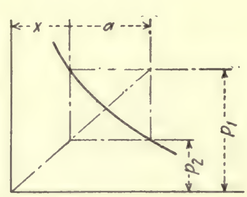


FIG. 77.

It may also be located graphically as shown by the dotted lines. This line will enable us to draw the curve, but may not be the actual clearance line.

The compression curve likewise fails to locate the point of exhaust, and a similar method may be resorted to as shown in Fig. 76. The continuation of the compression curve up to e gives the volume of steam re-

9

tained in the cylinder by exhaust closure, raised to initial pressure. This is known as *cushion steam.* The additional steam admitted to the cylinder up to the point of cut-off is called the *cylinder feed* and its volume is given by the line *eb*. The horizontal distance between the expansion and compression curves at any pressure represents the volume of cylinder feed at that pressure, as *e'b'*. Then had the cylinder feed, after filling the portion of the clearance space not occupied by the cushion steam, been received at maximum pressure, the cut-off would have been the commercial cut-off just described.

It is perhaps more satisfactory for a general non-mathematical discussion to assume the curves *bc* and *ed* to be actual steam curves. Mathematical calculations dealing with them can be considered as approximate only.

56. Mean Effective Pressure.—This may be found from an actual diagram by dividing the area of the diagram in square inches as found by a planimeter, by its length in inches, and multiplying by the indicator spring scale. With the conventional diagram the mean ordinate in terms of pressure may be found by calculation and this process will now be followed step by step.

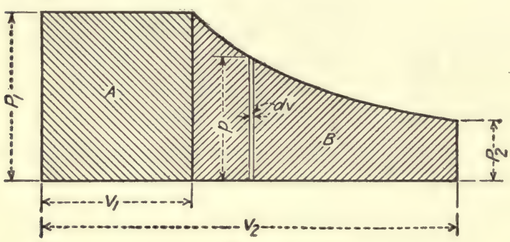


Fig. 78.

The hyperbola, being most used, will alone be considered. The area of the diagram may be best determined by first taking the total area between steam line, expansion curve and the coördinate axes, then subtracting the portions which do not belong to the diagram proper. Notation is given at the beginning of chapter and on the diagrams.

In Fig. 78, area $A = P_1V_1$. From the equation of the rectangular hyperbola with the asymptotes as coördinate axes:

$$pv = P_1V_1 = \text{constant}$$

or:

$$p = \frac{P_1V_1}{v} \qquad \text{also} \qquad r = \frac{V_2}{V_1}.$$

Then:

$$\text{Area } B = \int_{V_1}^{V_2} p.dv = P_1V_1\int_{V_1}^{V_2}\frac{dv}{v} = P_1V_1(\log_E V_2 - \log_E V_1)$$

$$= P_1V_1 \log_E \frac{V_2}{V_1} = P_1V_1 \log_E r.$$

Then:

$$\text{Total area} = \text{area } A + \text{area } B = P_1V_1(1 + \log_E r)$$

The mean ordinate in terms of pressure is:

$$P_M = \frac{\text{total area}}{V_2} = \frac{P_1V_1}{V_2}(1 + \log_E r) = \frac{P_1(1 + \log_E r)}{r}$$

The back pressure of all engines is greater than zero, and neglecting clearance and compression, is represented by a straight line of uniform pressure as in Fig. 79; then:

$$P_M = \frac{P_1(1 + \log_E r)}{r} - P_B \qquad (1)$$

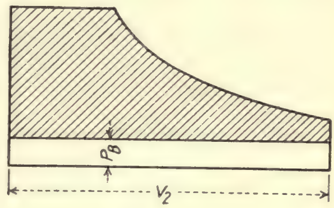


Fig. 79.

Because of its simplicity this formula is often used, r being taken as the reciprocal of the cut-off. This leads to considerable error with early compression.

It is impossible to operate engines without clearance, and practically all engines have a certain amount of compression; therefore, it is more satisfactory to employ a conventional diagram which includes these items. Fig. 80 is such a diagram, being a reproduction of Fig. 78 so far as boundary lines and total area are concerned. The areas A, B and C are subtracted from the total area, leaving the effective area. Substituting the new

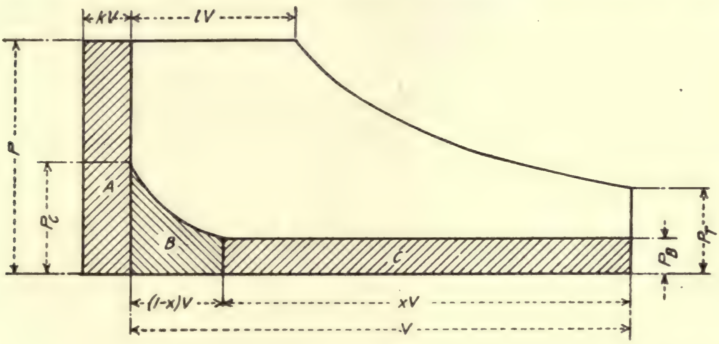


Fig. 80.

notation in the formulas for ratio of expansion and total area of Fig. 80 gives:

$$r = \frac{V + kV}{lV + kV} = \frac{1 + k}{l + k} \qquad (2)$$

and:

$$\text{Total area} = P(lV + kV) + P(lV + kV)\log_E r = PV(l + k)(1+\log_E r).$$

Also:

Area $A = PkV$. Area $C = P_B xV$. Area $B = P_C kV \log_E \frac{(1-x)V + kV}{kV}$.

As the curves are hyperbolas it is obvious that:

$$P_C kV = P_B[(1-x)V + kV] \tag{3}$$

Then:

$$\text{Area } B = P_B V(1 + k - x) \log_E \frac{1 + k - x}{k}.$$

The ratio in this expression is the ratio of compression; or:

$$r_C = \frac{1 + k - x}{k} \tag{4}$$

The effective area of the diagram then is:

$$E = PV[(l + k)(1 + \log_E r) - k] - P_B V[x + kr_C \log_E r_C].$$

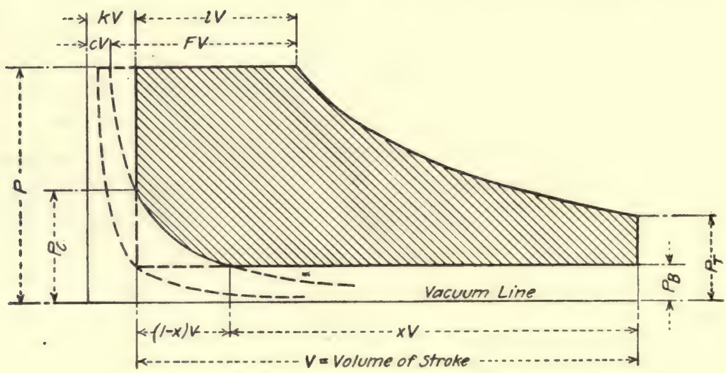


Fig. 81.

Then:

$$P_M = \frac{E}{V} = P[(l + k)(1 + \log_E r) - k] - P_B[x + kr_C \log_E r_C] \tag{5}$$

From (2):

$$l + k = \frac{1 + k}{r}.$$

Then (5) may be written:

$$P_M = P\left[\frac{1 + k}{r}(1 + \log_E r) - k\right] - P_B[x + kr_C \log_E r_C] \tag{6}$$

a form sometimes convenient.

From (3), if x is assumed;

$$P_C = P_B \frac{1 + k - x}{k} = P_B r_C \tag{7}$$

Or if P_C is assumed:

$$x = 1 - k\left[\frac{P_C}{P_B} - 1\right] = 1 - k(r_C - 1) \tag{8}$$

Fig. 81 is a diagram in which the effective area is shaded. The continuation of the compression curve up to initial pressure shows the *cushion steam* cV and the *cylinder feed* FV. It may be easily shown that the product of cylinder feed and pressure is a constant for a given diagram when the curves are hyperbolas, but as this is not a general case it is of little importance.

It is often convenient to know the cut-off which will produce a certain m.e.p. when other data are known. This is especially true when the

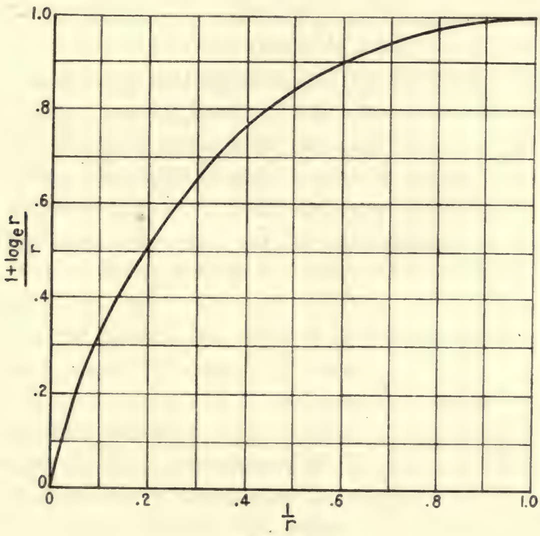


Fig. 82.

engine is of a type having constant compression. Formula (6) may be transposed to read:

$$\frac{1 + \log_E r}{r} = \frac{P_M + Pk + P_B\theta}{P(1 + k)} \tag{9}$$

where

$$\theta = x + kr_C \log_E r_C.$$

With values of

$$\frac{1 + \log_E r}{r}$$

as ordinates and $1/r$ as abscissas, Fig. 82 has been plotted. In using the curve the ordinate is found from (9), the right-hand member of which

contains only known quantities; referring to the chart, $1/r$ is found, and from (2):

$$l = \frac{1}{r}(1 + k) - k \tag{10}$$

As pv is constant for the hyperbola it is obvious from (2) that:

$$r = \frac{P}{P_T} \tag{11}$$

The difference between P_T and P_B is known as *terminal drop*. It is clear that the limiting value of $P_T - P_B$ for operating conditions is zero, and it is usually considered that this should never be less than the m.e.p. which would be required to run the engine without load, or friction m.e.p. This places a maximum limit upon the ratio of expansion. The minimum limit, giving maximum cut-off, depends upon the type of valve gear employed, and if the gear has no limit, upon the conditions of service for which the engine is designed. The cut-off giving maximum economy is usually between $\frac{1}{5}$ and $\frac{1}{3}$, exceeding these limits in some cases. If greatly exceeded in either direction, loss of economy ensues, as may be seen from Fig. 83, which is a water rate curve of a Corliss engine.

Assuming an economical cut-off for the rated power, the chart in Fig. 82 may be used to determine the cut-off required for a certain per cent. of overload.

Example.—Assume an initial pressure of 125 lb. per sq. in. gage and a back pressure of 15 lb. absolute, a clearance of 4 per cent. and compression 0.9 stroke. For constant compression as with a Corliss engine, find the cut-off for 50 per cent. overload if the rated cut-off is $\frac{1}{4}$ stroke (the various ways of expressing the ratios—clearance, compression and cut-off are given here as these are so used in practice; they must all be reduced to fractions or decimals for numerical use in the formulas).

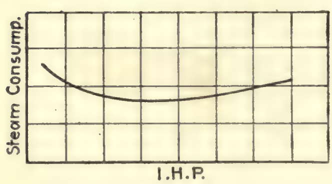


Fig. 83.

The m.e.p. from (5) is:

$$P_M = 140[(0.29 \times 2.278) - 0.04] - 15[0.9 + (0.04 \times 3.5 \times 1.2525)]$$
$$= (140 \times 0.62) - (15 \times 1.075) = 70.7 \text{ lb.}$$

For 50 per cent. overload:

$$P_M = 70.7 \times 1.5 = 106 \text{ lb.}$$

From (9):

$$\frac{1 + \log r}{r} = \frac{106 + 4.32 + 16.1}{145.5} = 0.869.$$

From the chart, the corresponding value of $1/r$ is 0.562, which placed in (10) gives:

$$l = (0.54 \times 1.04) - 0.04 = 0.522$$

This is a longer cut-off than can be obtained under governor control with a single-eccentric Corliss gear, as will be explained in Chap. XX, so a double-eccentric gear, or some other type must be used.

Uniflow Engines.—In applying the formulas of this paragraph to the uniflow engine, the clearance must be determined which will give a compression pressure equal to the initial pressure. Solving for k in (8) gives:

$$k = \frac{1 - x}{\dfrac{P_C}{P_B} - 1}.$$

In determining k for the condensing engine, it is well to limit the value of P_B to not much less than 6 or 7 lb. absolute, even though a higher vacuum is to be used; otherwise, excessive compression pressure is experienced if the vacuum is decreased for any reason. Should a better vacuum be obtained, the cut-off will shorten somewhat. In determining the maximum thrust, the best vacuum (minimum back pressure) should be used.

57. Diagram Factors. *Method* 1.—The A.S.M.E. Rules for Conducting Steam Engine Tests (Code of 1902) contains the following: "The diagram factor is the proportion borne by the actual mean effective pressure measured from the indicator diagram to that of a diagram in which the various operations of admission, expansion, release and compression are carried on under assumed conditions. The factor recommended refers to an ideal diagram which represents the maximum power obtainable from the steam accounted for by the indicator diagrams at the point of cut-off, assuming first that the engine has no clearance; second, that there are no losses through wire-drawing the steam either during the admission or the release; third, that the expansion line is a hyperbolic curve; and fourth, that the initial pressure is that of the boiler and the back pressure that of the atmosphere for a noncondensing engine, and of the condenser for a condensing engine.

"The diagram factor is useful for comparing the steam distribution losses in different engines, and is of special use to the engine designer, for by multiplying the mean effective pressure obtained from the assumed theoretical diagrams by it he will obtain the actual mean effective pressure that should be developed in an engine of the type considered. The expansion and compression curves are taken as hyperbolas, because such curves are ordinarily used by engine builders in their

work, and a diagram based on such curves will be more useful to them than one where the curves are constructed according to more exact law.

"In cases where there is considerable loss of pressure between the boiler and the engine, as where steam is transmitted from a central plant to a number of consumers, the pressure of the steam in the supply main should be used in place of the boiler pressure in constructing the diagrams."[1]

Only the first paragraph is given in the revised code of 1915, in vol. 37, but the diagrams are reproduced from vol. 24.

According to this definition and the accompanying diagrams, the m.e.p. of the reference diagram would be due to the expansion of the cylinder feed to a volume equal to the volume of stroke, and would be calculated by Formula (1), using for the ratio of expansion, the value:

$$r = \frac{1}{l + k - c}.$$

From Fig. 82:

$$P_c = P_B(1 + k - x), \text{ or } c = \frac{P_B}{P}(1 + k - x)\cdot$$

Then:

$$r = \frac{1}{l + k - \frac{P_B}{P}(1 + k - x)} \tag{12}$$

Method 2.—While the simple formula (1) is used in the preceding method to find the m.e.p. of the reference diagram, the value of r given by (12) requires as full a knowledge of the valve setting as the more exact formula (5), and the labor of calculation is reduced but little. Furthermore, Formulas (1) and (12) do not represent actual cylinder operations and there appears to be little logical reason for the assumptions, as it is a physical impossibility to realize the maximum work from the cylinder feed in this way. Neither does this method bear any closer relationship to efficiency when it is remembered that in the use of saturated steam a goodly percentage of the steam actually fed into the cylinder is condensed by the time cut-off is reached, the actual cylinder feed being known only from an elaborate test.

For these reasons the author favors Fig. 7 as a reference diagram, and Formula (5) or (6) for determining the m.e.p. Cut-off and compression are assumed to be as given under commercial cut-off.

Tables of diagram factors are given in engineering handbooks and are usually not the results of recent practice. Kent, Mechanical Engi-

[1] *Trans.* A.S.M.E., vol. 24, p. 753.

neers' Pocket Book, 8th edition, p. 931, following the derivation of the m.e.p. formula says: "The actual indicator diagram generally shows a mean pressure considerably less than that due to the initial pressure and the rate of expansion. The causes of loss of pressure are: (1) Friction in the stop valves and steam pipes. (2) Friction or wire-drawing of the steam during admission and cut-off, due chiefly to defective valve gear and contracted steam passages. (3) Liquifaction during expansion. (4) Exhausting before the engine has completed its stroke. (5) Compression due to early closure of exhaust. (6) Friction in the exhaust ports, passages and pipes.

"Re-evaporation during expansion of the steam condensed during admission, and valve leakage after cut-off, tend to elevate the expansion line and increase the mean pressure.

"If the theoretical mean pressure be calculated from the initial pressure and the rate of expansion on the supposition that the expansion curve follows Mariott's law, pv = a constant, and the necessary corrections are made for clearance and compression, the expected mean pressure in practice may be found by multiplying the calculated results by the factors (commonly called the "diagram factor") in the following table, according to Seaton.

Particulars of engine	Factor
Expansive engine, special valve gear, or with a separate cut-off valve, cylinder jacketed	0.94
Expansive engine having large ports, etc. and good ordinary valves, cylinder jacketed	0.9 to 0.92
Expansive engines with the ordinary valves and gear as in general practice, unjacketed	0.8 to 0.85
Fast running engines of the type and design usually fitted in war ships	0.6 to 0.8."

The portion of the table related to compound engines is reserved for the next chapter.

Method 3.—A diagram giving the maximum possible m.e.p. for steam of any quality expanded to a given terminal pressure is furnished by the Rankine cycle with incomplete expansion. The m.e.p. is given by Formula (11), Chap. VII, and is reproduced here with change of subscripts, as follows:

$$P_M = 5.4\frac{C - C_T}{v_T - \sigma} + P_T - P_B \tag{13}$$

C and C_T are heat contents for initial and terminal pressures respectively and v_T the specific volume at pressure P_T. These values may be found in Peabody's entropy table, or calculated as explained in Chap. VII. The value of σ may be taken as 0.017 and is so small that it may be neglected.

The effect of compression is ignored and no idea given of valve setting other than that required to give the assumed or measured value of P_T.

This method is mentioned more as a matter of interest than with any idea of inviting its adoption, although it possesses advantages if maximum theoretical work in a given cylinder, with a practical economical terminal pressure is desirable as a standard of comparison.

The m.e.p.'s of the reference diagrams for the three methods outlined may best be compared by numerical examples, assuming a certain valve setting.

Example 1.—Let $P = 140$, $P_B = 15$, $l = 0.25$, $k = 0.05$, $x = 0.85$ and the steam be dry saturated at point of cut-off.

Method 1.—From (12), $r = 3.59$ and from (1), $P_M = 74$.

Method 2.—From (2), $r = 3.5$ and from (4), $r_C = 4$. Then from (5), $P_M = 70.9$.

Method 3.—From (11), $P_T = 40$. Then from the entropy table, $C = 1188$.

$C_T = 1092$ and $v_T = 9.695$. From (13), $P_M = 78.8$.

Example 2.—Data the same as before except $x = 0.1$ as for a uniflow engine and $P_C = P$.

Method 1.—From (8), $k = 0.108$. Then $r = 4$ and $P_M = 68.6$.

Method 2.—$k = 0.108$, $r = 3.09$ and $r_C = 9.34$. Then $P_M = 56.1$.

Method 3.—$P_M = 78.8$ as before.

Example 3.—Same as example 1 except that there is no compression and $x = 1$.

Method 1.—$r = 3.39$ and $P_M = 76.7$.

Method 2.—$r = 3.59$, $r_C = 1$ and $P_M = 73.8$.

Method 3.—$P_M = 78.8$ as before.

There is no doubt that in all cases the results of Method 2 are more nearly like the values from an actual indicator diagram; the diagram factor would be nearer unity and less error would follow if too high a value were selected.

58. Governing.—In practically all stationary engines the speed is constant except for the slight variation necessitated by the practical operation of the governor. Neglecting this, the m.e.p. is proportional to the power of the engine, and possible load variations may be predetermined by the design of the conventional indicator diagram, which must precede valve-gear design.

Small engines are often governed by *throttling* the steam; this changes the initial pressure, the valve events being the same for all loads. The conventional diagram for several different loads is given in Fig. 15, Chap.

III. The maximum load is limited by the maximum initial pressure, or pressure in the steam line; the minimum load by the allowable terminal pressure P_T discussed in Par. 56. The total range of load may thus be determined by finding the maximum and minimum m.e.p., and some intermediate load selected as the rated load, which will allow a certain overload. The initial pressure required for the rated load may be found by solving for P in Formula (5); or:

$$P = \frac{P_M + P_B(x + kr_C \log_E r_C)}{(l + k)(1 + \log_E r) - k} \tag{14}$$

As the cut-off is fixed, it is usually later in the stroke than is conducive to the best economy, in order to increase the maximum capacity of a cylinder of given size. To have equal capacity with an automatic cut-off engine of equal size, speed and pressures, the fixed cut-off of a throttling engine must be the same as the maximum cut-off of the automatic engine; this may be about $\frac{3}{4}$ stroke. When but small overload capacity is required, a shorter cut-off may be used, resulting in better economy—in some cases exceeding that of the automatic cut-off engine of the same power, especially at light loads.

The *automatic cut-off* engine is governed by changing the cut-off. In the Corliss engine and some four-valve engines with two eccentrics, the compression is constant, the cut-off only being changed to suit the load. This is illustrated in Fig. 16, Chap. III. The limits of allowable load are the maximum cut-off and minimum terminal pressure discussed in Par. 56.

The chart of Fig. 82 may then be used to determine the cut-off for the rated load which will allow a certain overload; if this cannot be fixed within the economical limits—usually from $\frac{1}{5}$ to $\frac{1}{3}$ cut-off—a compromise must be made or a different type of gear chosen.

With single-valve engines and four-valve engines with a single eccentric, the compression is changed with the cut-off; as both affect the area of the diagram in the same way, less change of cut-off is necessitated for the same change of load. A diagram for this type is given in Fig. 17, Chap. III. The compression having been decided upon for a given cut-off, it can only be found for another cut-off by the use of a valve diagram, therefore the chart in Fig. 82 can not be used for determining the cut-off for rated load, and it must be found by trial and error.

A combination of throttling and change of cut-off is sometimes used, but as the relation between cut-off and initial pressure may not be predetermined except for maximum load, such engines must be designed for maximum power and correct relations obtained by adjustment

after the engine is built. This combination is sometimes obtained unintentionally, the throttling being due to restricted port opening at short cut-off. This method is shown in Fig. 18, Chap. III.

As more fully explained in Chap. XIX, slight changes of speed accompany changes of load; assuming constant boiler and exhaust pressures, speed increases with decrease of load, and decreases with increase of load for both throttling and automatic cut-off engines. This is not on the principle of the work of a horse, which can run faster with a light load and must slow down when given a greater load; it is simply because the governor must run at a given speed to set the valve gear for the cut-off—or with the throttling engine, throttle the steam pressure—necessary to carry a certain load. It is obvious that not only change of load, but anything which influences the setting of the valve gear may produce these slight speed changes; if an engine changes while running, from condensing to noncondensing without change of load, it will slow down slightly, because the back pressure has been increased, and to keep the same m.e.p. the cut-off must be lengthened; to effect the required position of the gear the governor must run slower. In other words, removing part of the work-producing force is equivalent to increasing the load as far as it affects the valve-gear adjustments. Likewise, lowering the steam pressure slows the engine for the same reason and not because there is not sufficient pressure to carry the load, provided the change is not outside the limit of adjustment.

59. Piston Thrust.—An important consideration in engine design is the maximum force exerted by the piston, or piston thrust. This is the product of the maximum unbalanced pressure per square inch p and the effective piston area A in square inches. The unbalanced pressure is the difference between the pressure on the two sides of the piston and is measured between the steam or expansion line, which together form the forward-pressure line, of one diagram, and the back-pressure line of the other. This is shown in Fig. 84, the full lines representing the pressures on opposite sides of the piston during one stroke, and the dotted lines during the other. Such a diagram is called a *stroke diagram.*

The maximum piston thrust is then:

$$P_X = pA \tag{15}$$

If D is the diameter of the cylinder in inches, and the sectional area of the piston rod be neglected as being on the safe side:

$$P_X = p \cdot \frac{\pi D^2}{4} = (P - P_B)\frac{\pi D^2}{4} \tag{16}$$

The force required to accelerate the reciprocating parts of an engine, known as the inertia of the reciprocating parts, offsets a portion of the steam pressure. This is discussed in Chap. XVI but is shown by Fig. 85, which is drawn by plotting the distance between the two full lines of Fig. 84 from a horizontal line. The curve ABC is the inertia curve, and the effective pressures transmitted to engine parts are measured between

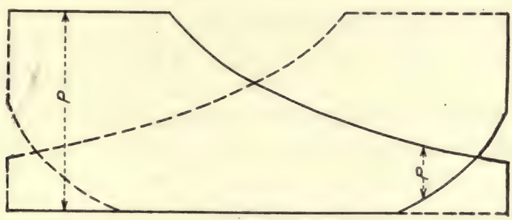


Fig. 84.

the two lines as shown by p_E. It may be seen that if cut-off is long enough to lie over or beyond B, as shown by the dotted line—which is usual in practice for overloads—the inertia has no offsetting effect upon the steam pressure, and the maximum unbalanced pressure is as given by (15) and (16), or even greater.

The effect of inertia is quite fully discussed in Chaps. XVI and XXI (the last paragraph of the latter).

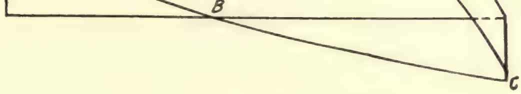


Fig. 85.

60. Indicated Horsepower.—In Par. 23 of Chap. VI, general Formula (5) is derived for the i.h.p. of one working cylinder end of any piston heat engine.

The formula is:

$$H = \frac{2 \times 144}{33{,}000} \cdot \frac{P_M v_S N}{m} \tag{17}$$

If L and D are stroke and diameter of piston respectively:

$$v_S = \frac{LA}{1728}$$

and as m is 2 for steam engines, (17) becomes:

$$H = \frac{P_M LAN}{396,000} \tag{18}$$

Letting H and C denote the head and crank ends respectively, the total power of a single-cylinder engine is:

$$H = H_H + H_C = (P_{MH}A_H + P_{MC}A_C)\frac{LN}{396,000} \tag{19}$$

Formula (19) is general for steam engines; if the engine is single-acting P_{MC} is zero. In double-acting engines the piston rod reduces the piston area at the crank end, and if a tail rod is used the head-end area is also reduced. Formula (19) must be used for determining the power from a test.

For the purpose of design it is more convenient to assume the areas A_H and A_C equal. The piston rod may be neglected and the discrepancy provided for by the diagram factor; or a percentage of the piston area may be deducted which will be approximately correct for most practical cases. In special cases with exceptionally large rods, more careful calculation must be made.

In general, if d_H is the diameter of the tail rod, or extension for tandem engine, and d_C is the diameter of the main rod; and if P_M is the same in both ends of the cylinder, which is always assumed in designing, an equivalent area which would give the same total power is:

$$A_E = \frac{\pi}{4}\left[D^2 - \frac{d_H^2 + d_C^2}{2}\right] = \frac{\pi}{4}\delta D \tag{20}$$

Then (19) becomes, for double-acting steam engines:

$$H = \frac{P_M L A_E N}{198,000} = \frac{\delta P_M D^2 LN}{252,100} \tag{21}$$

Piston speed is usually more convenient than stroke and r.p.m in. design. If this is denoted by S:

$$S = \frac{LN}{6} \tag{22}$$

Then (21) becomes:

$$H = \frac{\delta P_M D^2 S}{42,000} \tag{23}$$

If $d_H = d_C = \frac{D}{5}$, which is good proportion for average practice,

$$\delta = 0.96 \text{ with tail rod,}$$
$$\delta = 0.98 \text{ without tail rod.}$$

Assuming a mean of these two values to apply to double-acting engines with or without tail rods with close approximation, and $\delta = 1$ for single-acting engines, special equations may be written.

Case 1.—Single-acting, single-cylinder.

$$H = \frac{P_M D^2 LN}{504,200} = \frac{P_M D^2 S}{84,000} \tag{24}$$

From which:

$$D = 710\sqrt{\frac{H}{P_M LN}} = 290\sqrt{\frac{H}{P_M S}} \tag{25}$$

Case 2.—Double-acting, single-cylinder.

$$H = \frac{P_M D^2 LN}{260,000} = \frac{P_M D^2 S}{43,300} \tag{26}$$

From which:

$$D = 510\sqrt{\frac{H}{P_M LN}} = 208\sqrt{\frac{H}{P_M S}} \tag{27}$$

The relation between S, L and N may be determined from (22); if N is fixed by such conditions as direct-connection to an electric generator, S or L may be assumed. Should the ratio L/D not prove desirable, S and L may be changed and a new value of D found.

If P_M is the theoretical value it should be multiplied by a diagram factor.

The effect of piston speed, ratio of stroke to diameter and the factors affecting P_M upon economy is discussed in Par. 47, Chap. IX. Their effect upon capacity may be seen from Formulas (24) and (26). For a given piston speed, long stroke means less capacity for a given weight; for a given rotative speed, long stroke means reduced cylinder diameter and piston thrust, and greater capacity for a given weight. Conservative designers prefer to keep the piston speed within 800 ft. per min., this having been considered a maximum a short time ago; but with materials now available there is little doubt but that 1000 ft. may now be considered conservative for well-constructed engines of good design. This value has been far exceeded in special cases for stationary engines, while locomotives develop as high as 1700 ft. per min.

Rotative speeds vary greatly with size and type, limitations as to the latter applying generally to releasing valve gears; however, certain builders list releasing-gear engines with speeds up to 200 r.p.m.

Ratios of stroke to diameter vary for simple engines from 1 to 3, sometimes slightly exceeding these limits.

High steam pressure is discussed in Chap. IX; this is largely a question of boiler construction and operation.

From Formulas (5) and (6) it is apparent that a given valve setting will produce a certain theoretical m.e.p. Then from (26) it appears that with a given valve setting and cylinder diameter the i.h.p. will increase with the piston speed.

This is true within certain limits, and the range of speed over which this applies depends largely upon the capacity of steam passages and ports to provide for easy steam flow; but in any case, even though there were no constructional limitations, a very high speed would result in wire-drawing through ports which would decrease the m.e.p., and a point would be reached where the product $P_M S$ would no longer increase, but on the contrary, may decrease.

Practical considerations which involve both port and boiler capacity thus limit the horsepower of locomotives. The tractive power, which is the mean effort at the rail, not including friction, is computed by the American Locomotive Co. for a piston speed of 250 ft. per min., and a mean cylinder pressure 85 per cent. of the boiler pressure. For higher speeds this is multiplied by speed factors less than unity. The product of speed factor and piston speed—and therefore the horsepower—increases for saturated-steam locomotives, up to 700 ft. piston speed, remaining constant to 1000 ft., after which it gradually decreases. For superheated-steam locomotives the maximum is reached at 1000 ft. and maintained up to 1600 ft., which is as high as the table goes. The maximum i.h.p. for saturated steam locomotives is:

$$H = 0.0212 P_1 A$$

and for superheated steam:

$$H = 0.0229 P_1 A$$

where P_1 is boiler pressure by gage and A the area of one piston in square inches.

At 1600 ft. piston speed the superheated locomotive has 25 per cent. greater capacity than the saturated locomotive, to say nothing of the increase in economy.

In designing stationary engines, the best guarantee against reduction of power by wire-drawing is to proportion the ports so that excessive steam velocity is not necessary. This will be discussed in Chap. XX.

61. Theoretical steam consumption may be computed from either an actual or theoretical diagram. It is the measure of the cylinder feed, neglecting condensation and quality, although it may be applied in case of initial superheat. The total weight of steam in the cylinder is usually measured at cut-off, while the cushion steam is taken at back pressure. Such a computation is of little practical value as it is only a

partial indication of economy; a correction factor, involving as it does so many variables, is also of doubtful value. However, theoretical steam consumption indicates whether a certain steam distribution aids or counteracts any measure for improvement of economy.

Let w = weight per cubic foot of steam at initial pressure.

w_B = weight per cubic foot of steam at back pressure.

W = weight of steam per i.h.p.-hr.

The weight of steam per cycle of two strokes for one cylinder end is:

$$w(l + k)\frac{LA}{1728} - w_B(1 + k - x)\frac{LA}{1728}.$$

As there are $60N$ cycles per hour, the total weight per hour is:

$$\frac{LAN}{28.8}[w(l + k) - w_B(1 + k - x)].$$

Dividing this by the i.h.p. of a single-acting engine given by (18) gives the water rate per i.h.p.-hr.; or

$$W\ \frac{13{,}750}{P_M}\left[w(l + k) - w_B(1 + k - x)\right] \tag{28}$$

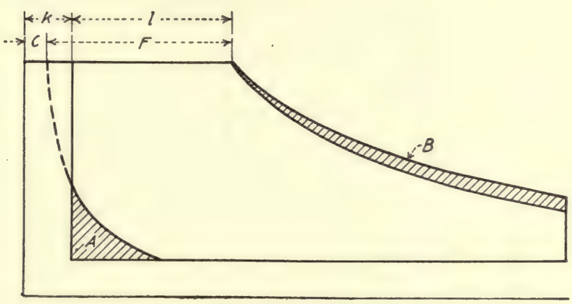


Fig. 86.

If P_M is for a conventional diagram it should first be multiplied by a diagram factor.

62. Compression.—The capacity of an engine cylinder of given size is not affected by clearance if the expansion of the clearance steam is equal to the unbalanced work of compression, or if the two shaded areas of Fig. 86 are equal. The volume of the stroke is unity. The lower expansion line is without clearance while the upper one is for clearance. Then, assuming expansion and compression to be hyperbolic, the unbalanced work of compression is:

$$\begin{aligned} A &= P_C k \log_E r_C - P_B(r_C k - k) \\ &= P_B k[1 + r_C(\log_E r_C - 1)] \end{aligned}$$

The work of clearance steam is:

$$B = P\left[(l+k)\log_E \frac{1+k}{l+k} - l\log_E \frac{1}{l}\right]$$

Equating and solving for a term containing r_C gives:

$$r_C(\log_E r_C - 1) + 1 = \frac{P}{kP_B}\left[(l+k)\log_E \frac{1+k}{l+k} - l\log_E \frac{1}{l}\right] \quad (29)$$

The right-hand member of (29) contains only known quantities. The corresponding values of r_C may be found from Fig. 87.

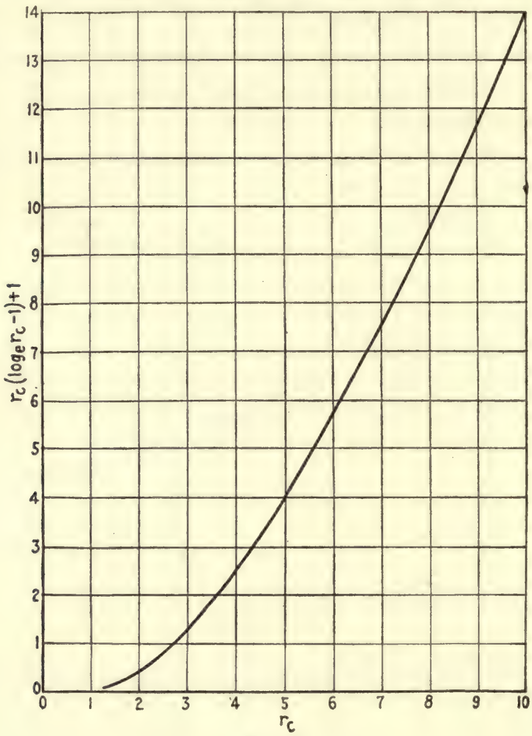


FIG. 87.

Theoretical indicated economy is the ratio of work done to steam supplied, or more properly, to heat supplied. For given pressures and conditions of steam this may be expressed thus:

$$E = \frac{P_M}{F} \quad (30)$$

From Fig. 86:

$$F = l + k - c = l + k\left(1 - \frac{P_B}{P}r_C\right) = l + k\left(1 - \frac{P_C}{P}\right) \quad (31)$$

Some numerical examples, the results of which are contained in Tables 22, 23 and 24, give some idea of the way E is affected by different conditions, but it must be remembered that the equations of this paragraph, as in the one previous, only show whether a certain cycle of valve events tends for or against economy; other features, mentioned in Chap IX, may more than offset the effects apparent from these equations. Table 22 gives values of r_C from Formula (29) and the corresponding values of E for three cut-offs; also E for the imaginary condition of zero clearance, and the ratio of compression to initial pressures. The economy for the clearance assumed (4 per cent.) is 95 per cent. of that for zero clearance in all cases. The work, of course, is the same.

If cut-off were full stroke when k is zero, E would be 125, which, compared with Table 23, incidentally shows the advantage of using steam expansively.

Table 23 gives E for 4 per cent., 8 per cent. and for zero clearances and (for the former two) r_C from Formula (29); also for the case of no compression, or $r_C = 1$, and when compression is carried to the initial pressure. When r_C is calculated from (29), giving the same work as

TABLE 22

l	$P=140$ $P_B=15$			
	$k=0.04$, and r_C from (29)		$k=0$	P_C/P
	r_C	E	E	
0.2	6.70	275	290	0.717
0.3	6.36	245	258	0.574
0.4	3.95	218	230	0.423

TABLE 23

r_C	$P=140$	$P_B=15$	$l=0.3$
	E for $k=$		
	0.04	0.08	0
r_C from (29)	245.00	232.00	
$r_C = 1$	239.00	222.00	258
$r_C = P/P_B$	242.50	224.00	

zero clearance, E is the greatest, and from calculations with r_C a little on either side of those given by (29) it is apparent that the results of this equation give conditions of maximum economy for a given clearance; also, when compression is thus determined, clearance has less effect

upon economy; *i.e.*, for this case, the 8 per cent. clearance gives 95 per cent. of the economy of the 4 per cent. clearance and with no compression this ratio is 93 per cent.; when the compression is carried to initial pressure the ratio is 90 per cent., which points to certain advantages of the uniflow principle which enable it to overcome this apparent drawback. However, with shorter cut-offs, such as obtained for rated load for the uniflow, the high compression is not so far from that given by (29); nevertheless, the equation indicates that if the clearance can be reduced without counteracting other economical features, a gain in economy might be expected.

In Chap. IX certain tests were mentioned in which economy was improved by carrying the compression pressure to 45 per cent. of the initial pressure. From Table 22 it may be seen that (29) gives this ratio for a cut-off some less than 0.4 stroke when the clearance is 4 per cent.

TABLE 24

l	$P=140$	$P_B=15$	$k=0.06$	$P_C=P$ when $l=0.05$
	From (29)		By valve gear	
	x	r_C	x	r_C
0.1	0.53	8.80	0.56	8.22
0.2	0.66	6.56	0.64	6.90
0.3	0.75	5.17	0.72	5.66
0.4	0.80	4.25	0.77	4.83
0.5	0.85	3.45	0.82	4.08
0.6	0.89	2.85	0.86	3.33
0.7	0.94	2.00	0.87	2.90

Table 24 gives the theoretical values of r_C and x for a single-valve engine with cut-offs ranging from 0.1 to 0.7 stroke; also in actual values produced by the valve gear for the conditions named in the table, assuming a shifting eccentric with constant lead and neglecting the angularity of the connecting rod. The method of determining these values is given in Chap. XX.

The value of E varies but little for quite considerable deviations of r_C, so that it appears from Table 24 that a single-valve engine or its equivalent in steam distribution, provides for the maximum value of E for a given cut-off, if not offset by other conditions mentioned in Chap. IX; however, the fact that in Table 23 the maximum deviation of E from the value given by (29) is only 2.5 per cent. when the clearance is 4 per cent., and but 4.2 per cent. for 8 per cent. clearance, may account for the

superior economy of certain types of engines with a constant compression which may not agree with that given by (29) for loads usually carried by the engine.

The indications of this paragraph are probably more nearly fulfilled in large cylinders, or when steam jackets or superheated steam is used. In small cylinders the larger ratio of surface to volume makes high compression less desirable, especially if the engine is not of high rotative speed.

63. Standard Engines.—Most steam-engine parts may be standardized to the great advantage of both the engineering department and shops. This may be done by designing a line of engines to carry some standard maximum unbalanced pressure per square inch which will be denoted by p_S. This pressure acting upon the piston of a standard cylinder of diameter D_S will give the same maximum thrust upon the piston rod as some actual pressure p upon the piston rod of a cylinder of actual diameter D. This thrust is given by (16).
Equating gives:

$$P_X = p_S\frac{\pi D_S{}^2}{4} = p\frac{\pi D^2}{4}.$$

From this:

$$D_S = D\sqrt{\frac{p}{p_S}} = KD \qquad (32)$$

Then all engine parts excepting those pertaining directly to the cylinder would be the same as for the standard engine with a cylinder diameter of D_S. The end of the frame pattern containing the flange for attachment to the cylinder may be a loose piece, so that the frame may be attached to cylinders of different diameter by using other flange pieces.

To illustrate, let it be desired to determine dimensions of engine parts for a cylinder 24 in. in diameter, carrying a pressure of 150 lb. gage and exhausting to atmosphere. Then p is 150. Let the standard pressure be 125 (or $p_S = 125$). Then from (32):

$$D_S = 24 \times \sqrt{\frac{150}{125}} = 26.3 \text{ in.}$$

When the standard cylinders are not in fractions of an inch, it is safer to select the next largest whole number; this would give D_S as 27 inches. Should the standards be based upon diameters which are multiples of 2, the next larger is 28 inches, but it is probable that 26 inches might be used in this case, the usual factor of safety taking care of the slight discrepancy. Then for the 24-in. engine for a pressure of 150 lb., the parts would be the same as for the standard 26-in. engine.

Pins, rods, etc., may be designed and tabulated for the full line of cylinders varying by 1 or 2 inches, but the frame pattern may cover a number of sizes when the advantage of reducing the number of patterns outweighs the cost of using a heavier frame than is necessary for the smaller engines using the same pattern. In this case the frame must be designed for the largest standard cylinder with which it is to be used. The diameter of the bearings may be varied by the box patterns, and the length, by varying the boss on the wheel side of the bearing; this is sometimes necessary even if the pressure is not changed, as may be seen from Chap. XXVIII.

The same crank and crosshead patterns may also be used for the range of sizes covered by the frame. The smaller this range, the better.

64. Application of the formulas derived in this chapter will be made in the following example. In such work it is sufficiently accurate to take 15 lb. as atmospheric pressure.

Example.—Design a simple, noncondensing Corliss engine to develop 450 i.h.p. with an initial gage pressure of 125 lb. and a piston speed of 800 ft. per min. Let the cut-off be ¼ stroke, the compression 0.8 stroke and and assume the clearance to be 4 per cent. Let the diagram factor be 0.9.

From (5):

$$P_M = 140[(0.29\times 2.278) - 0.04] - 15[0.8 + (0.04\times 6\times 1.79)] = 68.4 \text{ lb.}$$

Let 70 per cent. overload be provided for; then:

$$P_M = 1.7 \times 68.4 = 116 \text{ lb.}$$

From (9):

$$\frac{1 + \log_E r}{r} = \frac{116 + 4.32 + 18.4}{145.5} = 0.953.$$

From the chart, Fig. 82, $1/r$ is 0.71; then l is 0.7, and a long-range cut-off must be used for such an overload. The limit of cut-off and corresponding overload for a single-eccentric Corliss engine will be explained in Chap. XX.

From (27), including the diagram factor:

$$D = 208\sqrt{\frac{450}{0.9 \times 68.4 \times 800}} = 19.85, \text{ say } 20 \text{ in.}$$

A good ratio of L to D is obtained if L is 48; then from (22) N is 100, and the size of the engine is:

$$20'' \times 48'' - 100.$$

This may be taken as one of a series of standard engines designed for a standard unbalanced pressure of 125 lb.

In tabulating engines for power it is convenient to omit the diagram factor, as this may vary for different conditions. Initial pressure is usually taken as boiler pressure, but if a long steam line is used there will be a pressure drop and the diagram factor will be less. Taking P_m as the theoretical m.e.p. in (26), we may write, where f is the diagram factor:

$$H_T = \frac{H}{f} = \frac{P_M D^2 S}{43{,}300}$$

where H is the actual required i.h.p. and H_T the tabular value; the diagram factor may be chosen to suit any particular case.

If 125 lb. is taken as standard unbalanced pressure, the value of K in (32) may be tabulated for other pressures as in Table 25.

TABLE 25

p	100	110	120	130	140	150	160	170	180	190	200
K	0.895	0.938	0.980	1.02	1.06	1.10	1.13	1.17	1.20	1.24	1.27

CHAPTER XIII

THE COMPOUND STEAM ENGINE

Notation.

P = absolute pressure in pounds per square inch.

P_H = m.e.p. in high-pressure cylinder.

P_L = m.e.p. in low-pressure cylinder.

P_K = an arbitrary pressure used in Formula (10); value from 3 to 6.

P_X = maximum total unbalanced pressure transmitted by piston rod to engine parts.

p = terminal pressure in pounds per square inch, absolute. Also pressure in general.

p_S = standard unbalanced pressure per square inch used in designing standard simple engines.

V = volume of stroke used on diagrams.

v = volume used in general discussion.

d = terminal drop in pounds per square inch.

k = ratio of clearance in one end of cylinder to volume of stroke.

l = ratio of stroke up to cut-off to entire stroke.

x = ratio of stroke up to exhaust closure to entire stroke.

c = ratio of cushion steam in one end of cylinder at initial pressure to volume of stroke.

a = difference in volume of low-pressure and high-pressure cushion steam at receiver pressure.

f = diagram factor (see Par. 57, Chap. XII).

m = ratio of maximum absolute pressure in low-pressure cylinder to pressure at cut-off.

q = ratio of receiver volume to volume of stroke of low-pressure cylinder.

R = cylinder ratio—ratio of low-pressure to high-pressure volume of stroke.

r_T = total ratio of expansion of cylinder feed.

r = ratio of expansion in one cylinder.

r_C = ratio of compression in one cylinder.

H = i.h.p.

D = cylinder diameter in inches.

D_S = diameter of standard simple engine cylinder, which with

pressure p_S will give the same maximum thrust on piston rod as compound engine.

S = mean piston speed in feet per minute.

W = theoretical water rate—pounds per h.p.-hr.

w = weight per cubic foot of steam.

Log_E = hyperbolic, natural or Naperian logarithm.

Subscripts H and 1 refer to high-pressure cylinder while L and 2 refer to low-pressure cylinder. For triple-expansion engines, 1 and 2 refer to intermediate cylinder while L and 3 refer to low-pressure (see diagrams for notation).

65. Indicator Diagrams.—Generally speaking, the economy of the compound engine is so much superior to that of the simple engine that it is used in most important engine installations notwithstanding its greatly increased first cost. A description of the compound engine, with a statement of the general principle of operation is given in Par. 3, Chap. III, while a discussion of its economical advantages is given in Par. 47, Chap. IX; these may be considered as introductory to this chapter, although in some measure dependent upon a knowledge of a portion of its contents.

A simple but practical method of determining pressure and volume relations will be given first, based upon the following assumptions:

(1) That the expansion and compression curves are hyperbolas.

(2) That the work is equally divided between the high-pressure and low-pressure cylinders.

(3) That cut-off, compression and clearance are the same in both cylinders.

(4) That the receiver volume is so large that change of pressure due to exhaust from the high-pressure cylinder and intake of steam of the low-pressure cylinder may be neglected, these lines being straight horizontal lines which coincide.

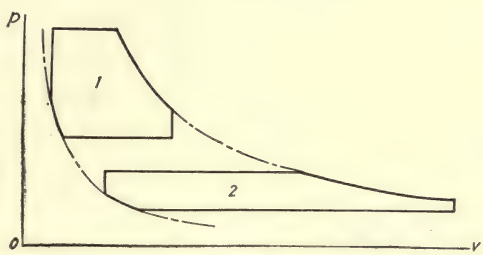


Fig. 88.

With diagrams and formulas established upon this basis, practical modifications may be made. It is of course assumed that a condition of equilibrium obtains between cylinder and receiver pressures, the effect upon these due to starting and change of load being considered later. Conventional diagrams for the foregoing conditions may be arrived at as follows:

If all pressures of diagram 1, Fig. 88 are multiplied by a constant and

the corresponding volumes divided by the same constant, and the values plotted to the same scale, diagram 2 would be produced with an area equal to that of diagram 1. If this constant be so chosen that the lower boundary line of diagram 1 coincides with the upper line of diagram 2, the required conventional diagram will be formed, as shown in Fig. 89.

The cylinder ratio, or ratio of volume of stroke of low-pressure to that of the high-pressure cylinder is:

$$R = \frac{V_2}{V_1}.$$

It is obvious that R is the constant previously referred to, and that:

$$\frac{P_1}{R} = P_2 \tag{1}$$

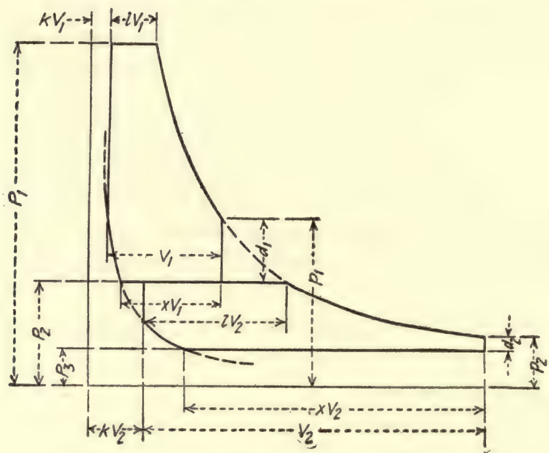


FIG. 89.

and:

$$\frac{P_2}{R} = P_3 = \frac{P_1}{R^2} \tag{2}$$

From (2):

$$R = \sqrt{\frac{P_1}{P_3}} \tag{3}$$

The receiver pressure is given by (1), after having found the cylinder ratio from (3). The total ratio of expansion is found from the pressure ratio (for the hyperbola), p_2 being usually assumed in compound engine design; then:

$$r_T = \frac{P_1}{p_2} = \frac{RP_2}{p_2} = Rr \tag{4}$$

From which:

$$r = \frac{r_T}{R} \tag{5}$$

and from (10), Chap. XII:

$$l = \frac{1 + k}{r} - k \tag{6}$$

After assuming the value of x, the m.e.p. of the high-pressure cylinder is:

$$P_H = P_1\left[\frac{1 + k}{r}(1 + \log_E r) - k\right] - P_2\,[x + kr_C \log_E r_C] \tag{7}$$

The value of r_C may be determined from (4), Chap. XII.

From (1) and (2), and the fact that the quantities in brackets are the same in both high-pressure and low-pressure cylinders, it is clear that:

$$P_L = \frac{P_H}{R} \tag{8}$$

The terminal drop in the high-pressure cylinder is:

$$d_1 = Rd_2 \tag{9}$$

66. Condensing Engines.—For these engines the value of P_3 is sometimes so small that the value of R given by (3) is excessive, necessitating the selection of some other value. To preserve uniformity, Formula (3) may be used by replacing P_3 by some limiting value, the actual value of P_3 to be used should it equal or be greater than the limiting value, which may be denoted by P_K, and which may have a range of from 3 to 6, according to the judgment of the designer; then:

$$R = \sqrt{\frac{P_1}{P_K}} \tag{10}$$

Let the ratio found by (3) be denoted by R_K. Then:

$$R_K = \sqrt{\frac{P_1}{P_3}} \tag{11}$$

Should P_2, the receiver pressure be found as in Par. 65, using the value given by (10), it would be too high; if by (11), too low. If the latter value were used and multiplied by

$$\sqrt[3]{\frac{R_K}{R}}$$

it will result in nearly equal work in high-pressure and low-pressure cylinders; this gives for the receiver pressure:

$$P_2 = \frac{P_1}{R_K}\sqrt[3]{\frac{R_K}{R}} = \frac{P_1}{\sqrt[3]{RR_K^2}} \tag{12}$$

This is an empirical expression and should be carefully checked, especially outside the range of P_K given (3 to 6); in all cases it is safer to check the relation of high-pressure to low-pressure work by (33).

If equal compressions are retained the compression curves will not lie on the same hyperbola. This affects the cut-off and terminal pressure in the low-pressure cylinder if the high-pressure cut-off remains the same, and while perhaps not of great importance numerically, a brief consideration of these relations may aid in a better understanding of compound engine operation.

Going on assumptions 1 and 4, and approximately assumption 2 of the previous paragraph, Fig. 90 has been constructed for any relation of

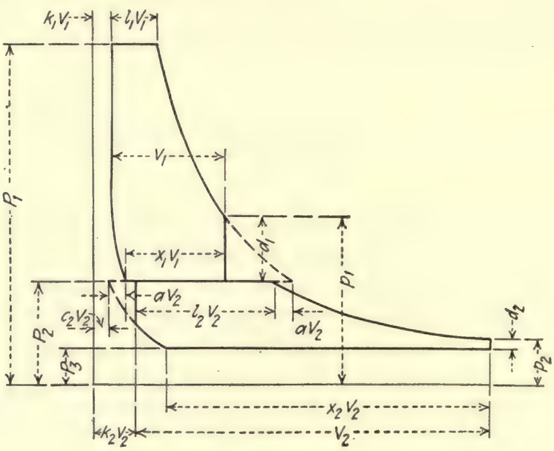


FIG. 90.

compression in the two cylinders. The volume of cushion steam in the low-pressure cylinder at pressure P_2 is c_2V_2; then:

$$P_2c_2V_2 = P_3V_2(1 + k_2 - x_2)$$

or:

$$c_2V_2 = \frac{P_3}{P_2}V_2(1 + k_2 - x_2)\cdot$$

From Fig. 90:

$$aV_2 = V_1(1 + k_1 - x_1) - c_2V_2$$

or:

$$a = \frac{1 + k_1 - x_1}{R} - \frac{P_3}{P_2}(1 + k_2 - x_2) \tag{13}$$

If a is negative the low-pressure curves lie to the right of the high-pressure curves.

Cylinder feed is the distance between the expansion and compression curves, and its weight is assumed to be the same in both cylinders; then at receiver pressure, a pressure common to both cylinders, the volume is the same, and the quantity aV_2 is also the horizontal distance between the high-pressure and low-pressure expansion curves. Equating the product of pressure and volume at P_2 gives:

$$P_1V_1(l_1 + k_1) = P_2V_2(l_2 + k_2 + a) \tag{14}$$

To facilitate application a summary of the foregoing method is given.

Summary.

$$\text{If } P_3 \geqq P_K, \quad R = \sqrt{\frac{P_1}{P_3}} \tag{15}$$

$$\text{If } P_3 < P_K, \quad R = \sqrt{\frac{P_1}{P_K}} \tag{16}$$

and:

$$R_K = \sqrt{\frac{P_1}{P_3}} \tag{17}$$

Then:

$$P_2 = \frac{P_1}{\sqrt[3]{RR_K{}^2}} \text{ approx.} \tag{18}$$

If x_1 and x_2 are assumed:

$$a = \frac{1 + k_1 - x_1}{R} - \frac{P_3}{P_2}(1 + k_2 - x_2) \tag{19}$$

If p_2 is assumed, as is usual for rated load:

$$r_2 = \frac{P_2}{p_2} \tag{20}$$

and:

$$l_2 = \frac{1 + k_2}{r_2} - k_2 \tag{21}$$

Then from (14):

$$l_1 = \frac{P_2}{P_1} R(l_2 + k_2 + a) - k_1 \tag{22}$$

and:

$$r_1 = \frac{1 + k_1}{l_1 + k_1} \tag{23}$$

Also:

$$r_T = \frac{P_1}{p_2} \tag{24}$$

If l_1 is known, as for an overload:

$$l_2 = \frac{l_1 + k_1}{R} \cdot \frac{P_1}{P_2} - (k_2 + a) \tag{25}$$

Or, if l_2 is fixed as in some engines:

$$P_2 = \frac{P_1}{R} \cdot \frac{l_1 + k_1}{l_2 + k_2 + a} \tag{26}$$

From (4), Chap. XII:

$$r_{C1} = \frac{1 + k_1 - x_1}{k_1} \tag{27}$$

and:

$$r_{C2} = \frac{1 + k_2 - x_2}{k_2} \tag{28}$$

Then from (6), Chap. XII:

$$P_H = P_1\left[\frac{1 + k_1}{r_1}(1 + \log_E r_1) - k_1\right] - P_2\left[x_1 + k_1 r_{C1} \log_E r_{C1}\right] \tag{29}$$

and:

$$P_L = P_2\left[\frac{1 + k_2}{r_2}(1 + \log_E r_2) - k_2\right] - P_3\left[x_2 + k_2 r_{C2} \log_E r_{C2}\right] \tag{30}$$

Terminal drop in high-pressure cylinder is:

$$d_1 = \frac{P_1}{r_1} - P_2 \tag{31}$$

In investigating an engine already designed or built:

$$R = \left(\frac{D_L}{D_H}\right)^2 \tag{32}$$

This assumes the stroke of high-pressure and low-pressure cylinder to be the same—which is practically always true—and may be used in place of (15) or (16), the remainder of the calculations being as before.

Neglecting the influence of the receiver, the method of this paragraph is general for the assumptions made. The pressure at which high-pressure compression begins is slightly different when the receiver is considered; otherwise all lines are the same except the back-pressure line of the high-pressure diagram and the steam line of the low-pressure diagram.

For preliminary calculations for noncondensing engines, and for condensing engines when P_3 is but little different from P_K, the method of Par. 65 may be used and greatly simplifies the work. In fact, much less refined methods than this are often used in practice—perhaps advisedly; but a better grasp of principles and a sounder knowledge of engine operation are obtained by a more exact analysis, especially with a wide range of valve setting.

If the work is equal in high- and low-pressure cylinders:

$$P_H V_1 = P_L V_2 \quad \text{or,} \quad P_H = P_L R$$

The ratio of low-pressure to high-pressure power is therefore:

$$\frac{H_L}{H_H} = \frac{RP_L}{P_H} \tag{33}$$

The statement that no hard-and-fast rule can be made for determining cylinder ratios is a well-known and well-worn one, and undoubtedly true. Custom has usually settled the question, and the high ratio advocates have raised the figure to a certain extent, but there is still not much uniformity. The question is briefly discussed from a thermal standpoint in Chap. IX, Par. 47, and there seems to be advantages in a reasonably high ratio.

A high ratio lengthens the cut-off in the high-pressure cylinder, and for a single-eccentric Corliss gear—at one time largely used on high-pressure cylinders even after the double eccentric had been used for some time on the low-pressure gear—the overload capacity under governor control is reduced. This, no doubt, has had some influence and the resulting low ratio became custom.

With double-eccentric Corliss and other long-range gears, it has sometimes been considered that a large high-pressure cylinder was necessary for large overload capacity, but Example 1, Par. 73 shows that such is not the case, and that practically 100 per cent. overload may be carried at ¾ cut-off with a cylinder ratio of 6.42. Such a ratio gives a good cut-off at rated load, and in the author's opinion, has a number of advantages. At any rate, why use a large cylinder when a smaller one will do the work as well, and with at least as good economy?

67. Influence of the Receiver.—While it may not be considered of great practical value to treat the subject of compound engines more elaborately, a better understanding of the principles of compounding may be gained thereby: therefore sets of formulas will be derived for four cases which occur in practice with 2-cylinder, double-acting engines with double expansion, showing the influence of the receiver on the back-pressure line of the high-pressure diagram and the steam line of the low-pressure diagram. The angularity of the connecting rod is neglected to avoid complicated calculation; however, this may be easily accounted for by plotting the diagrams to a large scale and measuring the volumes from them.

The method employed is a modification of that used by Prof. Unwin in his Machine Design, so arranged that the valve events and connection with receiver to both ends of each cylinder may easily be seen for the entire cycle. Equilibrium of operation is assumed, the changes of receiver pressure during starting and change of load being reserved for the

following paragraph. Pressures and volumes are denoted by the small letters p and v, the subscripts referring to points so numbered on the indicator diagrams; v_H, v_L, c_H and c_L indicate volumes of stroke and clearance of high- and low-pressure cylinders respectively; v_R is the receiver volume.

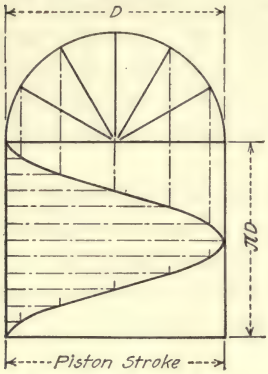


Fig. 91.

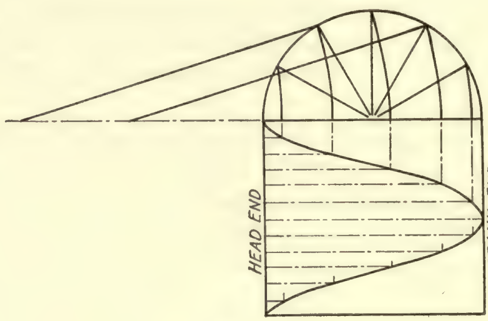


Fig. 92.

In numerical computation, either v_H or v_L may be considered as unity, the other values of v being in proportion. For simplicity, release and admission are assumed to occur at dead center, and the expansion and compression curves are hyperbolas.

The diagram employed to show the relation of events consists of a curve of piston displacement plotted on a rectified crank circle, as in Fig. 91.

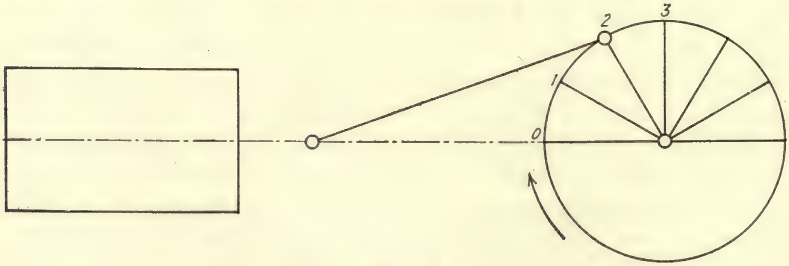


Fig. 93.

If angularity of connecting rod is to be taken into account, this may be done as in Fig. 92. This is neglected in the diagrams which follow.

In order to avoid confusion, the relation of cylinder and crank is the same as in Chap. XX, the cylinder being at the left of the crank as shown in Fig. 93. The crank rotates clockwise and crank positions are numbered from the *head-end dead center of the high-pressure engine.* For the

tandem compound this coincides with the low-pressure engine and may be shown in the form given in Unwin's Machine Design, which is also introductory to the method to be employed.

Tandem Compound.—Fig. 94 produces the head-end high-pressure diagram and the crank-end low-pressure diagram. Dividing v_R into equal parts to represent a rectified crank circle, the receiver pressure may be plotted, representing, however, the pressure changes for but one stroke of the pistons.

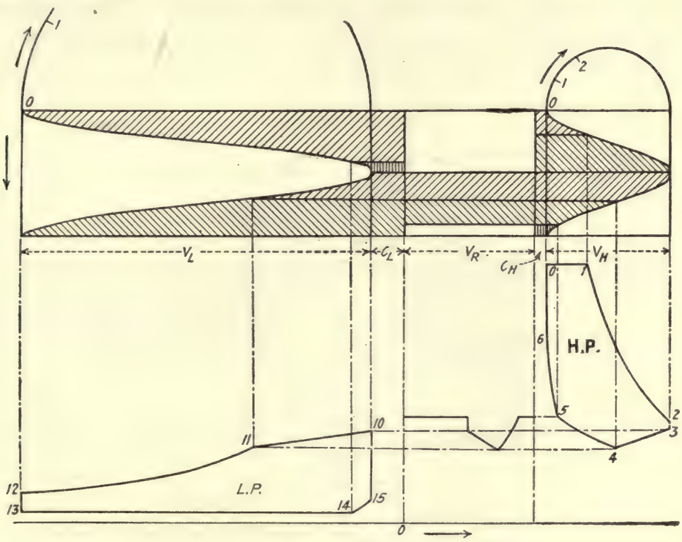


FIG. 94.

Beginning with the head end of both diagrams, operations may be traced. From 0 to 1 on the high-pressure diagram, steam is admitted to the cylinder. At 1, expansion of all steam including clearance steam, begins in the high-pressure cylinder; at the same time steam is being forced out of the low-pressure cylinder until 14 is reached, when compression begins and is finished at 15. At the same time high-pressure expansion ends at 2. High-pressure release and low-pressure admission are now assumed to occur simultaneously. Pressure p_{15} in the low-pressure clearance, pressure p_2 in the receiver and p_5 in the high-pressure cylinder are now all changed to p_3 $(= p_{10})$, as both cylinders are now open to the receiver.

The piston now starts on the return stroke from right to left, and as the low-pressure piston displaces volume as it takes steam from the receiver, more rapidly than the high-pressure piston as it exhausts into

the receiver, the pressure falls until low-pressure cut-off is reached at 11. Low-pressure expansion now begins, and at the corresponding point 4 of the high-pressure diagram, compression of steam into the receiver is begun by the high-pressure piston. This continues until the high-pressure exhaust valve closes at 5, shutting off the communication with the receiver. Compression in the high-pressure cylinder is complete at 6, and at the same time low-pressure expansion is complete.

Fig. 95 is a modification of this diagram, from which indicator diagrams may be traced for both ends of each cylinder. The diagrams used in forming the equations are in full lines. With this method the receiver volume may not be drawn to scale, but all shaded areas extending into

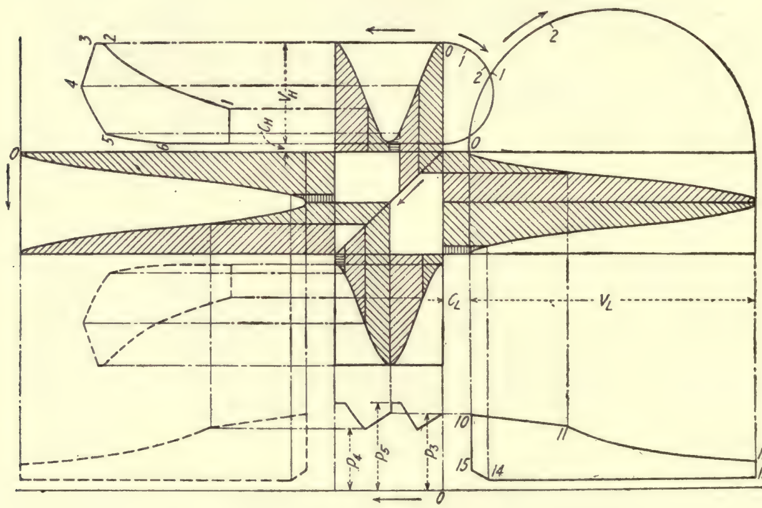


Fig. 95.

the central square indicate communication with the receiver. All such areas join a diagonal line as shown, and if shaded areas from two cylinder ends meet, they are both in communication with the receiver. The upper end of the diagonal corresponds to head-end dead center of the high-pressure crank, and following this line to its lower extremity, all events of the cycle may be traced.

The indicator diagrams of Figs. 95, 97, 100 and 103 are for a small receiver, exaggerating the pressure variation. The volume of the low-pressure cylinder is drawn one-half the computed volume to save space. No special work division is aimed at, the diagrams being used to illustrate the method. Figs. 96, 99, 102 and 105 are for condensing engines with a

cylinder ratio of 6.42, a back pressure of 2 lb. absolute and a receiver volume equal to the volume of the low-pressure cylinder.

From Fig. 95 the following equations may be written.

$$p_1v_1 = p_2v_2 \quad (a) \qquad p_2v_2 + p_5v_R + p_{15}c_L = p_3(v_2 + v_R + c_L) \quad (b)$$

$$p_3(v_2 + v_R + c_L) = p_4(v_4 + v_R + v_{11}) \quad (c) \quad p_3 = p_{10} \quad (d)$$

$$p_4 = p_{11} \quad (e) \qquad p_4(v_4 + v_R) = p_5(v_5 + v_R) \quad (f)$$

$$p_5v_5 = p_6c_H \quad (g) \qquad p_{11}v_{11} = p_{12}v_{12} \quad (h) \qquad p_{14}v_{14} = p_{15}c_L \quad (i)$$

From (a), (b), (c) and (i):

$$p_4(v_4 + v_R + v_{11}) = p_1v_1 + p_{14}v_{14} + p_5v_R \quad (j)$$

and from (f),

$$p_5 = \frac{p_4(v_4 + v_R)}{v_5 + v_R} \qquad (k)$$

Then from (j) and (k):

$$p_4(v_4 + v_R + v_{11}) - \frac{p_4v_R(v_4 + v_R)}{v_5 + v_R} = p_1v_1 + p_{15}c_L \qquad (l)$$

From (e), (l) and (i), an equation containing only known quantities is derived; or:

$$p_4 = p_{11} = \frac{p_1v_1 + p_{14}v_{14}}{(v_4 + v_R + v_{11}) - \dfrac{v_R(v_4 + v_R)}{v_5 + v_R}} \qquad (m)$$

The equations following give all other points on the diagram.

From (i):

$$p_{15} = \frac{p_{14}v_{14}}{c_L} \quad (n) \qquad \text{From } (h), \quad p_{12} = \frac{p_{11}v_{11}}{v_{12}} \quad (o)$$

$$\text{From } (f),\ p_5 = \frac{p_4(v_4 + v_R)}{v_5 + v_R} \quad (p) \qquad \text{From } (g), \quad p_6 = \frac{p_5v_5}{c_H} \quad (q)$$

$$\text{From } (a),\ p_2 = \frac{p_1v_1}{v_2} \quad (r) \qquad \text{From } (c) \text{ and } (d),\ p_3 = p_{10} = \frac{p_4(v_4 + v_R + v_{11})}{v_2 + v_R + c_L} \quad (s)$$

If l_L = low-pressure cut-off:

$$v_4 = v_2 - l_Lv_H \qquad (t)$$

Combined diagrams for the head end are shown in Fig. 96, in which the receiver volume equals the volume of the low-pressure cylinder. The dotted lines show the indefinitely large receiver assumed in Par. 66. Except for the back-pressure line of the high-pressure diagram and the steam line of the low-pressure diagram the lines are practically identical. The larger the receiver the more nearly will the lines mentioned approach the dotted lines.

It is plain that the influence of the receiver is to increase the maximum pressure in the low-pressure cylinder, increase the work done in this cylinder and decrease the work of the high-pressure cylinder.

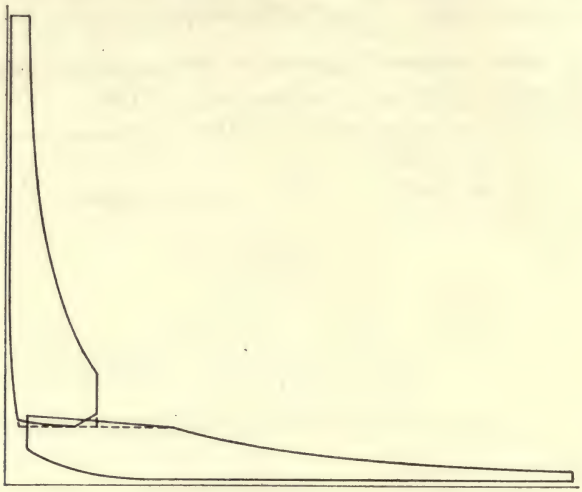


FIG. 96.

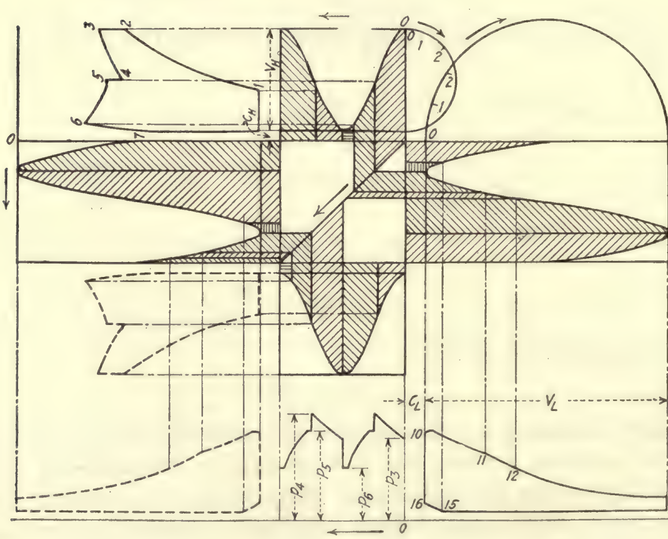


FIG. 97.

Cross Compound.—Diagrams for cross-compound engines will be drawn with the high-pressure crank leading the low-pressure crank by

90 degrees. They would be identical with those in which the low-pressure crank leads so long as angularity of the connecting rod were neglected.

There are three sets of diagrams required to show conditions commonly found in practice due to the action of the governor upon the valve gear.

Case 1.—In which low-pressure cut-off occurs after compression in one end of the high-pressure cylinder and before release in the other end. The diagram is shown in Fig. 97 and is similar to Fig. 94 except that the piston displacement curves for the low-pressure cylinder are not in phase with the high-pressure curves.

In Fig. 98,

$$v_4 = v_5 = \frac{v_H}{2} + c_H$$

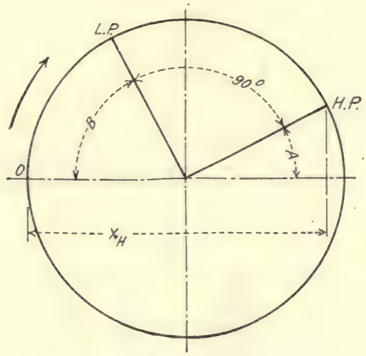


FIG. 98.

and v_{11} is determined by the position of the low-pressure crank when the high-pressure crank is in the compression position as in Fig. 98. Then:

$$\text{versin } A = \frac{2(v_6 - c_H)}{v_H} \cdot \quad B = 90 - A \quad \text{and} \quad v_{11} = \frac{v_L}{2} \text{ versin } B + c_L.$$

Or, if $x_H =$ the fraction of stroke up to high-pressure compression:

$$v_{11} = v_L(0.5 - \sqrt{x_H - x_H{}^2}) + C_L.$$

If $v_{11} > v_{12}$, use Case 2.

From Fig. 97 the following equations may be written:

$$p_1v_1 = p_2v_2 \quad (a) \qquad p_2v_2 + p_{12}v_R = p_3(v_2 + v_R) \quad (b)$$

$$p_3(v_2 + v_R) = p_4(v_4 + v_R) \qquad (c)$$

$$p_4(v_4 + v_R) + p_{16}c_L = p_5(v_4 + v_R + c_L) \qquad (d)$$

$$p_5(v_4 + v_R + v_L) = p_6(v_6 + v_R + v_{11}) \qquad (e)$$

$$p_{10} = p_5 \quad (f) \qquad p_{11} = p_6 \quad (g) \qquad p_6v_6 = p_7c_H \quad (h)$$

$$p_6(v_R + v_{11}) = p_{12}(v_R + v_{12}) \quad (i) \qquad p_{12}v_{12} = p_{13}v_{13} \quad (j)$$

$$p_{15}v_{15} = p_{16}c_L \qquad (k)$$

From (a) to (e):

$$p_6(v_6 + v_R + v_{11}) = p_1v_1 + p_{16}c_L + p_{12}v_R \quad (l)$$

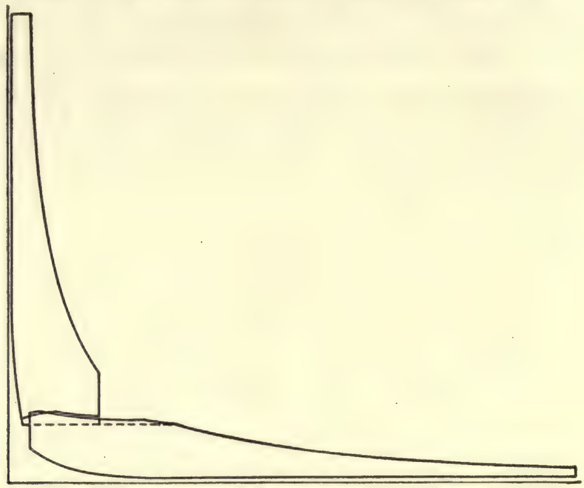


FIG. 99.

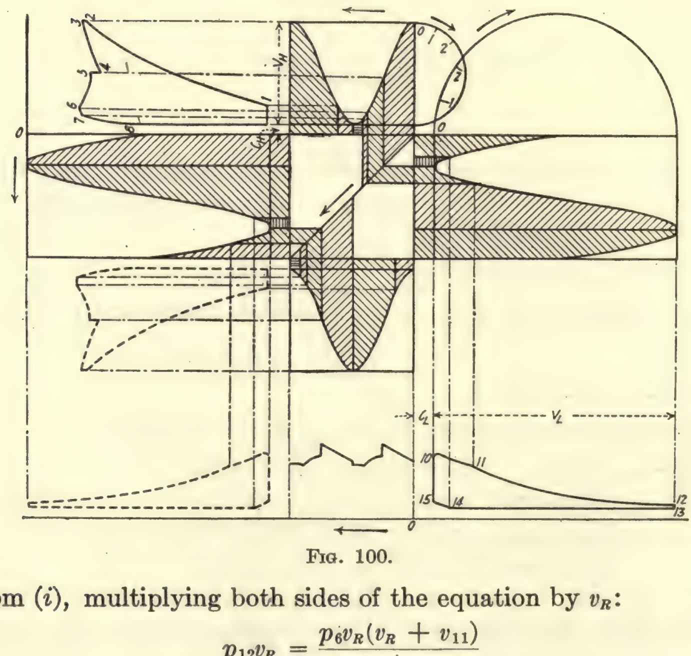


FIG. 100.

From (i), multiplying both sides of the equation by v_R:

$$p_{12}v_R = \frac{p_6v_R(v_R + v_{11})}{v_R + v_{12}} \qquad (m)$$

Substituting (k) and (m) in (l), solving for p_6 and equating with (g) gives an equation containing only known quantities:

$$p_6 = p_{11} = \frac{p_1 v_1 + p_{15} v_{15}}{(v_6 + v_R + v_{11}) - \dfrac{v_R(v_R + v_{11})}{v_R + v_{12}}} \qquad (n)$$

The equations which follow locate all other points on the diagram. From (h),

$$p_7 = \frac{p_6 v_6}{c_H} \qquad (o) \qquad \text{From } (i), \quad p_{12} = \frac{p_6(v_R + v_{11})}{v_R + v_{12}} \qquad (p)$$

$$\text{From } (j), \; p_{13} = \frac{p_{12} v_{12}}{v_{13}} \qquad (q) \qquad \text{From } (k), \quad p_{16} = \frac{p_{15} v_{15}}{c_L} \qquad (r)$$

$$\text{From } (e) \text{ and } (f), \quad p_5 = p_{10} = \frac{p_6(v_6 + v_R + v_{11})}{v_4 + v_R + c_L} \qquad (s)$$

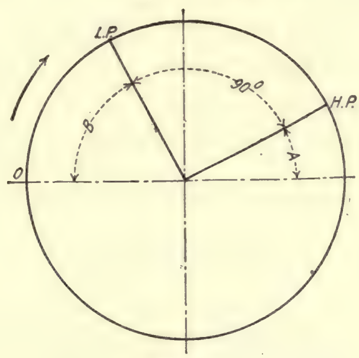


FIG. 101.

$$\text{From } (d) \text{ and } (k), \quad p_4 = \frac{p_5(v_4 + v_R + c_L) - p_{15} v_{15}}{v_4 + v_R} \qquad (t)$$

$$\text{From } (a), \quad p_2 = \frac{p_1 v_1}{v_2} \qquad (u) \qquad \text{From } (c), \quad p_3 = \frac{p_4(v_4 + v_R)}{v_2 + v_R} \qquad (v)$$

Combined diagrams for the head end are shown in Fig. 99, using the same data as in Fig. 96. The dotted lines, as before, show an indefinitely large receiver, and the variation of pressure and work due to a receiver of practical size may be seen.

Case 2.—In which low-pressure cut-off occurs before compression in the high-pressure cylinder. This is shown in Fig. 100, which is in other respects like Fig. 97.

In Fig. 100,

$$v_4 = v_5 = \frac{v_H}{2} + c_H$$

and v_6 is determined by the position of the high-pressure crank when the low-pressure crank is in the cut-off position, as in Fig. 101. Then:

$$\text{versin } B = \frac{2(v_{11} - c_L)}{v_L} \cdot A = 90 - B \text{ and } v_6 = \frac{v_H}{2} \text{versin } A + c_H.$$

Or, if $l_L =$ fraction of stroke up to low-pressure cut-off:

$$v_6 = v_H(0.5 - \sqrt{l_L - l_L^2}) + c_H.$$

If $v_6 < v_7$, use Case 1.

From Fig. 100, the following equations may be written:

$$p_1v_1 = p_2v_2 \quad (a) \qquad p_2v_2 + p_7v_R = p_3(v_2 + v_R) \quad (b)$$

$$p_3(v_2 + v_R) = p_4(v_4 + v_R) \quad (c)$$

$$p_4(v_4 + v_R) + p_{15}c_L = p_5(v_4 + v_R + c_L) \quad (d)$$

$$p_5(v_4 + v_R + c_L) = p_6(v_6 + v_R + v_{11}) \quad (e)$$

$$p_{10} = p_5 \quad (f) \qquad p_{11} = p_6 \quad (g) \qquad p_6(v_6 + v_R) = p_7(v_7 + v_R) \quad (h)$$

$$p_7v_7 = p_8c_H \quad (i) \qquad p_{11}v_{11} = p_{12}v_{12} \quad (j) \qquad p_{14}v_{14} = p_{15}c_L \quad (k)$$

From (a) to (e), and (k),

$$p_6(v_6 + v_R + v_{11}) = p_1v_1 + p_7v_R + p_{14}v_{14} \quad (l)$$

From (h), multiplying both terms of the equation by v_R and solving for p_7v_R,

$$p_7v_R = \frac{p_6v_R(v_6 + v_R)}{v_7 + v_R} \qquad (m)$$

Substituting (m) in (l), solving for p_6 and equating with (g) gives an equation containing only known quantities.

$$p_6 = p_{11} = \frac{p_1v_1 + p_{14}v_{14}}{(v_6 + v_R + v_{11}) - \dfrac{v_R(v_6 \times v_R)}{v_7 + v_R}} \qquad (n)$$

All other points are located by the following equations:

From (h), $p_7 = \dfrac{p_6(v_6 + v_R)}{v_7 + v_R}$ (o). From (i), $p_8 = \dfrac{p_7v_7}{c_H}$ (p)

From (j), $p_{12} = \dfrac{p_{11}v_{11}}{v_{12}}$ (q). From (k), $p_{15} = \dfrac{p_{14}v_{14}}{c_L}$ (r)

From (e) and (f), $p_5 = p_{10} = \dfrac{p_6(v_6 + v_R + v_{11})}{v_4 + v_R + c_L}$ (s)

From (d) and (k), $p_4 = \dfrac{p_5(v_4 + v_R + c_L) - p_{14}v_{14}}{v_4 + v_R}$ (t)

From (a), $p_2 = \dfrac{p_1v_1}{v_2}$ (u). From (c), $p_3 = \dfrac{p_4(v_4 + v_R)}{v_2 + v_R}$ (v)

Combined diagrams for the head end are shown in Fig. 102.

Case 3.—In which high-pressure release occurs before low-pressure cut-off, which is greater than one-half stroke. This is shown in Fig. 103.

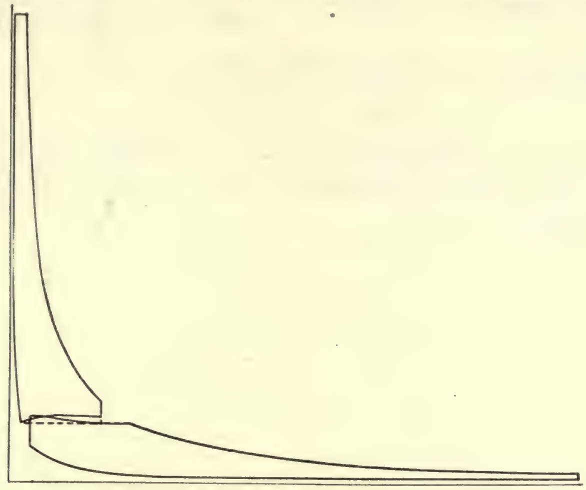


FIG. 102.

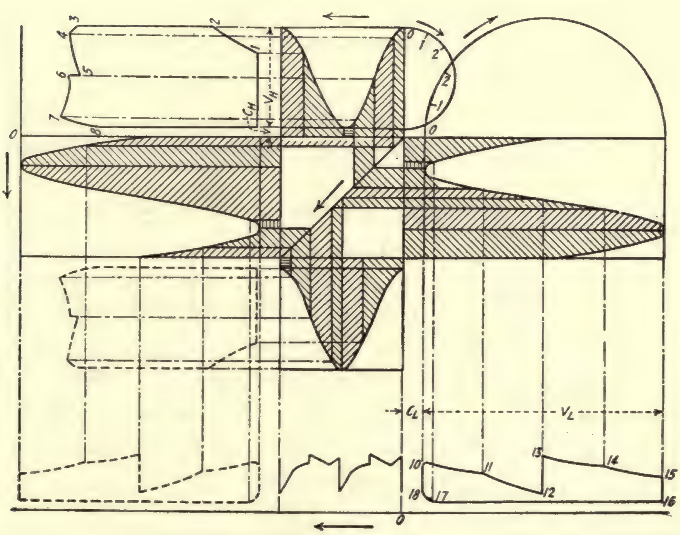


FIG. 103.

From Fig. 103,

$$v_5 = v_6 = \frac{v_H}{2} + c_H \qquad\qquad v_{12} = v_{13} = \frac{v_L}{2} + c_L$$

and v_4 is determined by the high-pressure crank position at the time of low-pressure cut-off as shown by Fig. 104.

Then:

$$\cos A = \frac{2(v_{15} - v_{14})}{v_L}. \quad B = 90 - A \quad \text{and} \quad v_4 = \frac{v_H}{2}(1 + \cos B) + c_H.$$

Or, if $l_L =$ fraction of stroke up to low-pressure cut-off:

$$v_4 = v_H(0.5 + \sqrt{l_L - l_L^2}) + c_H.$$

The value of v_{11} is determined by the low-pressure crank position at the time of high-pressure compression and is found as for Fig. 101, replacing v_6 by v_7, or:

$$v_{11} = v_L(0.5 - \sqrt{x_H - x_H^2}) + c_L$$

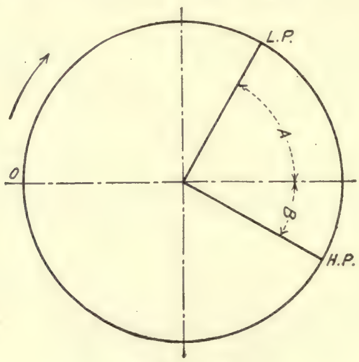


Fig. 104.

in which $x_H =$ fraction of stroke up to compression in the high-pressure cylinder.

From Fig. 103, the following equations may be written:

$$p_1v_1 = p_2v_2 \quad (a) \qquad p_2v_2 + p_{12}(v_R + v_{12}) = p_3(v_2 + v_R + v_{12}) \qquad (b)$$

$$p_3(v_2 + v_R + v_{12}) = p_4(v_4 + v_R + v_{14}) \quad (c) \quad p_{13} = p_3 \quad (d) \quad p_{14} = p_4 \quad (e)$$

$$p_4(v_4 + v_R) = p_5(v_5 + v_R) \quad (f)$$

$$p_5(v_5 + v_R) + p_{18}c_L = p_6(v_5 + v_R + c_L) \qquad (g)$$

$$p_6(v_5 + v_R + c_L) = p_7(v_7 + v_R + v_{11}) \qquad (h)$$

$$p_{10} = p_6 \quad (i) \qquad p_{11} = p_7 \quad (j) \qquad p_7v_7 = p_8c_H \quad (k)$$

$$p_{11}(v_{11} + v_R) = p_{12}(v_R + v_{12}) \qquad (l) \qquad p_{14}v_{14} = p_{15}v_{15} \quad (m)$$

$$p_{17}v_{17} = p_{18}c_L \quad (n)$$

From (a) to (c), $p_4(v_4 + v_R + v_{14}) = p_1v_1 + p_{12}(v_R + v_{12}) \quad (o)$

From (j) to (l), $p_{12}(v_R + v_{12}) = p_7(v_R + v_{11}) \quad (p)$

Substituting (p) in (o) and solving for p_4 gives:

$$p_4 = \frac{p_1v_1 + p_7(v_R + v_{11})}{v_4 + v_R + v_{14}} \qquad (q)$$

From (f) to (h), and (n):

$$p_4 = \frac{p_7(v_7 + v_R + v_{11}) - p_{17}v_{17}}{v_4 + v_R} \qquad (r)$$

Equating (q), (r) and (j) and solving for p_7 gives an equation containing only known quantities.

$$p_7 = p_{11} = \frac{p_1v_1(v_4 + v_R) + p_{17}v_{17}(v_4 + v_R + v_{14})}{(v_7 + v_R + v_{11})(v_4 + v_R + v_{14}) - (v_R + v_{11})(v_4 + v_R)} \qquad (s)$$

The following equations locate all other points.

From (h) and (i), $$p_6 = p_{10} = \frac{p_7(v_7 + v_R + v_{11})}{v_5 + v_R + c_L} \qquad (t)$$

From (k), $p_8 = \dfrac{p_7v_7}{c_H}$ (u). From (l), $$p_{12} = \frac{p_{11}(v_R + v_{11})}{v_R + v_{12}} \qquad (v)$$

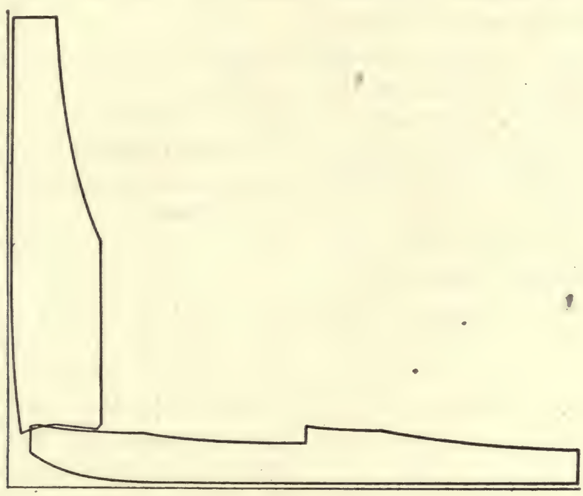


FIG. 105.

From (g) and (n), $$p_5 = \frac{p_6(v_5 + v_R + c_L) - p_{17}v_{17}}{v_5 + v_R} \qquad (w)$$

From (n), $p_{18} = \dfrac{p_{17}v_{17}}{c_L}$ (x). From (e) and (f), $$p_4 = p_{14} = \frac{p_5(v_5 + v_R)}{v_4 + v_R} \qquad (y)$$

From (m), $$p_{15} = \frac{p_{14}v_{14}}{v_{15}} \qquad (z)$$

From (a), $p_2 = \dfrac{p_1v_1}{v_2}$ (A). From (c) and (d), $$p_3 = p_{13} = \frac{p_4(v_4 + v_R + v_{14})}{v_2 + v_R + v_{12}} \qquad (B)$$

Combined diagrams for the head end are shown in Fig. 105.

The foregoing discussion covers all practical cases of tandem and cross-compound engines, the latter with cranks 90 degrees apart. It is obvious that the same analysis may be applied to any arrangement of cylinders

and cranks, to 3-cylinder compounds, or to the large variety of cases found in triple- and quadruple-expansion engines; but due to the large number of formulas involved and their infrequent application, it was thought best to limit the treatment to the 2-cylinder compound engine.

The method of this paragraph is helpful in the study of compound-engine diagrams, the ability to rightly interpret which being impossible without the knowledge due to such an analysis. For instance, the sloping line of the low-pressure diagram has often been attributed to wire-drawing, and in one instance to the author's knowledge, the point in the steam line of the low-pressure diagram of a cross-compound engine caused by high-pressure exhaust closure was thought by a consulting engineer to be the point of cut-off.

The distorted diagrams sometimes taken from triple-expansion engines are not the result of improper valve setting or too small ports, but are due to compression and expansion of receiver steam.

The sharp corners do not appear on actual diagrams, and with large receivers, both high- and low-pressure diagrams often closely resemble those from simple engines. If plotted to different scales for combining, however, the general form is more like the conventional diagrams.

68. Governing.—The speed regulation of compound engines is accomplished in two ways: (1) by direct governor control of both cylinders; (2) by direct control of the high-pressure cylinder only. In method (1) the increased or decreased weight of steam exhausted into the receiver is provided for in the low-pressure cylinder by a lengthened or shortened cut-off, thus maintaining a nearly constant receiver pressure, as shown in Figs. 99, 102 and 105, in which the receiver pressure at the time of low-pressure cut-off is the same. Assuming an indefinitely large receiver, the receiver pressure would be constant for this method of governing if the low-pressure cut-off were rightly selected, as shown in Fig. 106. Both cylinders then respond to the governor, and the regulation should be practically as sensitive as with a simple engine.

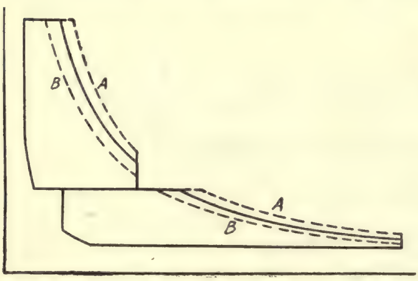


Fig. 106.

In method (2) the low-pressure cut-off is fixed, and for different loads on the engine, equilibrium requires different pressures as shown in Fig. 107, in which, for convenience, the compression curves are shown on

the same hyperbolas for all loads, necessitating different points of high-pressure exhaust closure.

If these receiver pressures were attained instantly upon change of high-pressure cut-off, the control would probably be as sensitive as with method (1) and perhaps more sensitive, as the high-pressure terminal drop being more uniform, the change of total diagram area is greater for a given change of high-pressure cut-off; but a number of strokes are required to raise or lower the receiver pressure to meet the new load, during which time the change of high-pressure cut-off must be greater than for method (1) with a consequent increased speed fluctuation. As the receiver pressure changes, the cut-off gradually changes to that required for equilibrium at the new load. It is obvious that the larger the receiver the longer the time required for pressure adjustment, which, however, means a more uniform receiver pressure, an advantage gained by a large receiver in any case. Slightly better economy is obtained by method (2) according to some engine builders.

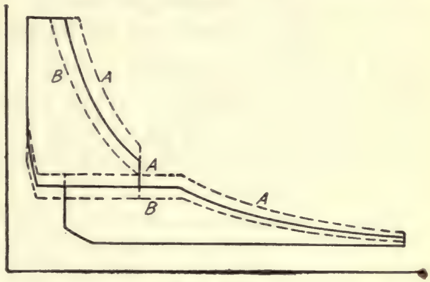


Fig. 107.

Were it not for the great labor of determining the changing high-pressure cut-off during receiver-pressure adjustments to correspond to a new load, the equations of the previous paragraph could be used for this purpose. Some idea of the pressure changes may be given by assuming a constant new cut-off. Then taking the data from which Fig. 96 for a tandem compound is plotted, the pressure at low-pressure cut-off is:

$$p_4 = p_{11} = 20.2 \text{ lb.}$$

The maximum receiver pressure is:

$$p_5 = 22.15 \text{ lb.}$$

Assuming the low-pressure cut-off as given and the high-pressure cut-off increased to 0.5 stroke, the new value of p_{11} for equilibrium is, from (m):

$$p_4 = p_{11} = 43.3 \text{ lb.}$$

Then finding p_5 from (k) and p_4 again from (j) by substituting the value of p_5 just found, and so on, alternating between (j) and (k), p_4 gradually rises and is shown in Fig. 108, in which ordinates are changes of pres-

sure from 20.2 to 43.3, and abscissas are the number of strokes since the change of high-pressure cut-off. The calculations were made with a slide rule, but it is obvious from the nature of the operation that theoretically, the curve would never touch the upper line.

Under the assumptions of constant high-pressure cut-off, the engine load must gradually increase as the receiver pressure is built up. For constant load, the high-pressure cut-off would gradually shorten, lengthening the time required to regain equilibrium, as less steam is exhausted into the receiver with each stroke.

On engines governing by method (2) there is usually a hand adjustment for the low-pressure cut-off. If there is a considerable load change, being fairly constant for quite a period during the day, the cut-off may then be adjusted so as to keep a more uniform receiver pressure and a more even division of work between the cylinders.

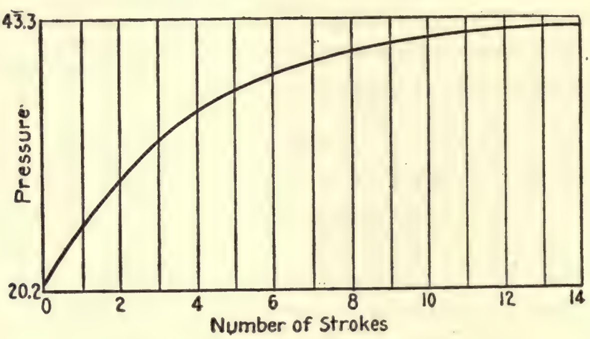


FIG. 108.

In starting a compound engine, the initial pressure in the receiver is approximately that of the atmosphere. The pressure is gradually built up as in the case of changing load, but requires a longer time. To hasten pressure increase, the low-pressure cut-off may be shortened by the hand adjustment, and gradually lengthened again as the pressure rises, giving a better work distribution between the cylinders.

In engines governing by method (1) there is usually a provision for adjusting the relative high-pressure and low-pressure cut-offs, making it possible to obtain any desired work division. On some engines there is also an independent adjustment for starting, by means of which the low-pressure gear connections may be temporarily broken and a short low-pressure cut-off obtained; then when equilibrium is attained the connection is again made and both cylinders controlled by the governor. These various appliances are illustrated in Chap. XX.

The relative merits of the two methods depend somewhat upon conditions. Both are used on high-grade engines. For engines with large overload capacity, especially when the overload may come on unexpectedly and remain for an appreciable period, method (1) is probably preferable. For comparatively small ranges of load, it is simpler to govern only the high-pressure cylinder. With a large receiver and a sensitive governor this method will give good results even with fluctuating loads.

For the rated power, the terminal pressure in the low-pressure cylinder is usually selected, presumably so as to give maximum economy. This may vary from 2 to 4 lb. above back pressure, but should not be below 5 lb. absolute.

69. Maximum Thrust.—In Figs. 96 and 99 it may be seen that the maximum pressure in the low-pressure cylinder is greater than the pressure at cut-off, which, returning to the notation used previous to Par. 68, is denoted by P_2. Let the ratio of the maximum absolute pressure to P_2 be denoted by m, then the maximum piston thrust may be determined.

Tandem Compound. The thrust of both high-pressure and low-pressure pistons is taken by the main piston rod and transmitted to the other engine parts. Let this be denoted by P_X; then, neglecting the effect of inertia of reciprocating parts:

$$\begin{aligned} P_X &= \frac{\pi D_H{}^2}{4}(P_1 - mP_2) + \frac{\pi D_L{}^2}{4}(mP_2 - P_3) \\ &= \frac{\pi D_L{}^2}{4}\left[\frac{P_1 - mP_2}{R} + mP_2 - P_3\right] \qquad (34) \end{aligned}$$

Cross-compound.—The maximum thrust of the high-pressure piston is, from Par. 66:

$$P_X = (P_1 - P_2)\,\frac{\pi D_H{}^2}{4} \qquad (35)$$

The influence of the receiver is to reduce this somewhat, as may be seen in Figs. 99 and 102, but may be neglected on the side of safety.

The maximum low-pressure thrust is:

$$P_X = \frac{\pi D_L{}^2}{4}(mP_2 - P_3) \qquad (36)$$

The influence of the receiver may not safely be neglected and is provided for by the factor m, to be derived presently.

The parts of the high- and low-pressure engines are alike, therefore, the greater value given by Formulas (35) and (36) must be used. Except for small values of R or when very large receivers are employed, the maximum value of P_X is given by (36), and should this be much in excess, the high-pressure piston rod may be determined to resist the thrust given by (35).

With tandem engines with the low-pressure cylinder next the frame—the more usual arrangement—(35) may be used for the high-pressure rod; should the high-pressure cylinder be next the frame, the low-pressure rod must be found from (36) and the main rod from (34).

The value of m for a tandem engine may be found by taking the ratio p_3/p_4, Fig. 95, from (*s*). Substituting the notation of Par. 66 and letting q be the ratio of the receiver volume to volume of stroke of the low-pressure cylinder:

$$m = 1 + \frac{l_2\left(1 + \frac{1}{R}\right)}{\frac{1 + k_1}{R} + k_2 + q} \tag{37}$$

It is obvious from (37) that m increases with l_2 but not in the same ratio; the maximum low-pressure cut-off must therefore be assumed.

Table 26 is computed from (37) for ½ cut-off, assuming that $k_1 = k_2 = 0.04$, and with values of q ranging from 0.25 to 1.5, and R from 3 to 8. Table 27 is for a ¾ cut-off.

TABLE 26

R	q				
	0.25	0.5	0.75	1	1.5
3	1.53	1.38	1.30	1.24	1.18
4	1.69	1.47	1.36	1.29	1.21
5	1.81	1.54	1.41	1.32	1.23
6	1.90	1.59	1.44	1.35	1.25
7	1.98	1.63	1.46	1.37	1.26
8	2.05	1.67	1.48	1.38	1.27

TABLE 27

R	q				
	0.25	0.5	0.75	1	1.5
3	1.80	1.57	1.45	1.36	1.27
4	2.04	1.71	1.54	1.44	1.32
5	2.22	1.81	1.62	1.48	1.35
6	2.35	1.89	1.66	1.53	1.38
7	2.47	1.95	1.69	1.56	1.39
8	2.58	1.99	1.72	1.57	1.41

For a cross-compound engine the maximum value of m is found under Case 1, by the ratio of (s) to (p), which gives, when $l_2 \gtreqless 0.5$:

$$m = \frac{\left[\dfrac{1 + k_1 - x_1}{R(q + k_2 + 0.5 - \sqrt{x_1 x_1^2})} + 1\right](q + k_2 + l_2)}{\dfrac{k_1 + 0.5}{R} + q + k_2} \tag{38}$$

Comparing Fig. 97 with Fig. 103, it is obvious that the maximum value of m, neglecting angularity of the connecting rod, occurs when $l_2 = 0.5$; substituting this in (38) and assuming that $k_1 = k_2 = 0.04$, and that $x_1 = 0.8$, Table 28 may be computed for different values of q and R.

TABLE 28

R	q				
	0.25	0.5	0.75	1	1.5
3	2.03	1.63	1.45	1.36	1.25
4	2.15	1.69	1.49	1.38	1.27
5	2.23	1.73	1.52	1.40	1.28
6	2.30	1.76	1.54	1.41	1.29
7	2.35	1.78	1.55	1.42	1.29
8	2.38	1.80	1.56	1.43	1.30

The slight rise in pressure shown in the cross-compound low-pressure diagrams at the beginning of the stroke may be neglected. This rise is due to the fact that the high-pressure piston is near mid-stroke, traveling at about maximum velocity while the low-pressure piston is leaving the end of the stroke; the high-pressure piston is displacing volume more rapidly for a short time than the low-pressure piston, causing the rise in pressure, usually not more than 1 pound.

The effect of inertia of the reciprocating parts has the same effect as with the simple engine. In the latter it was neglected when maximum thrust was considered as explained in Par. 59, Chap. XII. In the compound it tends to offset the effect of m, but except in high-speed engines it may be neglected, which is usually on the side of safety. In any engine, the inertia of the piston only should be deducted from the effect on the piston rod, the inertia of piston and rod from the effect on the crosshead and so on. In view of this, and the possibility of more or less erratic action of a compound engine, inertia may usually be neglected except in its effect upon turning effort.

Formulas (34) to (36) may be used in determining the engine parts

of compound engines, selecting m from Tables 26, 27 or 28; should R lie between two tabular values the next higher may be taken.

It is sometimes said that a tandem compound requires no receiver other than the connecting piping; a comparison of Tables 26 and 27 with 28 will show that if extreme maximum thrust is to be avoided, the requirements are practically the same as for a cross-compound engine, especially for long-range cut-off.

70. Indicated Horsepower.—If the power is to be determined from a test, general Formula (17), Chap. XII may be applied to each cylinder, the sum being the total power. For the purpose of design, (22) to (25), Chap. XII may be applied.

Case 1.—Single-acting, 2-cylinder compound.

$$H = \frac{P_H D_H{}^2 S}{84{,}000} + \frac{P_L D_L{}^2 S}{84{,}000}$$

$$= \left(\frac{P_H}{R} + P_L\right) \frac{D_L{}^2 S}{84{,}000} \tag{39}$$

From which:

$$D_L = 290 \sqrt{\frac{H}{S\left(\frac{P_H}{R} + P_L\right)}} \tag{40}$$

From (32):

$$D_H = \frac{D_L}{\sqrt{R}} \tag{41}$$

Case 2.—Double-acting, 2-cylinder compound.

$$H = \left(\frac{P_H}{R} + P_L\right) \frac{D_L{}^2 S}{43{,}000} \tag{42}$$

From which:

$$D_L = 213 \sqrt{\frac{H}{S\left(\frac{P_H}{R} + P_L\right)}} \tag{43}$$

D_H may be found from (41).

From (22), Chap. XII:

$$S = \frac{LN}{6} \tag{44}$$

where L is the stroke in inches.

If P_H and P_L are from theoretical diagrams they should be multiplied by a diagram factor. In Par. 57, Chap. XII, a quotation from Kent's Mechanical Engineers' Pocket Book is given, with a portion of a table of diagram factors from Seaton. The remainder of the table, applying to marine engines is as follows:

Particulars of engine	Factor
Compound engines with expansion valve to h.-p. cylinder, jacketed, with large ports, etc.	0.9 to 0.92
Compound engines with ordinary slide valves, cylinders jacketed, good ports, etc.	0.8 to 0.85
Compound engines as in early practice in the merchant service, without jackets and expansion valves	0.7 to 0.8
Fast running engines of the type and design usually fitted in war ships	0.6 to 0.8.

Creighton gives the following diagram factors for stationary compound engines:

Kind of engine	Factor
High speed, short stroke, unjacketed	0.60 to 0.80
Slow rotative speed, unjacketed	0.70 to 0.85
Slow rotative speed, jacketed	0.85 to 0.90
Corliss	0.85 to 0.90
Triple-expansion	0.60 to 0.70

Bauer and Robertson give for marine compounds:

Kind of engine	Factor
Large engines up to 100 r.p.m.	0.60 to 0.67
Small engines greater than 100 r.p.m.	0.55 to 0.60
Triple-expansion—war vessels with high r.p.m.	0.53 to 0.54
Triple-expansion—merchant vessels up to 100 r.p.m.	0.56 to 0.61

71. Theoretical steam consumption may be calculated as in Par. 61, Chap. XII. The weight of steam is measured by the high-pressure cylinder; then, referring the low-pressure m.e.p. to the high-pressure cylinder, (26), Chap. XII becomes:

$$W = \frac{13{,}750}{P_H + RP_L}[w(l_1 + k_1) - w_B(1 + k_1 - x_1)] \tag{45}$$

where W is the water rate in pounds per horsepower-hour, w, the weight per cubic foot of steam at initial pressure and w_B its weight at back pressure.

72. Standard Engines.—Standard engine parts may be selected for compound engines in the same manner as for simple engines working under different pressures, as explained in Par. 63, Chap. XII, by equating the maximum thrust of the standard engine with that of the compound.

For the tandem compound:

$$P_X = \frac{\pi D_S^2 p_S}{4} = \frac{\pi D_L^2}{4}\left[\frac{P_1 - mP_2}{R} + mP_2 - P_3\right]$$

From which:

$$D_S = D_L \sqrt{\frac{\frac{P_1 - mP_2}{R} + mP_2 - P_3}{p_S}} = KD_L \tag{46}$$

For the cross-compound:

High-pressure cylinder from (35),
$$D_S = D_H\sqrt{\frac{P_1 - P_2}{p_S}} = KD_L \tag{47}$$

Low-pressure cylinder from (36),
$$D_S = D_L\sqrt{\frac{mP_2 - P_3}{p_S}} = KD_L \tag{48}$$

As explained in Par. 69, the greater value must be used.

Values of K may be tabulated for given pressure ranges, receiver ratios, etc., and it will be found that cylinder ratio has little effect on power if within practical limits.

73. Application of the formulas of this chapter to design will be shown by a number of examples. As noncondensing compounds may be designed by the simple method of Par. 65, the examples will be confined to the more important condensing compound.

Example 1.—Let $P_1 = 165$, $P_3 = 2$, $P_K = 4$, $p_2 = 6$, $x_1 = 0.9$, $x_2 = 0.8$ and $k_1 = k_2 = 0.04$.

From (16):
$$R = \sqrt{\frac{165}{4}} = 6.42\cdot$$

From (17):
$$R_K{}^2 = 82.5.$$

From (18):
$$P_2 = \frac{165}{\sqrt[3]{6.42 \times 82.5}} = 20.4 \text{ lb.}$$

From (19):
$$a = \frac{0.14}{6.42} - \frac{2 \times 0.24}{20.4} = -0.0017.$$

From (20):
$$r_2 = \frac{20.4}{6} = 3.4.$$

From (21):
$$l_2 = \frac{1.04}{3.4} - 0.04 = 0.266.$$

From (22):
$$l_1 = \frac{20.4 \times 6.42 \times 0.304}{165} - 0.04 = 0.201.$$

From (23):
$$r_1 = \frac{1.04}{0.24} = 4.3.$$

From (24):
$$r_T = \frac{165}{6} = 27.5$$

From (27):

$$r_{C1} = \frac{0.14}{0.04} = 3.5.$$

From (28):

$$r_{C2} = \frac{0.24}{0.04} = 6.$$

From (29):

$$P_H = 165[(0.24 \times 2.458) - 0.04] - 20.4[0.9 + (0.04 \times 3.5 \times 1.2528)]$$
$$= 69.4 \text{ lb.}$$

From (30):

$$P_L = 20.4[(0.306 \times 2.224) - 0.04] - 2[0.8 + (0.04 \times 6 \times 1.791)]$$
$$= 10.6 \text{ lb.}$$

The high-pressure terminal drop given by (31) is:

$$d_1 = \frac{165}{4.3} - 20.4 = 18 \text{ lb.}$$

The total m.e.p. referred to the low-pressure cylinder is:

$$\frac{P_H}{R} + P_L = \frac{69.4}{6.42} + 10.6 = 21.4.$$

If $H = 1000$, $S = 800$ and the diagram factor is 0.85, (43) gives: $d_L = 56''$, and from (41), $D_H = 22''$. If L be taken as 48″, $N = 100$. The size of the engine is then:

22″ and 56″ by 48″ − 100.

The ratio of low-pressure to high-pressure work is given by (33), and is:

$$\frac{H_L}{H_H} = \frac{6.42 \times 10.6}{69.4} = 0.982.$$

The cylinder ratio in Example 1 is above average practice. High ratios have been objected to as lacking overload capacity. Assuming a cut-off of ¾ in each cylinder, P_2, found from (26), is 24.54. Substituting these values in (29) and (30), gives: $P_H = 133.6$ and $P_L = 21.34$. The ratio of low- to high-pressure work is:

$$\frac{H_L}{H_H} = \frac{21.34 \times 6.42}{133.6} = 1.025.$$

The m.e.p. referred to the low-pressure cylinder is:

$$\frac{133.6}{6.42} + 21.34 = 42.14$$

Then:

$$\frac{42.14}{21.4} = 1.97$$

and the overload capacity is 97 per cent., which seems ample.

Assuming a receiver volume equal to 0.5 the volume of stroke of the low-pressure cylinder, the maximum value of m from Table 28 is 1.78 for a cross-compound engine. Then the standard engine having the same parts as the compound, has from (47) for the high-pressure side, a diameter of:

$$D_S = D_H\sqrt{\frac{144.6}{125}} = 1.075D_H$$

if p_S, the standard pressure is 125. As

$$D_H = \frac{D_L}{\sqrt{6.42}},\ D_S = 0.423D_L.$$

For the low-pressure side, from (48):

$$D_S = D_L\sqrt{\frac{(1.78 \times 20.4) - 2}{125}} = 0.524D_L.$$

This is larger and should be used. A small receiver was taken for safety, as such a receiver might be used; but for special cases, if standard compound engine tables are not to be used, actual values of q may be taken. If $q = 1$:

$$D_S = D_L\sqrt{\frac{(1.42 \times 20.4) - 2}{125}} = 0.464D.$$

This value is still larger than for the high-pressure cylinder and should be used in this case if q is no smaller than 1.

For the tandem compound with a maximum cut-off of $\frac{3}{4}$ stroke and $q = 0.5$, $m = 1.95$; and from (46):

$$D_S = D_L\sqrt{\frac{\frac{165 - (1.95 \times 20.4)}{6.42} + (1.95 \times 20.4) - 2}{125}} = 0.677D.$$

If $q = 1$, $D_S = 0.636D_L$.

Example 2.—Assume the same data as before except P_K, which is 3—the minimum extreme given. Without taking step by step as was done in Example 1, the principal results are:

$R = 7.42$, $P_2 = 19.45$, $l_2 = 0.281$, $l_1 = 0.237$, $P_H = 77.7$, $P_L = 10.34$.

Also:

$$\frac{H_L}{H_H} = 0.988 \qquad \frac{P_H}{R} + H = 20.82 \qquad d_1 = 24.55.$$

For $q = 0.5$, $D_S = 0.513D_L$ for cross-compound, and $0.656D_L$ for tandem.

Example 3.—The same as 1 and 2 except P_K is 6, the maximum extreme given.

Then: $R = 5.25, P_2 = 21.8, l_2 = 0.246, l_1 = 0.161, P_H = 57.6, P_L = 10.82$.

Also:

$$\frac{H_L}{H_H} = 0.986. \qquad \frac{P_H}{R} + H_L = 21.77. \qquad d_1 = 10.1.$$

For $q = 0.5$, $D_S = 0.54D_L$ for cross-compound, and $0.72D_L$ for tandem. If q is 1.5 for this cylinder ratio, the high-pressure cylinder gives a greater thrust than the low-pressure; it is safe to try both (47) and (48) for cross-compound engines.

It may be seen that throughout the entire range, Formula (18) gives very close results, and that the maximum variation of the total m.e.p. referred to the low-pressure cylinder is 0.95 lb., or 4.5 per cent. of the smallest value. Due to reasons explained in Par. 47, Chap. IX, the actual variation would probably be even less; this shows that the power of a compound engine depends mainly upon the low-pressure cylinder, the cylinder ratio affecting it but little. The m.e.p.'s of the examples given must be multiplied by a diagram factor before using in the power formula.

Taking $D_L = 56$ as just calculated, $q = 0.5$, and the nearest safe whole number for D_S, Table 29 is given so that comparison may be easily made.

TABLE 29

Example	P_K	R	D_H	D_L	D_S	
					C.C.	Tandem
2	3	7.42	20	56	29	37
1	4	6.42	22	56	30	38
3	6	5.25	24	56	31	41

D_S increases as the value of R decreases, and this is a measure of the weight and cost of the engine; it also probably has some influence on the friction. A slightly greater maximum power may be obtained from the lower ratio when the low-pressure is of the same diameter in all cases, but ample overload may be carried by engines of higher ratio as shown in Example 1.

Example 4.—A comparison will now be made with a Nordberg engine referred to in their Bulletin. It is a 19 and 44 by 42 in. cross-compound engine, and the cylinder ratio from (32) is:

$$R = \left(\frac{D_L}{D_H}\right)^2 = 5.37.$$

For the particular test mentioned: $P_1 = 169.18$, $P_3 = 1.91$ and the receiver pressure, which would be somewhere between P_2 and mP_2, was 20.75 lb. Actual m.e.p.'s were

$$f_1P_H = 52.605, \text{ and } f_2P_L = 9.557.$$

The total m.e.p. referred to the low-pressure cylinder was 19.352. Clearance and compression are not given, and it will be assumed that $k_1 = 0.07$, $k_2 = 0.05$, $x_1 = 0.9$, and $x_2 = 0.8$; also that $p_2 = 6$ lb.

With the above data, the results from the formulas of Par. 66 are: $P_2 = 21.7$, $l_2 = 0.24$, $l_1 = 0.14$, $P_H = 61.4$ and $P_L = 11.21$. Using a diagram factor of 0.85:

$$f_1P_H = 52.2 \text{ and } f_2P_L = 9.55.$$

The total m.e.p. referred to the low-pressure cylinder is 19.28, differing from the test value by less than 0.5 per cent.

Assuming a receiver volume 1.5 times the volume of stroke of the low-pressure cylinder, (38) gives: $m = 1.028$. The maximum receiver pressure is then:

$$mP_2 = 1.028 \times 21.7 = 22.3.$$

This is 1.55 lb. greater than the actual receiver pressure given, a result to be expected due to shrinkage of the diagram as shown by the diagram factor.

The data of Example 1 is plotted in Figs. 96 and 99, the dotted lines for an indefinitely large receiver and the full lines showing the effect of a receiver of practical volume.

CHAPTER XIV

THE INTERNAL-COMBUSTION ENGINE

74. Introduction.—In the treatment of this chapter a knowledge of the contents of Chaps. V and VI is assumed. A thorough understanding of the principles of operation of the internal-combustion engine can not be had without following through the power formulas based upon heat quantities; but certain practical formulas have value and are derived in this chapter; their constants are also given in terms of the quantities in the more theoretical formulas.

Notation.

h = heating value (high or low) in B.t.u. per pound of liquid fuel, or per cubic foot of gaseous fuel at standard temperature and pressure.

a = actual air supply in cubic feet per pound of liquid fuel, or per cubic foot of gaseous fuel, at standard temperature and pressure.

e_M = mechanical efficiency of engine.

e = indicated thermal efficiency of engine.

e_B = thermal efficiency referred to brake horsepower; called by Güldner, economic efficiency.

e_V = volumetric efficiency—the ratio of the volume of the charge (fresh air and fuel) at standard temperature and pressure, to the volume of stroke. See Chap. VI, Par. 27.

P_M = mean effective pressure in pounds per square inch.

p = unbalanced pressure in pounds per square inch at any point of stroke.

p_X = the maximum value of p.

P = total unbalanced pressure at any point of stroke.

P_X = maximum value of P.

D = diameter of cylinder bore in inches.

L = length of piston stroke in inches.

N = r.p.m.

S = mean piston speed in feet per minute.

m = number of strokes per cycle.

H = horsepower in general.

H_I = indicated horsepower per working cylinder end.
H_B = brake horsepower per working cylinder end.
H_M = maximum horsepower, indicated or brake.
H_R = rated horsepower, indicated or brake.
$q = \frac{H_M}{H_R}$.
w_F = liquid fuel used in pounds per hour.
v_F = gaseous fuel used in pounds per hour, at standard temperature and pressure.

75. Mean Effective Pressure.—The absence of accurately predetermined pressures possible in the steam engine makes the study of indicator diagrams for the internal-combustion engine more difficult, and a good collection of diagrams more desirable. These diagrams are useful for determining the proper valve setting, timing of ignition, best fuel mixture, and for finding the output and efficiency of the engine. However, the uncertainty of limiting pressures, the lack of uniformity of assumptions for a theoretical diagram and the wide deviation of the actual from such a reference diagram, makes the use of the diagram factor (see Chap. XII, Par. 57) for determining the m.e.p. less satisfactory than for the steam engine and other methods of calculation are more in favor.

In Par. 29, Chap. VI, the following formulas are derived from a more general discussion in Par. 23 of the same chapter.

For gaseous fuel $$P_M = 5.4ee_V \frac{h}{a + 1} \tag{1}$$

For liquid fuel. $$P_M = 5.4ee_V \frac{h}{a} \tag{2}$$

These apply to both 2-stroke and 4-stroke cycles, the difference in the values of P_M for the two cycles being usually due to the different values of e and e_V, the indicated thermal efficiency and volumetric efficiency respectively.

As explained in Chap. VI, this is the mean pressure which, if exerted throughout one stroke of the piston, would do the work of the cycle for one working cylinder end.

76. Horsepower.—In Chap. VI, Par. 23, the following general formula for indicated horsepower is derived for one cylinder end:

$$H = \frac{2 \times 144}{33{,}000} \cdot \frac{P_M v_S N}{m} \tag{3}$$

where N is the r.p.m., m the number of strokes per cycle, and v_S the volume of stroke in cubic feet.

If L is the length of stroke in inches, and D the diameter of the cylinder bore in inches:

$$v_S = \frac{\pi}{4} \cdot \frac{D^2}{144} \cdot \frac{L}{12}$$

Substituting in (3) gives:

$$H_I = \frac{P_M D^2 L N}{252{,}100\, m} = \frac{P_M D^2 S}{42{,}000\, m} \tag{4}$$

where S is the mean piston speed in feet per minute, which is:

$$S = \frac{2LN}{12} = \frac{LN}{6}. \tag{5}$$

The brake horsepower is:

$$H_B = e_M H_I = \frac{e_M P_M D^2 L N}{252{,}100\, m} = \frac{e_M P_M D^2 S}{42{,}000\, m} \tag{6}$$

For gaseous fuel, substituting (1) in (4) and (6) gives:

$$H_I = e e_V \frac{h}{a+1} \cdot \frac{D^2 L N}{46{,}700\, m} = e e_V \frac{h}{a+1} \cdot \frac{D^2 S}{7790\, m} \tag{7}$$

And as $e_M e = e_B$:

$$H_B = e_B e_V \frac{h}{a+1} \cdot \frac{D^2 L N}{46{,}700 m} = e_B e_V \frac{h}{a+1} \cdot \frac{D^2 S}{7790 m} \tag{8}$$

From (7) and (8):

$$D = 88.25 \sqrt{\frac{m H_I (a+1)}{e e_V S h}} = 88.25 \sqrt{\frac{m H_B (a+1)}{e_B e_V S h}} \tag{9}$$

For liquid fuel, substituting (2) in (4) and (6) gives:

$$H_I = e e_V \frac{h}{a} \cdot \frac{D^2 L N}{46{,}700 m} = e e_V \frac{h}{a} \cdot \frac{D^2 S}{7790 m} \tag{10}$$

$$H_B = e_B e_V \frac{h}{a} \cdot \frac{D^2 L N}{46{,}700 m} = e_B e_V \frac{h}{a} \cdot \frac{D^2 S}{7790 m} \tag{11}$$

From (10) and (11):

$$D = 88.25 \sqrt{\frac{m H_I a}{e e_V S h}} = 88.25 \sqrt{\frac{m H_B a}{e_B e_V S h}} \tag{12}$$

Although derived in a different manner, these equations give results identical with the formulas of Güldner, which are in terms of brake horsepower.

Formulas (9) and (12), being used for design, are in terms of piston speed rather than stroke and revolutions. Some discussion of piston speed is given in Par. 48, Chap. IX; with proper design there seems to be no restrictions within reasonable limits, other than those of gas velocity, which will be discussed in Chap. XX.

The relation of S, L and N may be adjusted by means of (5) when the cylinder diameter has been determined; if N is fixed by reason of service considerations, L may be found. If not, the question is one of the ratio of stroke to diameter; this ranges from 1 to 2, Diesel engines commonly having the ratio 1.5. Some discussion of this ratio is given in Par. 47, Chap. IX, under Design of Cylinders, and insofar as it concerns the weight of the engine parts it applies to internal-combustion engines. For an extensive discussion of the subject, the reader is referred to Güldner's Internal-combustion Engines.

The heating value of fuel and theoretical air supply are given in Table 30. Also see Par. 42, Chap. VIII. The actual air supply must be greater than the theoretical to insure perfect combustion; aside from this there should be an excess of air to provide for an increase of fuel supply for overloads. Even if an engine is rated at its maximum capacity there are enough uncertain factors in its design to make it unsafe to allow too small a margin over the air theoretically required for perfect combustion.

Table 30 is abridged from Güldner's Internal-combustion Engines, and

TABLE 30

Fuel	h (low) per		a(theoretical) per		a (actual) per		e_B for H_B per cylinder end of				
	Cu. ft.	Lb.	Cu. ft.	Lb.	Cu. ft.	Lb.	5	10	25	50	100 and over
Illuminating gas—lean	505	.....	5.5	.....	7.5	...	0.20	0.22	0.24	0.26	0.27
Illuminating gas—rich	675	.....	6.5	.....	10.0	...	0.20	0.22	0.24	0.26	0.27
Producer gas—anthracite	141	.....	0.85	.....	1.1 to 1.4	...	0.17	0.19	0.21	0.23	0.24
Blast-furnace gas	106	.....	0.75	.....	1.0 to 1.2	...	...	0.18	0.20	0.22	0.24
Coke-oven gas	505	.....	5.3	.....	7.0	...	....	0.17	0.19	0.21	0.23
Kerosene	...	18,900	...	185.0	...	257 to 353	0.11	0.12	0.13		
Crude oil—Diesel	...	18,000	....	176.0	...	288 to 323	0.25	0.26	0.27	0.30	0.315
Gasoline	...	18,500	....	176.0	...	240 to 323	0.19	0.21	0.23		
Alcohol (90 per cent. vol.)	...	10,300	...	96.5	...	128 to 193	0.22	0.24	0.26		

may be used as a guide in the selection of h, a and e_B. Values of H_B are for one cylinder end. Both theoretical and practical values of air supply

are given, the latter being based upon an overload capacity of from 15 to 20 per cent.

Volumetric efficiency is discussed in Par. 27, Chap. VI; some practical values from Güldner are given in Table 31. The values from Güldner's

TABLE 31

Type of engine	e_V
Slow-speed, mechanically operated inlet valve	0.88 to 0.93
Slow-speed, automatic inlet valve	0.80 to 0.87
High-speed, mechanically operated inlet valve	0.78 to 0.85
High-speed, automatic inlet valve	0.65 to 0.75

tables are based upon a standard pressure of 14.7 lb. per square inch and 32 degrees F., although he recommends 28.92 in. of mercury and 59 degrees F. The A.S.M.E. Code adopts 30 in. of mercury and 60 degrees F.

The mechanical efficiency e_M varies from 0.6 to 0.9 and is discussed in Chap. X.

Simple Formulas.—Thus far the formulas have been of a general character and these are necessary for a comprehensive study of engine power; but by making substitution of experimental values of P_M for rated power in (6), or of h, a, e_V and e_B in (8) and (11), a simple formula may be derived. E. W. Roberts, in The Gas Engine Handbook gives (the notation being changed):

$$H = \frac{D^2LN}{K} \tag{13}$$

in which the constant K differs for different fuel and with the type of engine. This may be written:

$$H = \frac{6D^2S}{K} \tag{14}$$

Then:

$$D = 0.41\sqrt{\frac{KH}{S}} \tag{15}$$

Comparing (14) with (6) and (8), the value of K may be expressed in terms of the quantities in these formulas; or:

$$K = \frac{252{,}100m}{e_MP_M} \tag{16}$$

also:

$$K = \frac{46{,}700m(a + 1)}{e_Be_Vh} \tag{17}$$

As $a + 1$ is but slightly different from a for liquid fuels, (17) may be applied to all internal-combustion engines.

Roberts gives for the average size of 4-cycle engine:

For natural gas .... $K = 16{,}000$;
For gasoline .... $K = 14{,}000$;

stating that K becomes smaller as the cylinder size increases. He cites a case of an automobile engine with exceptionally large valves and ball bearings in which K was 11,500. Smaller values have been obtained—notably in airplane engines (see Table 32).

For 2-cycle engines Roberts gives:

For gas .... $K = 12{,}000$;
For gasoline .... $K = 10{,}000$;
For large blowing engines .... $K = 8{,}400$.

It is claimed that these values are approximate.

The Bruce-Macbeth Engine Co. published a formula for 4-cycle engines which gives:

$$K = \frac{1{,}000{,}000}{P_B}$$

in which P_B varies as follows:

For natural gas, $P_B = 75$ lb., and $K = 13{,}330$;
For producer gas, $P_B = 60$ lb., and $K = 16{,}660$;
For gasoline, $P_B = 60$ lb., and $K = 16{,}660$.

A comparison of this formula with (16) shows that P_B is practically equal to $e_M P_M$, the brake m.e.p. The indicated m.e.p. is then equal to P_B divided by e_M.

Comparing the constants of 4-cycle and 2-cycle engines given by Roberts, the 2-cycle has but 40 per cent. greater capacity than the 4-cycle. Substituting the respective values of K in (17), assuming a and h the same for both engines, the value of the product of $e_B e_V$ for the 4-cycle engine is 43 per cent. greater than for the 2-cycle. This difference is probably in part due to e_B, but largely to e_V. Comparing the value of K for the large 2-cycle blowing engines mentioned by Roberts with that of the Bruce-Macbeth producer-gas engine gives practically equal values of the product $e_B e_V$. This is no doubt due to the greater value of e_V made possible by the separate charging cylinders of the large 2-cycle engine; this is probably greater than for the 4-cycle engine, while e_B is smaller.

Taking K from engine ratings or tests, and assuming e_B, e_V and h (or taking them from Table 30 or from tests), the air supply may be determined from (17); or:

$$a = \frac{K e_B e_V h}{46{,}700 m} - 1 \qquad (18)$$

A comparison of engine ratings for a given fuel may thus be made; the smaller the air excess the nearer to the maximum power is the engine rated, remembering that there must be some excess air at maximum power. Also by assuming e_M, the value of P_M may be found from (16).

The influence of e_V upon capacity, and undoubtedly upon efficiency, is considerable, and every practical means available for increasing this should be employed. Ample valve openings, easy passages and proper timing are probably the chief factors in a high value of e_V. From Par. 27, Chap. VI, it is obvious that the temperature of the charge directly affects capacity by determining in part the value of e_V; the other factor is the pressure of the charge, which depends upon valves, passages, etc. as just stated.

The value of e_B may be kept a maximum by proper compression, carburetion, ignition and lubrication, in addition to the factors just mentioned.

Table 32 contains some data pertaining to engines of different type and capacity, taken from builders catalogues, published results of tests, and information furnished the author directly. These may be used for comparison.

TABLE 32

	Type	m	Number of cyl.	H_B per cyl.	D (in.)	L (in.)	L/D	N	S	K	e_M	P_M
Gasoline	Stationary	4	1	1.5	3½	4	1.14	550	366	18,000	0.80	70
	Stationary	4	1	5.00	5½	7	1.27	425	496	18,000	0.80	70
	Stationary	4	1	7.00	6	8½	1.41	375	532	16,400	0.80	78
	Stationary	4	1	15.00	8⅝	14	1.62	250	584	16,400	0.80	78
	Automobile	4	4	10.00	3¼	5¼	1.61	2,800	2,450	15,500	0.80	81
	Automobile	4	6	5.00	3⅝	4	1.10	1,500	1,000	15,750	0.78	83
	Automobile	4	6	7.50	3¼	4½	1.38	2,600	1,950	17,500	0.78	74
	Automobile	4	8	8.75	3⅛	5⅛	1.64	2,400	2,050	13,700	0.80	92
	Boat	2	1	7.00	4¾	4	0.84	750	500	9,700	0.74	70
	Boat	2	1	12.00	5¾	5	0.87	675	563	9,300	0.76	71
	Boat	4	4	7.50	4⅛	5¼	1.27	1,000	875	11,900	0.80	106
	Boat	4	6	8.34	3¾	5¼	1.40	1,300	1,140	11,550	0.78	112
	Airplane	4	6	20.00	4¾	6½	1.37	1,350	1,465	9,900	0.78	130
Crude oil	Hot bulb	4	1	100.00	17	27½	1.62	200	917	15,900	0.85	75
	Hot bulb	4	1	140.00	20	34½	1.72	165	950	16,250	0.85	73
	Diesel	4	3	150.00	21	30	1.43	180	900	15,900	0.75	91
	Diesel	4	4	125.00	18⅞	28⅜	1.50	164	775	13,200	0.73	105
	Diesel	2	4	200.00	20	36	1.80	95	570	6,840	0.70	105
Gas	Natural gas	4	4	37.50	12¼	14	1.14	275	640	15,400	0.80	82
	Natural gas (D.A.)	4	2	112.00	20	36	1.80	125	750	16,100	0.81	78
	Blast-furnace gas (D.A.)	4	2	500.00	45	60	1.33	70	700	19,000	0.84	63

Values of K and P_M have been calculated, the latter by assuming a mechanical efficiency determined by Formulas (10) and (11), Chap. X.

Until recently but little information was available concerning the Diesel engine. Much valuable information is contained in publications of the A.S.M.E., a few notes from the paper mentioned at the end of Chap. V being given at this place:

Comparing the 4-cycle and 2-cycle Diesel, the latter gives from 70 to 80 per cent. more power for the same cylinder dimensions and speed. The 4-cycle has about 10 per cent. better efficiency and the mean temperature is lower. In this country, the 4-cycle engine is built in sizes up to from 700 to 1000 b. h.p., and above that the 2-cycle is used.

The Diesel engines built in the United States run from 150 to 300 r.p.m., with piston speeds from 600 to 900 ft. per minute. Some high-speed marine engines run as high as 480 r.p.m. In Europe the highest commercial speed is from 350 to 400 r.p.m., with as high as 550 for submarines.

Practically all Diesel engines are single-acting, and nearly all in this country have trunk pistons without crossheads.

The mechanical efficiency at full load is about 75 per cent. for 4-cycle and 70 per cent. for 2-cycle engines.

Some Diesel engine data are given in Table 32.

The m.e.p. of internal-combustion engines of small size and high speed is seldom known; it is sometimes calculated by assuming the mechanical efficiency. Some values of P_M, also of h are given in connection with compression pressures in Table 35, Par. 77, which may be compared with those already given in Table 32. E. W. Roberts, in The Gas Engine of October, 1917, gives values of $e_M P_M$ for a number of engine types, which represent recent practice in the United States. These values are given in Table 33; values of e_M are assumed, from which the values of P_M and K are computed and given in the table.

TABLE 33

Type	$e_M P_M$	e_M	P_M	K
Hot bulb, 2-cycle	33	0.70	47	15,300
Producer gas	57	0.80	71	17,700
Gasoline, 2-cycle	60	0.75	80	8,400
Modern high-speed gasoline	80	0.80	100	12,600
Modern airplane	105	0.80	130	9,600
Diesel, 4-cycle	75	0.73	103	13,500

It is to be assumed that the engines are 4-cycle unless otherwise stated.

The mechanical efficiency of the airplane engine is perhaps too small; indications are that a much better value has been obtained.

From the various data given it is obvious that while there is a general agreement, there is no definite standard of engine rating, and if there were it is not likely that capacities found from tests would always agree with the assumptions.

With new work it is well to determine D from (9) or (12), using data from Tables 30 and 31. This result may be checked by (15), taking K from Table 32 or 33; or by (6), taking P_M from Table 32, 33 or 35, and e_M from (10) and (11), Chap. X, or from Table 32 or 33.

With assumed values of e_B, e_V and h, the theoretical maximum capacity limit is when a is the theoretical air supply required for perfect combustion. The actual maximum power will be obtained with a value of a greater than this, the power always being less than the theoretical maximum. It is apparent that the more perfect the mixture and ignition, the smaller will be the excess of air required.

Should Formula(6) or (15) be used for determining cylinder dimensions, the air supply should be checked by (18), and if equal to, or less than the theoretical minimum, the engine will not develop the power desired—assuming that e_B, e_V and h have been rightly chosen.

The effect of changing the standard temperature from 32 to 60 degrees F., and in using higher instead of lower heating value of the fuel is easily found. From this it is clear that

$$\frac{h}{a+1} \cdot e_V \text{ and } \frac{h}{a} \cdot e_V$$

do not change with change of standard temperature, and that at any standard temperature, eh or e_Bh is constant for a given fuel whether higher or lower value is taken as the standard.

At very high speeds the power is limited by wire-drawing as discussed in Par. 60, Chap. XII, relative to steam engines; and in like manner the range over which power is proportional to piston speed may be increased by large valves, free passages, proper valve setting, etc. Maximum horsepower is reached at a certain speed, beyond which the power decreases. This does not mean, of course, that the power is not available at speeds much beyond that giving maximum power, as the power requirements may be much less than the maximum, as in automobile propulsion with good roads and other favorable conditions. Power and speed curves are given in Par. 22, Chap. V.

Fuel Consumption.—From equations (12) to (14), Chap. VIII, the following formulas are derived: let

w_F = the weight in pounds of liquid fuel used per hour.

v_F = the volume in cubic feet at standard temperature and pressure, of gaseous fuel used per hour.

h = the heating value per pound of liquid fuel or per cubic feet of gaseous fuel at standard temperature and pressure.

Then:

$$w_F = \frac{2545H}{eh} \tag{19}$$

and:

$$v_F = \frac{2545H}{eh} \tag{20}$$

If H is i.h.p. or b.h.p., e must be the corresponding efficiency. If h is the higher or lower value, e must be based upon it. To find the fuel per h.p.-hr., divide by H.

77. Compression.—In Chap. VI, Formulas(11) and (17) give the efficiencies of the constant-volume and constant-pressure cycles respectively. From these it is apparent that the higher the compression the greater

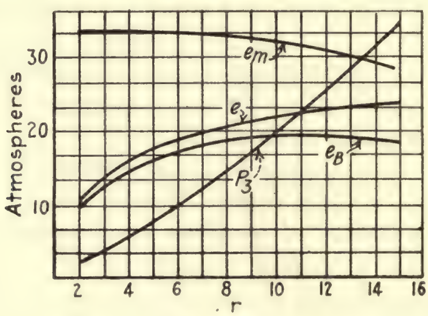


FIG. 109.

the efficiency. There are, however, practical limits. In engines compressing the mixture of fuel and air, ignition will occur prematurely from the heat generated thereby if compression is carried too far. The ignition point varies with different fuels and with different mixtures. Within certain limits compression may be increased to advantage by employing a lean mixture—poor in fuel; but above a certain limit, the m.e.p. decreases too rapidly, reducing the mechanical efficiency as explained in Chap. X. The theoretical thermal efficiency increases but slowly beyond a certain increase in compression, and a point is reached where the product of mechanical and thermal efficiency will decrease if compression is carried higher. This product, as already shown, is the thermal efficiency at brake, or true efficiency. This is shown in Fig. 109, taken from Güldner.

It is not well to approach too near this limit. The friction load is increased by high compression and unless there are positive advantages to be gained, moderation should be practised. Engines with moderate compression have sometimes shown remarkable economy, showing that other features of design offset this seeming advantage.

With the medium- and high-compression oil engines, compression is carried high enough to insure complete combustion. The mechanical efficiency of these engines is low, but is more than offset by the attainment of high compression without in any way sacrificing the charge weight.

In the methods of power determination employed in the preceding paragraph theoretical efficiency is not considered, therefore compression pressure does not enter into the discussion. By avoiding extremes, it is probable that considerable leeway may be allowed in the selection of compression pressure, but in the use of the formulas of Par. 75 it is assumed that a pressure suitable to the conditions of operation is employed.

The same engine may sometimes be used for several different fuels, but better efficiency or capacity might be obtained if the compression were changed.

Table 34 gives compression pressures in pounds per square inch, absolute, according to the authorities noted, while Table 35 was given by R. E. Mathot in Power, April 11, 1916. The m.e.p.'s in the latter table are maximum figures from well-designed engines working under favorable conditions with an overload capacity of from 10 to 15 per cent. Table 35 gives values from actual practice covering over 600 tests by Mr. Mathot, on about 40 different makes of European and American engines.

TABLE 34

Type of engine	Abs. comp. pressure	Authority
Gasoline	45 to 95	Lucke
Kerosene (hot bulb)	30 to 75	Lucke
Natural gas	75 to 130	Lucke
City gas	60 to 100	Lucke
Producer gas	100 to 160	Lucke
Blast-furnace gas	120 to 190	Lucke
Natural gas	95 to 110	Roberts
Illuminating gas	105	Roberts
Gasoline	95	Roberts
Kerosene	75	Roberts
Blast-furnace gas	142	Roberts
Producer gas	115 to 130	Roberts

TABLE 35

Kind of fuel	h (high)	Comp. press. absolute	p_M
Blast-furnace gas	100	185 to 215	70
Producer gas	135	155 to 185	75
Natural gas	900	155 to 165	90
Coke-oven gas	550	135 to 145	80
Illuminating gas (coal gas)	650	125 to 155	85
Kerosene	20,000	55 to 85	60
Kerosene (with water injection)	20,000	85 to 100	60
Benzol (industrial engine)	17,000	85 to 100	70
Benzol (automobile engine)	17,000	100 to 115	90
Alcohol (industrial engine)	13,000	115 to 145	85
Alcohol (automobile engine)	13,000	125 to 175	95
Crude oil (Diesel engine)	18,500	515	105
Crude oil (2-cycle, hot bulb engine)	18,500	155 to 175	45
Crude oil (4-cycle, hot bulb engine)	18,500	275 to 315	75
Tar oil (Diesel engine)	18,000	615	100

78. Governing.—There are two general methods employed to regulate the speed of internal-combustion engines. The first is the *intermittent impulse system* in which the working cycle is always the same, but at loads less than the maximum the number of cycles per minute is reduced by skipping. This is known as *hit-and-miss* governing. It is used on many small engines and is economical, as the combustion of each charge takes place under the best conditions. Due to the irregularity of its impulses, especially at light loads, it does not regulate closely; this may be in part offset by a very heavy flywheel, and hit-and-miss engines are sometimes used to operate small dynamos, but for larger powers where close regulation is required, and for driving alternators for parallel operation, they are impracticable.

The other general method is the *variable impulse system.* In this system the m.e.p. is changed as in the steam engine, but no cycles are skipped. This system has two subdivisions: the *quality method,* in which the proportion of fuel to air is varied; and the *quantity method,* in which a charge of constant quality is admitted during a portion of the suction stroke, or more commonly, is throttled during the entire stroke. Sometimes a combination of the quality and quantity methods is used.

The Quality Method.—In this method as applied to gas and light-oil engines, the air inlet valve admits a full charge; the fuel supply is under the control of the governor and is throttled at light loads. The compression pressure is constant, which is considered a theoretical advantage.

Fig. 110 is an indicator diagram from a gas engine using quality regulation, the full lines showing full load and the dotted lines lighter loads.

From medium to full load this method gives good results. For very light loads the mixture is so lean as to be difficult to ignite and slow burning, leading to the possibility of skipping and poor economy.

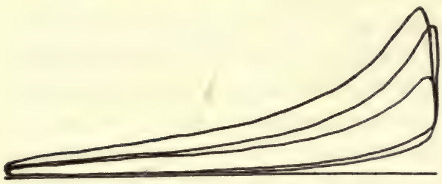


FIG. 110.—Quality regulation.

FIG. 111.

The quality method is used for the heavy-oil engine, but in this, ignition depends upon the high temperature of the highly compressed air, and all fuel delivered to the cylinder, no matter how small the quantity, is perfectly consumed. As theoretically indicated by Formula (17), Chap. VI, the shorter the combustion line—sometimes called the cut-off—the greater the thermal efficiency, and this is true in practice. This is, of course, the indicated efficiency, the reduced mechanical efficiency at very light loads offsetting this gain. Fig. 111 is an indicator diagram for a Diesel engine, the full lines being for rated load and the dotted lines for a lighter load.

The Quantity Method.—This method with variable admission, as usually employed, admits a charge of uniform quality during a portion of the stroke, when the valve is quickly closed by a trip mechanism similar in principle to the gear of a Corliss steam engine. Assuming a full-load diagram similar to Fig. 110, a light-load diagram (exaggerated for the purpose of illustration) is shown in Fig. 112. The charge is admitted along the suction stroke from a to b when the inlet valve suddenly closes. During the remainder of the suction stroke the charge is expanded from b to c, and theoretically, at least, passes over the same curve on the portion of the compression stroke from c to b. There is then no fluid-friction loss during this part of the cycle, the only loss of this character being the usual suction and back-pressure

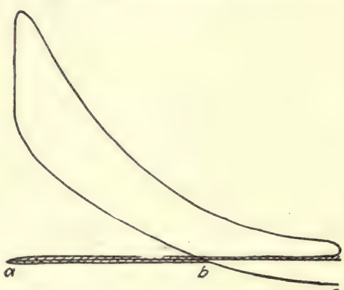


FIG. 112.—Quantity method with variable admission.

losses caused by the normal valve openings as shown by the shaded area of Fig. 112.

In the quantity method with throttling governor, a charge of uniform quality is throttled throughout the suction stroke. This is best shown by another exaggerated diagram in Fig. 113, assuming the same weight of charge—or the same pressure at the end of the suction stroke. The fluid-friction loss is indicated by the dotted area.

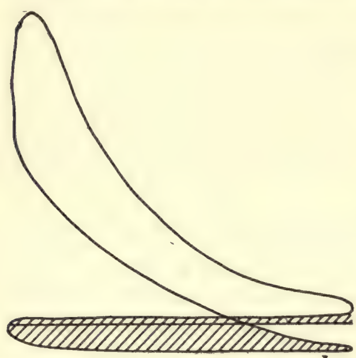


FIG. 113.—Quantity method with throttling governor.

In reality, the shaded areas in both Fig. 112 and Fig. 113 are relatively small, and while the cut-off method might be expected to give slightly better economy, the throttling method is more often used, due no doubt to the simpler mechanism required.

The usual argument against quantity governing is the low compression pressure at light loads; this tends to reduce the rate of combustion, but with a properly proportioned mixture this is not so noticeable.

Combined Methods.—By employing as lean a mixture as will give reasonable economy at the minimum load, and using the quantity method between this load and the lightest load obtained with maximum compression; then if the quality method be employed for loads greater than this, it is probable that better results may be obtained over a wider range of power. According to Power, this method is used with the 5000 horsepower gas engine of the Ford Motor Company.

The hit-and-miss method might—and probably has been applied to the lower pressure range, combined with either the quality or quantity method, and when the closest regulation is not required should give good economy over a wide range of power. This arrangement might be applicable to rolling-mill engines and should give as good regulation, surely, as the single-eccentric Corliss engines formerly used for such service.

More will be said about hit-and-miss governing in Chap. XVIII. The apparatus for effecting the different methods of governing will be explained in Chaps. XIX and XX.

79. Rating.—Internal-combustion engine rating is not as uniform as that of the steam engine. With the larger gas engines and oil engines, there is usually some provision for overload. As previously stated,

Güldner allows from 15 to 20 per cent. A number of manufacturers of Diesel engines allow 10 per cent. overload for a given time—such as two hours, and state that much larger loads have been carried for a limited time.

It is apparent that some small engines are rated at their maximum capacity, or at the maximum load at which they will operate continuously without giving trouble.

While desirable, it is not essential to have absolute uniformity in rating, so long as the conditions of rating are plainly stated. All internal-combustion engines have a maximum capacity at which they will run continuously without heating, and in most cases may exceed this capacity for a limited time. This condition may just fill the bill for certain work, and manufacturers should determine these limits as accurately as possible and use them conservatively in their specifications. The minimum capacity of satisfactory operation should also be known. With these data at hand, and a carefully plotted economy curve, the mill owner, consulting engineer, and the engine manufacturer himself will be able to handle power problems with accuracy and meet all guarantees.

The lack of overload capacity is often mentioned as an objection to the internal-combustion engine, and indeed, when the Diesel engine is rated to give 10 per cent. overload, and it is known that a steam engine with long range cut-off will run continuously and satisfactorily—barring economy—with an overload of from 50 to 100 per cent., the flexibility of the steam engine is certainly attractive for certain kinds of work.

Take for example, a 200-h.p. steam engine designed for 50 per cent. overload; a 200-h.p. Diesel engine as usually rated will carry a maximum load of 220 h.p., while the steam engine will carry 300 h.p. Is this a fair comparison? Both engines operate with maximum economy at 200 h.p., at least approximately. A Diesel engine rated at 273 h.p. would be required to develop a maximum power of 300 h.p.; then at 200 h.p., the rated power of the steam engine, the Diesel carries about 73 per cent. of its rating. But how does the economy at this load compare with the economy of the steam engine at the same load?

In Fig. 114 are plotted two curves showing B.t.u. per minute at different powers for a Diesel engine and a noncondensing, uniflow steam engine. Some assumptions had to be made in reducing the data so that a comparison might be made, and the chart may be considered as only suggestive. If 200 h.p. is assumed as the rated power of the Diesel, it will not be the most economical load, but it will give better economy than the steam engine at this load, and will have the same overload capacity

If the maximum power that may be developed continuously by an internal-combustion engine is denoted by H_M and the rated power by H_R, the ratio of the maximum to the rated load is:

$$q = \frac{H_M}{H_R}.$$

The fraction of the maximum load to give a certain rated load is:

$$H_R = \frac{1}{q}H_M.$$

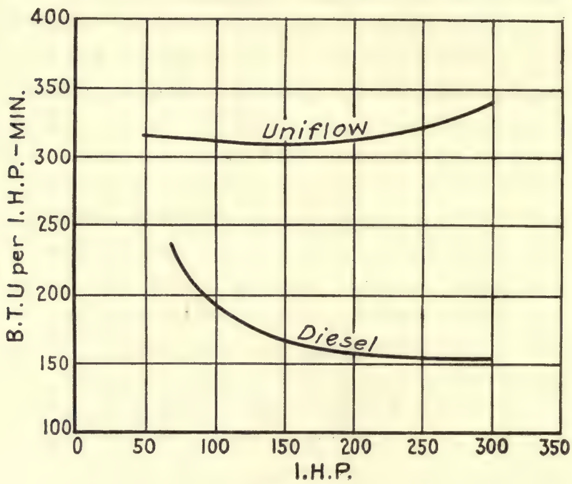


Fig. 114.

The overload capacity in per cent. is:

$$\text{Overload} = 100(q - 1)$$

or:

$$q = \frac{\text{Per cent. overload}}{100} + 1 \tag{21}$$

Table 36 gives the required percentage of the maximum capacity required for the rated load, to allow a given overload capacity.

Table 36

Overload in per cent. of rated = 100 $(q - 1)$	10.0	20.0	30.0	40.0	50.0	60.0	70.0	80.0	90.0	100.0
Rated load in per cent. of maximum = $100/q$	91.0	83.3	76.9	71.4	66.6	62.5	58.8	55.5	52.6	50.0

Then with a curve such as Fig. 115, showing the fuel consumption at different percentages of the maximum load, the rated-load economy for any overload requirement may be determined, and guarantees intelligently made.

Any means of increasing the economical load range, such as suggested in the preceding paragraph, will increase the possibility of providing for overloads with good economy for the rated and lighter loads.

80. Indicator Diagrams and Maximum Piston Thrust.—Actual indicator diagrams are of great value to the designer, and some knowledge of them absolutely essential; but, as with the steam engine, it is of great convenience to be able to plot conventional diagrams which approximate actual diagrams, and these may often be used as a basis of design.

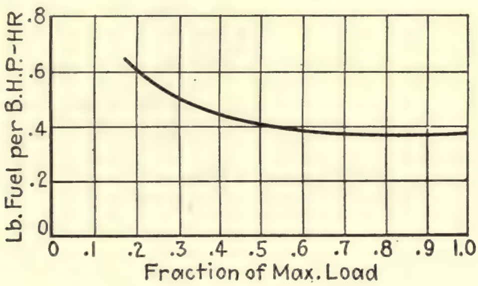


FIG. 115.

In Par. 30, Chap. VI, a method is given of plotting diagrams for the constant-volume and constant-pressure cycles. P_M may be found from Formulas (1) or (2), or assumed; then if the compression pressure of the constant-volume cycle is assumed the explosion pressure may be found by a simple calculation. The compression pressure must also be assumed for the constant-pressure cycle in order to determine the clearance volume; the volume at the end of combustion may then be found by trial and error.

The exponent of the curve must be chosen so as to give a curve as near the actual as possible; then in most cases with the constant-volume cycle, the clearance volume will be the practically required volume. The value of n varies in practice, and is not usually the same for both compression and expansion curves, although so assumed in order to simplify calculation. The value is usually taken from 1.3 to 1.35, but is sometimes as high as 1.5 for part of the curve. It is not constant in most cases but an average value may be assumed which will give close approximations. Roberts gives 1.3 as the exponent of the compression curve and 1.35 for the expansion curve. An actual constant-volume

diagram is given in Fig. 116, with a conventional diagram giving the same m.e.p. The diagrams are drawn separately as part of the outline coincides, but are to the same scale. A pencil tracing may be made of one, and by laying it over the other a comparison may be made.

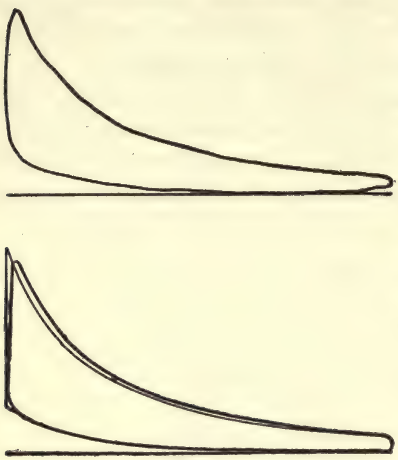


FIG. 116.

The m.e.p. of the actual diagram is 112 lb. Then from (36), Chap. VI, the pressure rise was found to be 344 lb., assuming the exponent to be 1.3; the compression pressure is 100 lb. per square inch gage and r is found from (31), Chap. VI. The suction pressure was taken as 14.7 lb. absolute, giving a ratio of clearance volume to volume of stroke of 0.26 (see Formula (33), Chap. VI).

Table 37 gives ratios of clearance to piston displacement for different gage compression pressures and absolute suction pressures, taken from a data card by the Bruce-Macbeth Engine Co. It is stated that 14.1 suction pressure is to be used from sea level to an altitude of 1000 ft.; 13.5 lb. from 1000 to 2000 ft.; 13 lb. from 2000 to 3000 ft., and 12 lb. from 4000 to 5000 ft. The value of the exponent is 1.3.

TABLE 37

Gage pressure in lb.	Ratio of clearance to piston displacement for suction of:			
	14.1	13.5	13	12
80	0.302	0.291	0.282	0.264
90	0.274	0.263	0.255	0.239
100	0.250	0.241	0.233	0.218
110	0.231	0.223	0.215	0.202
120	0.215	0.207	0.200	0.187
130	0.201	0.194	0.187	0.175
140	0.189	0.182	0.176	0.165
150	0.178	0.172	0.166	0.156
160	0.169	0.163	0.158	0.148
170	0.161	0.155	0.150	0.141
180	0.153	0.148	0.143	0.134
190	0.146	0.141	0.137	0.128
200	0.140	0.135	0.132	0.123
210	0.135	0.130	0.126	0.118
220	0.130	0.125	0.122	0.114

The conventional diagram of Fig. 116 was first drawn with a vertical explosion line; then the corners were rounded, the explosion line made slightly leaning, and from where it meets the maximum pressure line a new expansion curve is drawn with the same exponent. This increases the area, offsetting the rounded corners. The modified diagram is shown in heavy lines.

If desired, the explosion line may be left vertical and connected to the expansion line by a curve of small radius, and the maximum pressure retained for safety, as this diagram is mainly used for computing the forces acting on engine parts due to gas pressure.

Fig. 117 is a full-load diagram for a Diesel engine, and a conventional diagram. The m.e.p. is 86 lb., but at this rating the engine will carry an overload of 35 per cent.

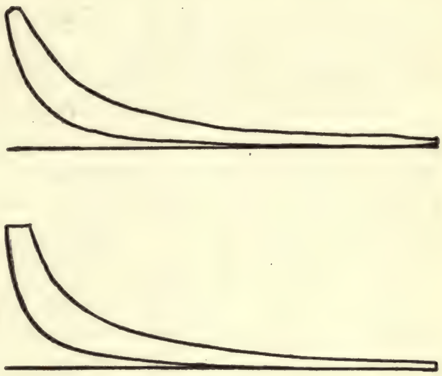


Fig. 117.

While it is sometimes stated that the exponent of Diesel curves may be as high as 1.5, the conventional diagram of Fig. 117 has rectangular hyperbolas for expansion and compression curves, and it appears that an exponent less than unity would give a somewhat closer agreement with the expansion curve. Although this diagram agrees with the actual diagram better than any with an exponent greater than unity, the equation of the hyperbola could not be used to determine the clearance volume, because the compression curve is more like an exponential curve.

Güldner says that for usual loads the value of ϵ (Par 30, Chap. VI) is 2.5, and the ratio of the "cut-off" (so called because it appears like the cut-off of a steam-engine diagram) to the entire stroke is 0.1. This gives a compression ratio of 16, and with a maximum compression pressure of 500 lb. gage and a suction pressure of 14.7 lb. absolute, the exponent for the compression curve is 1.28. Using this also for the expansion curve, the m.e.p. from (41), Chap. VI, is 119 lb. As this is from a conventional diagram it must be multiplied by a diagram factor. If this is 0.88, the m.e.p. is 105 lb., a value often found.

Fig. 118.

The actual compression curve of Fig. 117 is reproduced in Fig. 118

and compared with a curve having an exponent of 1.28, shown dotted.

For a force diagram it is likely that any of these curves will give accurate enough results.

In using (41), Chap. VI, the value of ϵ may best be found by assuming some value near 2.5 and solving for P_M. If this is greater than the required value, decrease to 2.4 and so on; a few trials will give a value near enough for practical use. If the hyperbola is used, the formulas in Par. 56, with Fig. 82 of Chap. XII may be used, in which x will be zero.

Maximum Diagram.—In design, the chief value of the indicator diagram is in the determination of pressure at different parts of the stroke; therefore, the maximum diagram for a given class should be determined. Lucke, in his Gas Engine Design uses 450 lb. per square inch gage as the maximum pressure in the cylinder; Güldner uses 356 lb. gage with such a factor of safety that the stresses may not be excessive should the pressure be 425 lb. With the maximum pressure of 356 lb., Güldner constructs a diagram which he uses in subsequent calculations. The diagram is for a producer-gas engine with a compression pressure of 128 lb. gage and m.e.p. of 96.5 lb. This diagram is given in Fig. 119 for comparison.

The maximum pressure in the Diesel engine is predetermined and is usually about 500 lb. per square inch above the atmosphere, although sometimes as high as 600 and as low as 450 lb. To allow for rounding corners the cut-off should be taken a little later than found by the actual m.e.p. In fact, with both constant-pressure and constant-volume diagrams the actual m.e.p. may be divided by a diagram factor before substituting in (36) or (41) of Chap. VI; the diagram factor may be about 0.9.

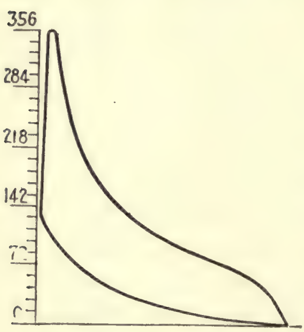


Fig. 119.

In cylinder design the maximum gage pressure must be employed. For other engine parts the maximum unbalanced pressure is required; this may include the effect of inertia, but maximum stresses should also be checked for maximum gas pressure, neglecting inertia.

Letting p_X denote the maximum unbalanced pressure per square inch with or without inertia of the reciprocating parts, and P_X the total maximum unbalanced pressure; then:

$$P_X = \frac{\pi D^2 p_X}{4} \tag{22}$$

For the total unbalanced pressure at any other part of the stroke:

$$P = \frac{\pi D^2 p}{4} \tag{23}$$

This value may be used in checking some of the combined stresses when the maximum may not occur at the point of the maximum direct thrust.

81. Application of Formulas.—This will be given by examples.

Example 1.—Design a single-acting, 4-cylinder, 4-cycle gas engine using anthracite producer gas, to develop a rated b.h.p. of 500 at a piston speed of 750 ft. per minute.

From Table 30, h = 141 B.t.u., a = 1.4 cu. ft. (the maximum) and $e_B = 0.24$; and from Table 31, e_V may be taken as 0.82. From (9), for one cylinder:

$$D = 88.25\sqrt{\frac{4 \times 125 \times 2.4}{0.24 \times 0.82 \times 750 \times 141}} = 21.2 \text{ in.}$$

This may be taken anywhere from 21 to 22 inches.

Comparing with (15), taking K from Table 33:

$$D = 0.41\sqrt{\frac{17{,}700 \times 125}{750}} = 22.25.$$

The engine size may be:

$$22'' \times 30'' - 150.$$

Example 2.—Design a single-acting, 3-cylinder, 4-cycle Diesel engine to develop a rated b.h.p. of 300 at a piston speed of 800 ft. per minute.

From Table 30, h = 18,000, a = 323 (maximum) and $e_B = 0.315$; and from Table 31, $e_V = 0.82$.

Then from (12):

$$D = 88.25\sqrt{\frac{4 \times 100 \times 323}{0.315 \times 0.82 \times 800 \times 18{,}000}} = 16.45 \text{ in.}$$

Comparing with (15), taking K from Table 33:

$$D = 0.41\sqrt{\frac{13{,}500 \times 100}{800}} = 16.85 \text{ in.}$$

The size may be taken as:

$$17'' \times 26'' - 185.$$

It is obvious that the values of K in Table 33 for these two cases are conservative, as they correspond with the maximum air supply given in Table 30.

Reference: The references at the end of Chap. V may be used in connection with this chapter.

CHAPTER XV

THE STEAM TURBINE

82. Introduction.—Chapter IV may be considered as introductory to the present chapter, in which turbine parts will be treated only in their relation to capacity and economy. A working knowledge of Thermodynamics is also essential, the flow of steam having direct application.

Aside from general treatment no originality is claimed. The excellent works of Peabody, Martin, Jude and Stodola have been studied, as well as catalogues and other material furnished by some of the leading American manufacturers of steam turbines. The prospective steam turbine designer is advised to fill in the outline here given by a study of the works mentioned.

Notation.

A = the reciprocal of Joule's equivalent = $1/778$.

k = the thickness factor—the fraction of the arc containing nozzles, which is available after deducting blade thickness, etc.

m = the ratio of blade length to pitch diameter of wheel or diaphragm.

y = fraction of kinetic energy used to overcome friction.

q = velocity coefficient—the ratio of actual velocity to what it would be without friction.

z = fraction of residual energy available for kinetic energy in next nozzles.

μ = fraction of increase of kinetic energy due to conservation of residual energy.

Δ = change of.

i = fraction of heat drop per stage utilized in reaction turbine.

j = fraction of exit energy from blade or guide of a reaction turbine utilized to produce kinetic energy in next guide or blade.

e = "indicated" thermal efficiency.

e_M = mechanical efficiency.

e_D = diagram efficiency.

e_B = blade efficiency of a reaction turbine.

F = heat factor—ratio of actual "indicated" thermal efficiency to Rankine efficiency. Sometimes referred to as "total" heat factor in multistage turbines. The "overall" heat factor is equal to $e_M F$, in which e_M may include turbine and generator.
F_S = heat factor per stage, or hydraulic efficiency.
F_R = reheat factor = F/F_S.
F_D = distribution factor.
R = distribution ratio.
V = velocity in feet per second in general; also denotes flow from nozzles or guides.
V_N = relative velocity of entrance.
V_X = relative velocity of exit.
V_R = residual velocity.
V_W = velocity of whirl.
V_F = velocity of flow.
S = velocity at pitch line of blades in feet per second.
W = steam consumption in pounds per horsepower-hour.
G = total steam consumption in pounds per hour.
w = weight of steam in pounds per second.
a = nozzle area in square inches.
v = specific volume of steam of any condition.
P = pressure in pounds per square inch absolute.
f_W = tangential force in pounds acting upon blades.
f_F = axial thrust upon blades in pounds.
E = energy in foot-pounds.
t = temperature in degrees F.
C = heat content of steam in B.t.u. above 32 degrees F., for any condition.
ΔC = adiabatic heat drop; any portion of $C_1 - C_2$ along the same entropy as that of C_1.
h = heat of the liquid.
L = latent heat of steam.
x = dryness fraction.
H = turbine horsepower.
kw = kilowatts.
D = diameter of pitch circle of wheel in inches.
d = radial length of blade in inches.
c = tip clearance in inches.
n_B = number of rows of moving blades.
n_V = number of velocity stages.
n_P = number of pressure stages.

N = r.p.m.
α = angle of nozzle or exit of guide.
θ = angle of entrance to moving blade.
ϕ = angle of exit from moving blade. Also entropy.
δ = angle due to flow of residual steam; also angle of entrance to guide.

83. The General Method.—A critical study of steam flow through a turbine, other than that based upon rather broad general principles, involves problems of great complexity and will not be attempted in this book. A few simple principles combined with the broader and better-known coefficients of performance, and a few of the simpler refinements to give a better understanding of the principles involved, will be the basis of procedure.

If in determining the power of a steam turbine a method parallel to that of the steam engine were employed, that is: if the sum of the work done upon each blade were determined in detail, there would enter into the calculation so many uncertainties that the accumulation of errors would render the determination useless. It is obvious that the large surface of the engine piston acted upon by accurately known steam pressures offers a different problem than the turbine with its multitude of blades, even though the mechanical principles involved in the latter are simple. A more general method is therefore best adapted to the turbine and will be outlined.

As it is impossible to take indicator diagrams from a turbine, there can be no indicated horsepower. An equivalent to this, the power developed by the action of the steam upon the blades, is called the *turbine horsepower* and will be denoted by H. The thermal efficiency is denoted by e; C is the heat content for steam of any condition and h the heat of the liquid as in Chap. VII. The steam consumption per hour is denoted by G and the consumption per h.p.-hr. by W.

Letting subscript 1 refer to initial absolute pressure and 2 to absolute exhaust pressure—being that of the atmosphere or condenser—Equation (4), Chap. VIII may be written:

$$W = \frac{2545}{e(C_1 - h_2)} \qquad (1)$$

From (9), Chap. VIII, and (8), Chap. VII:

$$e = F\frac{C_1 - C_2}{C_1 - h_2} \qquad (2)$$

where F is the heat factor, or the ratio of actual "indicated" efficiency to that of the Rankine cycle.

From (1) and (2):

$$W = \frac{G}{H} = \frac{2545}{F(C_1 - C_2)} \tag{3}$$

Or:

$$H = \frac{GF(C_1 - C_2)}{2545} \tag{4}$$

This formula also applies to the steam engine but is not of practical use in power determination. $C_1 - C_2$ is always the expression for heat drop due to adiabatic expansion, therefore C_1 and C_2 must always have the same entropy.

The heat factor F, sometimes called the efficiency ratio, is the product of several factors, such as blade efficiency due to form, and factors due to friction of nozzles, blades and discs, and to radiation losses. Experiment and experience have fixed some of these factors approximately, but their product must always check with F as found from test; the value of F found from tests is therefore more reliable.

As many steam turbines are employed for driving electric generators, they are often rated in kilowatts; all calculations in this book will be made in turbine horsepower which is obtained from kilowatt rating thus:

$$H = \frac{1.34kw}{e_M} \tag{5}$$

where e_M is the mechanical efficiency of turbine and generator.

Steam consumption per kw.-hr. is:

$$\frac{G}{kw} = \frac{1.34W}{e_M} \tag{6}$$

As complete expansion is always assumed in turbine operation, F must be based upon the Rankine cycle with complete expansion, given by (10), Chap. VIII, which is:

$$F = \frac{2545}{W(C_1 - C_2)} \tag{7}$$

Equating (4) and (5) gives:

$$e_MF = \frac{3410}{C_1 - C_2} \cdot \frac{kw}{G}.$$

This is sometimes known as the "overall" heat factor.

The heat factor is fairly well known for turbines of given power, and ranges from 0.45 to 0.75, and may be higher.

Formula (3) may be written:

$$G = \frac{2545H}{F(C_1 - C_2)} \tag{8}$$

Assuming F, finding C_1 and C_2 from entropy table or chart for the selected

pressures and quality, (8) gives the weight of steam per hour which must be passed through a turbine of given power. It now remains to proportion the various turbine parts to handle this steam with a minimum loss.

84. Nozzles and Other Passages.—The general formula for frictionless, adiabatic flow of one pound of steam is given by (12), Chap. VII, and is:

$$V = \sqrt{\frac{2g}{A}}\sqrt{C_1 - C_2}$$

$$= 223.7\sqrt{C_1 - C_2} \tag{9}$$

where V is the velocity in feet per second when the initial velocity is zero. The heat content C may be calculated from Formulas (2), (3) and (4) of Chap. VII, for dry saturated, wet, and superheated steam respectively, with the help of an ordinary steam table; but may be more conveniently taken from Peabody's entropy table or a Molier diagram.

In Equation (9) and in what follows, the subscript 2 does not necessarily refer to the exhaust pressure as assumed in Par. 83, but may indicate any pressure less than p_1 at which calculations are required.

The weight of steam in pounds per second which will flow through an orifice is, taking the area of the orifice in square inches:

$$w = \frac{aV}{144v} \tag{10}$$

The specific volume v may be found in Peabody's entropy table. Both V and v are the values corresponding to the area considered.

It may be shown mathematically that when a maximum weight of gas passes through an orifice there is a certain definite ratio between the initial pressure and the pressure in the orifice; and that the latter does not decrease nor the rate of flow increase, however much the pressure against which the orifice discharges is lowered. The calculated ratio for air is 0.5274. Experiments on saturated steam give a value of about 0.58, sometimes taken as 0.6.

There are a number of empirical formulas for flow through an orifice and the agreement with (9) is very close. These formulas may be found in engineering handbooks and will not be given here.

Formula (10) may be written:

$$a = 144w \cdot \frac{v}{V} \tag{11}$$

It is well known that if any fluid flows at high velocity through an orifice with sharp edges as in Fig. 120, the cross-sectional area of the jet will be less than the area of the orifice, forming a *vena contracta*. If the orifice

is formed as in Fig. 121 the jet will fill the orifice, in which case Formula (11) applies, and indicates that the velocity increases at a higher rate than the volume, the ratio being higher at the entrance. Fig. 121 is known as a *converging* nozzle and is in general the form used for the entrance of

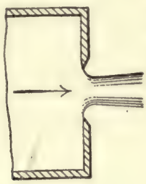


Fig. 120.

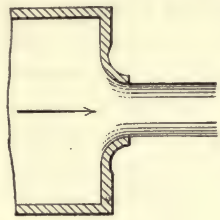


Fig. 121.

all machined nozzles. Nozzles in lower stages of compound turbines, sometimes called guide vanes, are often formed by "casting in" steel plates as in Fig. 122. Assuming the radial width uniform, the nozzle converges whether 1–2 or 1–3 be considered the width of entrance. The

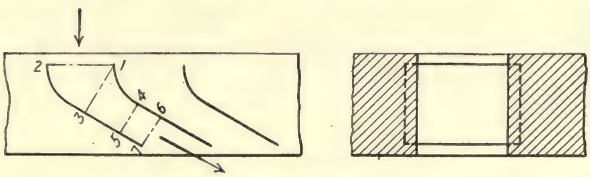


Fig. 122.

distance 4–5 is the minimum width and this may continue a short distance to 6–7; if this distance is made too great, frictional loss will result. The exit of the nozzle is at 6–7, the jet being formed by the time it reaches this cross section; and if the pressure against which it flows is not less than 0.58 of the pressure at entrance, it will retain its sectional form, dimensions and direction after it leaves the nozzle.

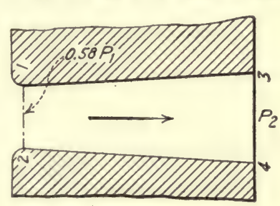


Fig. 123.

Neglecting friction, the velocity of flow will be that given by (9), the heat content C_2 being that obtained at a pressure 0.58 of the absolute pressure of the entering steam.

If the pressure against which the jet flows is appreciable less than 0.58 of the initial pressure, it will have a tendency to scatter and its kinetic energy will be dissipated if a converging nozzle is used, this effect being more marked the lower the discharge pressure; this makes it useless for turbine propulsion. Adding a diverging passage to the converging nozzle as in Fig. 123 overcomes this difficulty by con-

trolling the direction of flow until the pressure and specific volume correspond to the pressure against which the nozzle discharges; the result is a compact jet of the same cross-section as the nozzle exit. From (11), this indicates that in the diverging portion the specific volume increases more rapidly than the velocity. The relative rate of this increase has been the subject of theoretical speculation, and curves of various form have been tried, connecting the smallest area, or throat to the exit; but the straight line, commercially the simplest and cheapest, seems to give practically as good results as any other and is generally used.

The throat area is determined as for the exit of the converging nozzle, taking C and v corresponding to 0.58 of the initial pressure; the exit area is found by using C and v for the exit pressure. This calculation may be made in two stages, using the throat pressure for the initial pressure of the diverging portion, and adding the velocity so obtained to the throat velocity, but it is not so satisfactory.

Example.—Design a converging-diverging nozzle to expand adiabatically ¼ lb. of steam per sec. at 150 lb. gage, to atmospheric pressure.

Taking the nearest absolute pressures from Peabody's entropy table, C_1 is 1193.3 for an absolute pressure of 164.8 (entropy 1.56). At 96.1 lb. absolute—approximately 0.58 of 164.8—C_2 is 1149.8. Then from (9) the velocity at throat is:

$$V = 223.7\sqrt{43.5} = 1476 \text{ ft. per sec.}$$

The specific volume at throat from entropy table is 4.412. Then from (11):

$$a = \frac{144 \times 0.25 \times 4.412}{1476} = 0.108 \text{ sq. in.}$$

The corresponding diameter for a round section is nearly ⅜ in.

At exit, $C_2 = 1018$ at 14.7 lb. absolute and entropy 1.56. Then from (9):

$$V = 223.7\sqrt{175.3} = 2960 \text{ ft. per sec.}$$

At 14.7, $v = 23.13$, and from (11):

$$a = \frac{144 \times 0.25 \times 23.13}{2960} = 0.281 \text{ sq. in.}$$

For a round nozzle this is between 13/32 and ⅝ in. diameter.

If the nozzle is for a condensing turbine with 28-in. vacuum, the exit pressure nearest to this in the table is 1.005. $C_2 = 871.1$.

Then from (9):

$$V = 223.7\sqrt{322.2} = 4015 \text{ ft. per sec.}$$

At this pressure, $v = 256.8$, and from (11):

$$a = \frac{144 \times 0.25 \times 256.8}{4015} = 2.31 \text{ sq. in.}$$

For a round nozzle this is between $1\frac{11}{16}$ and $1\frac{3}{4}$ in. diameter.

Divergent nozzles formed by cast-in plates are like Fig. 122, but the portion between 4–5 and 6–7 is made longer; the area at 4–5 is the throat, and the divergence is made by increasing the radial width at the exit 6–7. This is usually done in a straight line. The form is not one of great accuracy and the sides formed by the casting are rough. Fig. 124 gives some idea of the general form.

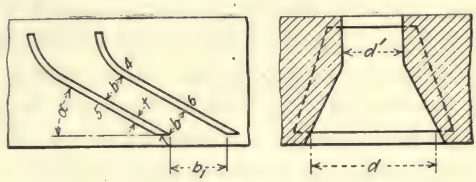


Fig. 124.

In Formula (11), a may be taken as the total nozzle area, being the product of the number of nozzles and the area of a single nozzle. For nozzles of the type shown in Figs. 122 and 124, it is often more convenient to take the total effective opening in the diaphragm. In Fig. 125 let l be the length, measured at the pitch line, of the opening containing cast-in plates, forming nozzles. This is the entire circumference in some cases. The effective area of exit of each nozzle is bd, and b is equal to $b_1 \sin \alpha$. Let:

$$\Sigma b_1 = kl$$

where k is less than unity and allows for the thickness of the plate; k may be $t/(b + t)$, or it may also allow for bridges used to strengthen the diaphragm when the nozzles extend around the entire circumference. Then the effective area with all dimensions in inches, is:

$$a = kdl \cdot \sin \alpha \qquad (12)$$

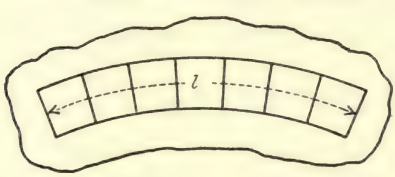


Fig. 125.

This also applies at the throat by using d' (Fig. 124) in place of d.

If $l = \pi D$, where D is the diameter of nozzle pitch circle in inches, and the ratio d/D be denoted by m, then (12) becomes:

$$a = \pi k m D^2 \cdot \sin \alpha \qquad (13)$$

Effect of Friction.—In the form of nozzle shown in Fig. 121, Prof. Rateau found the actual flow practically equal to the theoretical when the exit pressure was 0.58 of the initial; but when discharging into higher

pressures than this the flow decreased, the discrepancy between theoretical and actual being greater the less the pressure drop. Martin (Design and Construction of Steam Turbines) says that this was probably due to the formation of a *vena contracta*, reducing the effective area.

In diverging nozzles, the increased surface causes friction, which retards the flow. Various factors combine to make the actual discharge less than the theoretical, and it is well to provide for this. Let y be the fraction of the theoretically available heat energy dissipated by friction and radiation; also let it include any loss due to vena contracta or any influence resulting from an improper form of passage, since it is probably true in part that a correction of one will also reduce the other. Then the fraction of the heat drop available is $1 - y$, and the equation for flow is given by (13), Chap. VII, which is:

$$V = \sqrt{\frac{2g}{A}}\sqrt{(1 - y)(C_1 - C_2)}$$

$$= 223.7\sqrt{(1 - y)(C_1 - C_2)} \tag{14}$$

The expression $1 - y$ is known as the *nozzle efficiency.*

If the loss were wholly due to friction (which, for the usual commercial nozzles is probably an accurate enough assumption for the present purpose), a quantity of heat $y(C_1 - C_2)$ would be returned to the steam during its flow through the nozzle, increasing its entropy and volume. Formulas (14), (18) and (20) of Chap. VII give the resulting quality of steam at pressure p_2 when the heat quantity $y(C_1 - C_2)$ is added to the heat content resulting from adiabatic expansion to that pressure. The specific volume for this new value may be substituted in (11) and the area determined. The entropy table may be readily used for such problems.

Example.—Using the same data as in the previous example, determine throat and exit areas when the values of y are 0.05 and 0.15 respectively.

From the entropy table $C_1 - C_2$ for the throat is 43.5 as before; then from (14):

$$V = 223.7\sqrt{0.95 \times 43.5} = 1437.$$

The resulting heat content at 96.1 lb. is:

$$C_2 + y(C_1 - C_2) = 1149.8 + 2.175 = 1151.975 \text{ B.t.u.}$$

This occurs between the entropy 1.56 and 1.57 and we must interpolate to find the specific volume as follows:

1157.7	1151.97	4.451		4.4120
1149.8	1149.80	4.412		0.0107
7.9 is to	2.17 as	0.039	is to 0.0107	v = 4.4227

Then from (11):

$$a = \frac{144 \times 0.25 \times 4.4227}{1437} = 0.111 \text{ sq. in.}$$

This is less than 3 per cent. greater than before; the increase due to drying the steam is about 0.5 per cent.

At exit:

$$V = 223.7\sqrt{0.85 \times 175.3} = 2750$$

The resulting heat content at 14.7 lb. is:

$$1018 + 26.3 = 1044.3 \text{ B.t.u.}$$

The heat content at entropy 1.6 is 1044.9, which will be assumed as close enough. Then $v = 23.88$, and from (11):

$$a = \frac{144 \times 0.25 \times 23.88}{2750} = 0.312 \text{ sq. in.}$$

This is an increase of 11 per cent. over the area found by neglecting friction; about 3.3 per cent. of this is due to increased specific volume, the remainder to decreased velocity.

Using ordinary steam tables for determining v, x_N, the new dryness factor due to the degradation of heat may be found as follows: The value of x_2 at 14.7 lb. due to adiabatic expansion from dry steam, using the nearest value of initial pressure to 164.8, may be found by Formulas (5) and (6), Chap. VII, and is:

$$x_2 = \frac{0.524 + 1.037 - 0.3118}{1.4447} = 0.865$$

Then from (14), Chap. VII:

$$x_N = 0.865 \frac{0.15 \times 175.3}{970.4} = 0.892$$

From (16), Chap. VII:

$$v_2 = 0.017 + (0.892 \times 26.773) = 23.817 \text{ cu. ft.}$$

as compared with 23.88 found by the entropy table. All calculations were performed on the slide rule, and with the exception of the determination of volume at throat pressure from the entropy table, no interpolations were made.

The ratio of velocity reduced by friction to that due to adiabatic expansion is:

$$1 - y = \sqrt{0.85} = 0.922.$$

Formula (21), Chap. VII gives the heat returned to the steam in terms of this velocity ratio, which is there denoted by q, and is:

$$\text{Heat returned} = \frac{2960^2 (1 - 0.85)}{50{,}000} = 26.3$$

which is exactly the value found before.

The factor q is known as the *velocity coefficient* and its relation to y is:

$$q = \sqrt{1 - y} \tag{15}$$

or:

$$y = 1 - q^2 \tag{16}$$

Other Passages Used as Nozzles.—The fixed vanes, or guides, of a

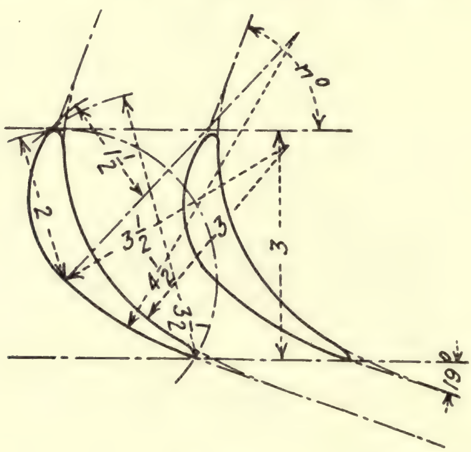


FIG. 126.

reaction turbine serve as convergent nozzles and are shown in Fig. 126, from Martin. This design is called the normal Parsons blading, and the shortest distance between the blades is made equal to ⅓ the pitch. According to Martin (Design and Construction of Steam Turbines), the discharge angle varies as the pitch is changed and cannot be determined by the tangent to the curve on the concave side, as the back of the adjacent blade is also curved and has a different tangent. In blades ⅜ in. wide (axial width), the pitch of the blades in the casing (guides) is ¼ in. and on the rotor the pitch is $\frac{3}{16}$ in. By experiment, the discharge angle of the guides is between 17 and 18 degrees, and for the blades on the rotor, between 18 and 19 degrees. Another form of blade, called the *wing blade* is used where large discharge angles are required and is shown

in Fig. 127. The discharge angle of wing blades is from 40 to 50 degrees. Semi-wing blades are normal blades specially spaced and set to have a discharge angle of from 28 to 30 degrees.

Due to the rounded backs of Parsons blades, the stream lines unite as shown by the dotted lines of Fig. 126, and according to Martin, no

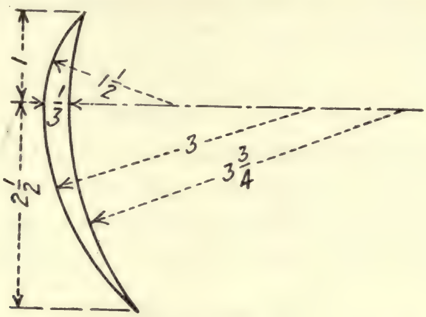


Fig. 127.

deduction is needed for thickness. Then, as these guides extend around the entire periphery, k in (12) and (13) is unity. Peabody says the necessary allowance for blade thickness is indefinite.

Guide blades of velocity-stage impulse turbines are used to change the direction of steam at constant pressure and are not considered as nozzles. The simplest form of guide is symmetrical—with inlet and dis-

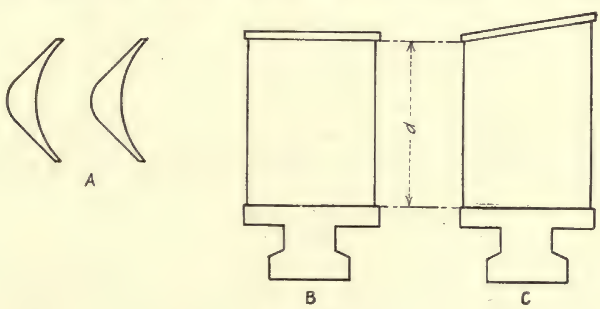


Fig. 128.

charge angles equal, as in Fig. 128. The width of the passage is constant and neglecting friction, the radial height d is constant. If friction is considered, there will be a slight increase in radial height on the discharge side as shown in Fig. 128C. Assuming q the velocity coefficient, the heat returned by friction is given by (21), Chap. VII. Putting this in place of $y(C_1 - C_2)$ in (14), Chap. VII (assuming wet steam), gives x_N; then find v

from (16), Chap. VII and substitute in (11) of this chapter, using qV_0 for V_0, the relative velocity at entrance. If the subscript N refers to the increased area:

$$\frac{a_N}{a} = \frac{v_N}{qv} = \frac{x_N}{qx} \tag{17}$$

as, neglecting the volume of water in the steam, the volume is directly proportional to the dryness factor. Then:

$$\frac{x_N}{x} = 1 - \frac{V_0^2(1 - q^2)}{50{,}000xL} \tag{18}$$

where L is the latent heat at the pressure in the guide. By similar substitution, conditions for superheated steam may be found from (18) and (20), Chap. VII, the latter being for initially wet steam becoming superheated. When the entropy table (Peabody's) is used a more direct way

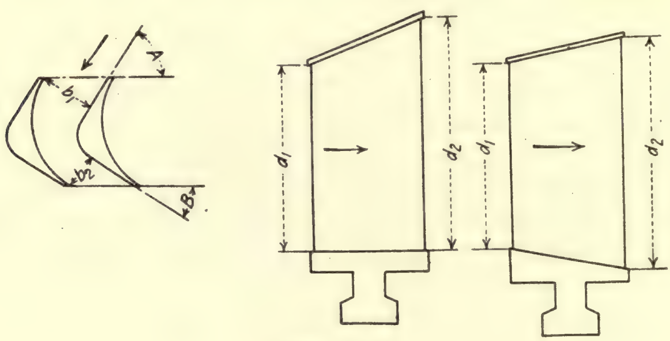


FIG. 129.

may be to add the heat returned as given by (21), Chap. VII to the heat content at the pressure considered, and then find x. The heat returned is:

$$Q_R = \frac{V_0^2(1 - q^2)}{50{,}000}.$$

Unsymmetrical guides are shown in Fig. 129. In this case the passage becomes narrower toward the exit, and for constant area of passage, the height d must increase so that the product of width and height is constant. This is approximated by making:

$$b_1d_1 = b_2d_2$$

or:

$$d_2 = \frac{b_1}{b_2}d_1 = \frac{\sin A}{\sin B} \cdot d_1.$$

Then the connecting line is made straight. This must now be multiplied by (17) if friction is to be considered. Then:

$$d_2 = \frac{x_N}{qx} \cdot \frac{b_1}{b_2} \cdot d_1 = \frac{x_N \sin A}{qx \sin B} \cdot d_1 \tag{19}$$

In some cases the ratio d_2/d_1, as given by (19) is excessive and would lead to impracticable blade dimensions; this may be remedied by increasing d_1 after d_2 has been determined. This is also discussed at the end of Par. 88. In applying (19) the angle A should be the angle determined by the velocity diagram and not the actual blade angle if increased as in Fig. 135 (to be explained later).

These equations may also apply to moving blades of impulse turbines when q refers to relative velocity.

The form of guide in Fig. 129 is sometimes used for convergent nozzles, in which case $d_2 = d_1$ and the area b_2d_2 must be determined from (11) as for any nozzle.

85. Practical Notes on Nozzles and Other Passages.—Nozzles should be used only with the range of pressure for which they are designed. *Over-expansion,* or carrying the expansion in the nozzle to a pressure below that into which the steam flows, causes a disturbance in the nozzle, with a loss of efficiency which increases rapidly with increase of back pressure. Experiments have shown that a slight amount of *under-expansion* in the nozzle—to a pressure slightly greater than the back pressure—is beneficial to blade efficiency. This may be accomplished by slightly reducing the exit area; possibly the neglect of the increase of specific volume due to friction would provide the right allowance, in which case all formulas for nozzle design are contained in this paragraph, and no reference to Chap. VII need be made. The decreased velocity given by (14) must not be overlooked.

Martin says that the entrance to the throat of a nozzle must be "very easy and well rounded" (referring to a round nozzle) or the discharge may be only 0.8 or 0.9 of the theoretical. Jude says that the inlet end should not have a large radius, but a small rounding off is advantageous. He further states that a round nozzle reasonably correct in shape, and not working with pressures entirely unsuitable, should give a velocity coefficient q of 0.95, but that undoubtedly q is a little lower for square or rectangular nozzles "on account of the natural internal instability of the jet." Also, "an appendage to a circular nozzle, to change the circular jet into a square or rectangular jet, also involves a loss of efficiency, rarely less than another 3 per cent. (velocity), and may often amount to 10 or 15 per cent., according to the way it is made, its continuity and the condition of the surfaces."

Martin says that curvature has an adverse effect on efficiency, especially when the curves leading to the throat are abrupt.

For experiments upon straight convergent-divergent nozzles with easy well-rounded entrance and taper not too rapid, Martin gives a formula for loss of energy, which in our notation is:

$$y = \frac{6(C_1 - C_2 - 45)}{10{,}000} \tag{20}$$

He says that experiments upon nozzles with curved entrance and in which a round section changed to rectangular (nearly square) at exit, gave values fully twice as great. For the example of nozzle design previously given, $C_1 - C_2 = 175.3$. Then from (20), $y = 0.078$, or $1 - y = 0.922$. From (15), $q = 0.96$; or the loss of velocity is 4 per cent. Twice the energy loss gives $1 - y = 0.844$, or $q = 0.918$; or the velocity loss is 8.2 per cent. These figures may be compared with those given by Jude.

Martin says that round nozzles lead to inefficiency in turbine practice. This is probably due to the entrance of a circular jet into the rectangular opening between the blades. However, round nozzles are much used, especially in the first stages of certain types.

No definite rules can be given for taper of nozzles. Too small a taper gives long nozzles with resulting friction; while if too large, the effect upon the jet is detrimental. A total taper (change of diameter) of from 1 in 6 to 1 in 12 is used for straight round nozzles. A smaller taper is sometimes used, especially when the increase of area in the divergent portion is small. Practical construction may sometimes call for a much greater taper than 1 in 6 for such nozzles as shown in Fig. 124, and in all cases judgment must be used.

From various experiments, Martin gives some practical values of y and q as follows: When $V = 800$ to 1200, $q = 0.92$ and $y = 0.15$. For nozzles similar to Figs. 122 and 124, the values were lower. The maximum values found for these were with a superheat of 130 degrees F. and with $V = 1900$ to 2100; then $q = 0.95$ and $y = 0.098$. When the superheat was but 4 degrees F., and $V = 1770$, $q = 0.915$ and $y = 0.16$. For Figs. 126 and 127 no direct experiments were made, but computed velocities from turbine tests indicate that for $V = 200$ to 300, $q = 0.95$ and $y = 0.1$.

In all *curved passages*, centrifugal force results, causing variation in pressure, the maximum being at the concave surface. If unconfined laterally the jet spreads, reducing the efficiency. Experiments indicate that the value of q for the passages between impulse blades for theo-

retical steam velocities of from 200 to 2600 ft. per sec., when the jet is unconfined laterally, is given by the formula:

$$q = 0.66 + \frac{V}{15{,}000} \tag{21}$$

When the jet is confined by shrouding as in Fig. 130, the value of q may be much increased. From (21) the velocity coefficient for unshrouded guides and blades is greater at high steam velocities, which is contrary to the usual effect of friction. Peabody gives for flow through guides and blades:

$$y = \frac{V}{10{,}000}$$

which gives:

$$q = \sqrt{1 - \frac{V}{10{,}000}} \tag{22}$$

This is no doubt intended for unworn, shrouded blades made by machining or drop forging.

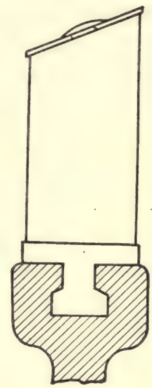


FIG. 130.

Jude says that the average maximum velocity coefficient for the best form of closed-vane passage is about 0.955, but that the average value of the better-known turbines appears to be about 0.92. However, various phenomena due to the steam surrounding the vanes and the transfer of steam from one passage to another give an equivalent to the reduction of this value, sometimes to as low as 0.6 or 0.7. Martin gives from 0.68 to 0.72, while Rateau gives 0.75 as a fair average. Jude states that he has confirmed Rateau's results, but that with special blades, he has obtained values as high as 0.8.

As we have taken q as the ratio of actual to theoretical velocity at the exit of the passage, and as the relative velocity in one vane passage of an impulse turbine is theoretically constant up to the entrance of the next vane passage, the intervening space with its losses may be considered as a part of the passage and these latter values of q as applying to this case, instead of the higher values found by experiment upon single rows of blades. These higher values may only be used with other factors which separately account for other losses, a detailed study of which may be found in the works cited. Jude says, however, that for velocity-stage blading it is probably more correct to take a higher value of q, such as 0.92, as an average in ascertaining the diagram efficiency. It is possible that as high as 0.85 may be taken for q in the case of velocity-stage turbines, but why it should be greater than for a simple impulse wheel is not altogether clear.

The constants for blade passages of the reaction turbine may best be treated under reaction blading. A number of practical nozzle designs are shown in Chap. XXXIII.

In guide vanes and nozzles formed by casting in plates the pitch should be as large as is consistent with properly directing the stream.

Methods of drawing nozzles, guides and blading are given in Chaps. XXXI and XXXIII.

86. Impulse Blading.—A simple arrangement of nozzles and blading for an impulse turbine is shown in Fig. 460, Chap. XXXIII. It will be seen that steam from the nozzle will sometimes impinge upon several blades at one time, but in our conception of the operation we may assume one pound acting upon a single blade.

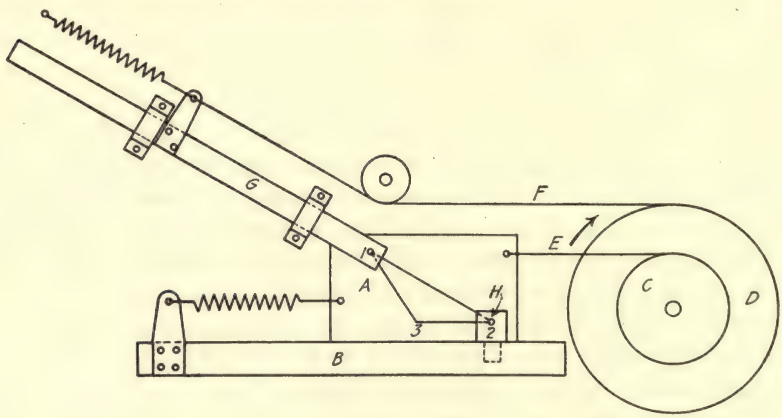


Fig. 131.

The means of determining the jet velocity has already been given by (14). The change of velocity and the consequent change of kinetic energy which the jet undergoes in its passage between the blades may best be explained by the *velocity diagram*, the fundamental form of which may be easily understood by the crude apparatus shown in Fig. 131, which needs little explanation.

A paper may be tacked to the board A which slides upon B. The sliding bar G holds a pencil in its lower end, and another pencil is held by H which is attached to B, but made removable. C and D may be grooved pulleys fastened together, the ratio of diameters being the ratio of jet to blade velocity.

To draw a diagram, disconnect the cord E and remove H; then turn the pulleys so as to slide bar G and draw line 1–2. This represents the direction and velocity of the jet from the nozzle. Connect cord E and

disconnect cord F after replacing G to its original position and removing the pencil. Place H so that its pencil point is at 2 and turn the wheel through the same angle as before, drawing line 2–3. This represents the direction of motion and velocity of the turbine blade. Move A back so the pencil in H is at 2 again, than remove H, connect cords E and F, place pencil of G on paper at 1 and turn wheel through same angle as the two previous times. The relation of the movements of G and A will be that of the jet to the moving blades, and the line 1–3 will be drawn on the paper. This is the direction of the pencil in G with reference to the paper, or, it represents the direction of the jet and its velocity in relation to the moving blade at the time the steam begins to enter the blade passage, and is known as *relative velocity*. If friction were absent and there were no other forces tending to change the velocity (such as expansion in the blades of the reaction turbine), the relative velocity would be constant, but its direction would change due to the influence of the blade passage which now controls its flow.

The lines 1–2 and 2–3 represent velocities with reference to a fixed object upon the earth's surface and are known as *absolute velocities*. As in marine service the turbine is in motion, the fixed parts of the turbine, such as the casing, may be considered the zero of motion from which absolute velocity is measured. By changing the angle of the rod G, or the relative diameters of C and D, velocity diagrams of different form may be drawn; it is simpler, however, to abandon the apparatus, which was only mentioned with the thought of making the matter plain to some who might not easily grasp the principle.

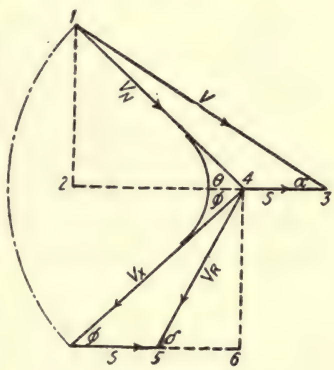


FIG. 132.

A simple velocity diagram, including the diagram for exit from the blade, is shown in Fig. 132. The concave surface of the blade against which the driving force is exerted is shown by the heavy line, the back of the blade being omitted. The blade is shown tangent to V_N, the relative velocity at entrance, so that the jet may enter without shock; this is the usual treatment, but more will be said upon the subject presently. The blade curve must be tangent to line V_X for impulse blades, to insure the direction of flow.

The blade shown in Fig. 132 is symmetrical, angles θ and ϕ being equal; and as friction is neglected, V_X, the relative velocity at exit, is

equal to V_N. S is the velocity of the blade (at the pitch circle) and must be equal in both entrance and exit diagrams.

If in Fig. 131 bar G were changed to the angle δ of Fig. 132 and the line V_R drawn with the paper stationary instead of the line 1–2 of Fig. 131, the movement of both bar and paper would draw the line V_X for the line 1–3, which must then be relative velocity as already stated. This shows that line V_R of Fig. 132 represents absolute velocity, and it is known as *residual velocity.* The reduction of the velocity of the steam in its passage through the blades is the difference between V and V_R, both being absolute velocities. The kinetic energy due to residual velocity V_R is the energy lost, and should be kept as small as practicable.

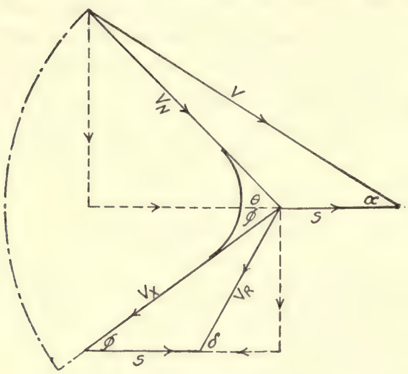


Fig. 133.

The absolute entrance velocity V of Fig. 132 may be resolved into two components, one parallel to the shaft center and represented by line 1–2, called the *velocity of flow;* the other in the direction of blade motion by line 2–3, called the *velocity of whirl.* Likewise the absolute exit velocity, or residual velocity may be resolved into 4–6, the *exit velocity of flow*, and 5–6, the *exit velocity of whirl.* The latter, in the diagram given is negative, being opposite in direction to the blade movement; but the momentum due to it drives the blade forward so is considered positive.

A more general treatment may be given by considering friction and assuming an unsymmetrical blade; such a diagram is Fig. 133, to which the same general definitions apply.

The kinetic energy supplied to the blade is due to the jet from the nozzle at velocity V, and is, in B.t.u. for 1 lb.;

$$A\frac{V^2}{2g} = \frac{V}{778 \times 64.32} = \left(\frac{V}{223.7}\right)^2$$

The rejected energy due to residual velocity is:

$$\left(\frac{V_R}{223.7}\right)^2$$

and the energy lost by friction is:

$$\left(\frac{V_N}{223.7}\right)^2 - \left(\frac{V_X}{223.7}\right)^2$$

or:

$$\left(\frac{V_N}{223.7}\right)^2 (1 - q^2)$$

The efficiency of the blades, or *diagram efficiency* e_D, is then given by:

$$e_D = \frac{V^2 - V_R^2 - V_N^2(1 - q^2)}{V^2} \tag{23}$$

The force of impact is the product of mass and velocity change per second. It is obvious that the change of tangential velocity of the steam produced by its impingement upon the turbine blade, is the difference between the tangential velocity, or velocity of whirl, at entrance and exit; which is:

$$V \cos \alpha - V_R \cos \delta.$$

The direction of blade motion having the plus sign, the value of $V_R \cos \delta$ is negative. To obviate the necessity of considering which sign this value has, we may, in view of the fact that the tangential component of V_X is always negative, and S always positive, write:

$$V_R \cos \delta = - V_X \cos \phi + S$$

Then the tangential velocity change is:

$$V \cos \alpha - (- V_X \cos \phi + S)$$

or:

$$V \cos \alpha + V_X \cos \phi - S$$

Then, substituting qV_N for V_X, the tangential force due the flow of w lb. of steam per sec. is:

$$f_W = \frac{w}{g} (V \cos \alpha + qV_N \cos \phi - S) = \frac{w}{g} V_W \tag{24}$$

This is the load on all blades receiving the steam; to find the load per blade divided by the number of blades.

Similarly, the axial pressure on the blades is due to the difference of the velocity of flow at entrance and exit, and is given by the expression:

$$f_F = \frac{w}{g} (V \sin \alpha - qV_N \sin \phi) = \frac{w}{g} V_F \tag{25}$$

This is known as *axial thrust* and must be provided for by a thrust bearing. If f_F is negative the thrust is in the opposite direction from the velocity of flow. Formulas (24) and (25) are general and apply to reaction blades also.

A graphical construction for the parenthetical quantities of (24) and (25) is given in Fig. 134, which may be drawn to scale and then measured as an alternative. Fig. 134 is a reproduction of Fig. 132 with the line V_X $(= qV_N)$ produced on the upper side of the center line of blades.

The useful work per second done upon the turbine wheel is $f_W S$ ft. lb.; and as $wV^2/2g$ is the ft. lb. of energy supplied by the nozzles per second, the diagram efficiency is:

$$e_D = \frac{f_W S}{\frac{wV^2}{2g}} = \frac{2S}{V^2}\,(V \cos \alpha + qV \cos \phi - S) \tag{26}$$

The quantity in parenthesis may be calculated, or measured from the diagram, Fig. 134. Formulas (23) and (26) give the same result, the latter being more convenient.

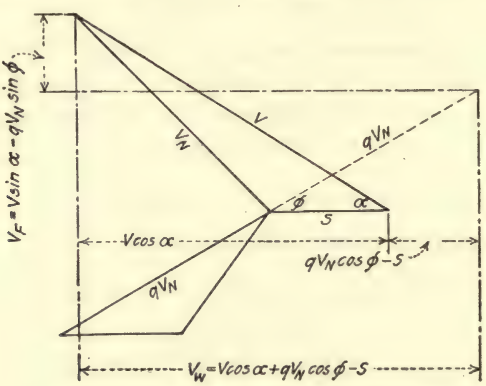


FIG. 134.

Formula (26) may also be written:

$$e_D = 2\,\frac{S}{V}\left(\cos \alpha + q\,\frac{V_N}{V}\cos \phi - \frac{S}{V}\right)$$

From Fig. 133:

$$V_N \cos \theta = V \cos \alpha - S$$

or,

$$\frac{V_N}{V} = \frac{\cos \alpha - \frac{S}{V}}{\cos \theta}$$

Substituting in Formula for e_D and rearranging gives:

$$e_D = 2\,\frac{S}{V}\left(\cos \alpha - \frac{S}{V}\right)\left(q\,\frac{\cos \phi}{\cos \theta} + 1\right)$$

$$= K\left(q\,\frac{\cos \phi}{\cos \theta} + 1\right) \tag{27}$$

Also $V_N \sin \theta = V \sin \alpha$

or,

$$\frac{V_N}{V} = \frac{\sin \alpha}{\sin \theta}$$

Equating with the value of V_N/V just given:

$$\tan \theta = \frac{\sin \alpha}{\cos \alpha - \frac{S}{V}} \tag{28}$$

When S/V and α are known K may be found, and $\cos \theta$ may be determined from (28). Table 38 gives values of K and $\cos \theta$ for different values of α and S/V; then by assuming different values of q and ϕ, e_D may be quickly determined. Within reasonable limits it is apparent that decreasing ϕ increases the efficiency; and that for a given value of $\cos \phi/\cos \theta$ and q the efficiency varies directly as K. Then it may be seen that decreasing the angle α increases the efficiency, and for maximum efficiency such a value of S/V must be chosen which will make K a maximum. This may be found by equating the first derivation of

$$\frac{S}{V} \cos \alpha - \frac{S^2}{V^2}$$

to zero, which gives:

$$\frac{S}{V} = \frac{\cos \alpha}{2} \tag{29}$$

TABLE 38

α	S/V											
	0.3			0.4			0.5			0.6		
	K	θ	$\cos \theta$	K	θ	$\cos \theta$	K	θ	$\cos \theta$	K	θ	$\cos \theta$
15	0.400	21°–13′	0.932	0.453	24°–37′	0.929	0.466	28°–54′	0.875	0.440	35°–11′	0.817
20	0.383	26°–53′	0.892	0.432	32°–25′	0.844	0.439	37°–55′	0.789	0.407	45°–51′	0.696
25	0.364	34°–58′	0.819	0.405	39°–54′	0.767	0.406	46°–8′	0.693	0.366	54°–5′	0.586

This will give the maximum value of K for any given value of α. Then S is one-half the velocity of whirl, and neglecting friction, Formula (27) for symmetrical blades reduces to:

$$e_D = \cos^2 \alpha$$

or with unsymmetrical blades:

$$e_D = \frac{\cos^2 \alpha}{2}\left(\frac{\cos \phi}{\cos \theta} + 1\right)$$

which may give a greater efficiency and tend to reduce the effect of friction.

87. Practical Notes on Impulse Blades.—The velocity coefficients for impulse blades and guides are given at the latter end of the preceding paragraph; these are for shrouded blades, which are practically always used with impulse turbines.

While the nozzle exit is reduced slightly from the area theoretically indicated, as stated in Par. 84, the blade and guide passages must not be restricted as this may lead to throttling. It is probably well to consider all effects of friction and use Formula (17) or (19) in determining exit areas. As the clearance space between nozzles and blades probably gives rise to friction loss with a possible slight spreading of the jet, the radial length of the blades may be slightly greater than the radial width of the nozzles; possibly if this were made equal to what the nozzle width would be by making allowance for the drying effect of friction, the condition would be properly met. Some designers increase the blade length beyond this rather arbitrarily to avoid any possibility of choking (see also velocity-stage blading); with round nozzles, however, the blade areas would exceed nozzle areas and choking would not be likely.

As stated earlier in this paragraph, the entrance angle of the blade is usually assumed equal to the angle θ at which the steam enters the blade; the idea is that with such a construction the steam slides into the blade passage without shock, and that friction is thus reduced to a minimum. According to Martin, this is an error, and better practical results have been obtained by making the blade angle some 5 to 15 degrees greater than the angle θ, Fig. 133. This is shown in Fig. 135. If this is not done, the decreasing relative velocity due to friction changes the direction in which the steam attempts to flow, exerting pressure on the blade surface forming the back of the passage and tends to retard the motion of the wheel. The trajectory diagrams, Fig. 150, are drawn with the entrance angle of the moving blades greater than θ, and it may be seen that the stream from the nozzle is deflected backward slightly upon joining the trajectory curves, showing a forward impulse. Had the entrance angle been equal to θ, the opposite would have been true, as friction is taken into account in these diagrams. This is explained in Par. 92.

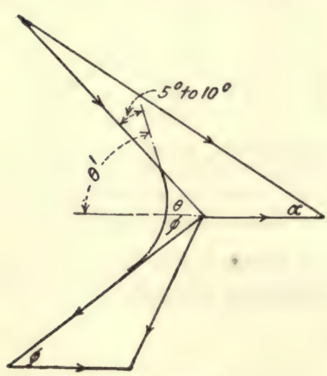


Fig. 135.

The advisability of making ϕ less than θ must be left to the designer's

judgment; it is usually done and no doubt increases the efficiency if not carried too far.

Jude says that the drying of the steam due to blade friction is improbable, as any water present is thrown to the outside of the curved path by centrifugal force and is difficult to reëvaporate.

No definite rule controls the blade length. For reaction turbines Martin gives a minimum length of $\frac{1}{25}$ the drum diameter for stationary turbines and 1/75 for marine turbines, the latter, however involving a loss of efficiency. The maximum length for the low-pressure end is given as 1/5 the pitch diameter. These ratios m may be used as a guide and should be as reliable for impulse turbines.

There seems to be no rational method of determining the pitch of impulse blading, which in practice ranges from 48 to 68 per cent. of the axial width. It would seem reasonable to make the pitch as large as possible and still properly direct the steam, thus reducing the friction surface, and it is probable that the greater the radius of curvature of the blade, the greater the pitch may be. A method of drawing blades is given in Chap. XXXI, by which the blades of Figs. 149 and 150 were drawn.

The speed at the pitch circle of simple impulse turbines such as the De-Laval ranges from 500 to 1400 ft. per second. For pressure-stage impulse turbines a range from 300 to 650 ft. is used, some large modern turbines having a blade-tip velocity as high as 950 ft. per sec.; in this case the blade length of the last wheel exceeds $\frac{1}{4}$ of the pitch diameter, or m is greater than $\frac{1}{4}$. The higher velocities necessitate great care in the designing of wheels and shafts which is discussed in Chaps. XXXI and XXXII.

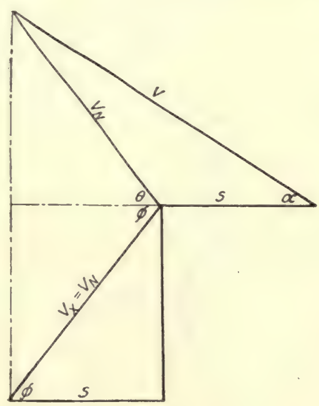


Fig. 136.

88. Velocity-stage Impulse Turbine.—For the purpose of studying the principle of speed reduction of the velocity-stage turbine, frictionless, symmetrical blades and guides will first be considered. A diagram for a single stage with zero exit velocity of whirl is given in Fig. 136. In this diagram, $\theta = \phi$, and it is obvious that the triangle formed by the exit diagram is equal to that formed by the dotted lines at the left of the entrance diagram. These are shown together in Fig. 137. Retaining the zero exit velocity of whirl, assume that the blade speed S is to be made one-half that shown in Fig. 137. This is done by dividing the base line

into two parts, forming four triangles as shown in Fig. 138. Disposing these triangles in a manner similar to Fig. 136 gives Fig. 139, which is a double diagram, having two rows of moving blades and one row of guides, which, except that there is no expansion in them, serve as nozzles for the

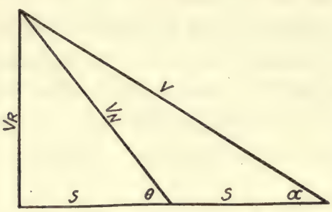


FIG. 137.

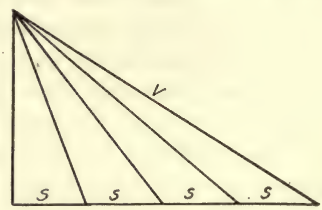


FIG. 138.

blades of the second wheel. Subscripts 1 and 2 are for the first and second stages respectively.

From Fig. 139 it may be seen that for the assumption made, the blade speed varies inversely as the number of velocity stages operated from one set of nozzles. In practice there is the ever-present friction, and the blades and guides are not always symmetrical, especially the latter; as when angle α_2 is reduced—sometimes to equal θ, in 2-stage wheels—the blades are not so flat in form. Fig. 140 is a diagram for a 2-stage wheel with these modifications.

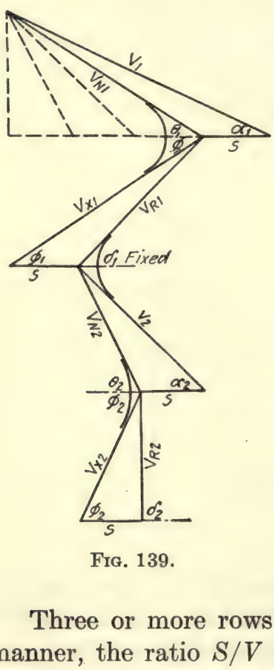


FIG. 139.

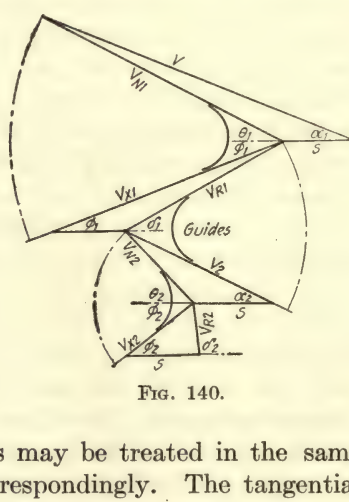


FIG. 140.

Three or more rows of moving blades may be treated in the same manner, the ratio S/V being reduced correspondingly. The tangential impulse on the blades may be found for each moving row by (24), or

graphically from Fig. 134, using subscripts 1, 2, 3, etc., for the successive stages. The total energy is given by:

$$S\ (f_{W1} + f_{W2} + f_{W3},\ \text{etc.})$$

Letting $V_W = V \cos \alpha + qV_N \cos \phi - S$, the parenthetical quantity of (24), the total energy is:

$$\frac{Sw}{g}\,(V_{W1} + V_{W2} + V_{W3}, \text{etc.})$$

and as the energy of the jet from the nozzles is $wV_1^2/2g$, the diagram efficiency is:

$$e_D = \frac{2S}{V_1^2}\,(V_{W1} + V_{W2} + V_{W3},\ \text{etc.}) \qquad (30)$$

From the equations of paragraph 86 it would seem that whatever the velocity coefficients were, maximum efficiency would be expected when the diagram is constructed so that:

$$S = \frac{V \cos \alpha}{2}$$

for the last stage, if this does not necessitate too small a value of α.

The tangential pressure on the blades may be found from (24), and this gives the proportion of work done by each set of moving blades. The axial thrust is the sum of the thrust on all sets of moving blades, and may be found from (25), using the proper subscripts.

In Fig. 139 it may be seen by inspection that $V_1 \cos \alpha_1 = 4S$, and $V_{N1} \cos \phi_1 = 3S$. Then $V_{W1} = 6S$; also $V_2 \cos \alpha_2 = 2S$, $V_{N2} \cos \phi_2 = S$. Then $V_{W2} = 2S$, or:

$$f_{W1} = 3f_{W2}$$

In like manner it may be shown that for symmetrical blades and guides, neglecting friction, the ratio of impulse, beginning with the first row is:

For 3 stages....................5, 3, 1
For 4 stages....................7, 5, 3, 1
For 5 stages....................9, 7, 5, 3, 1

This is also the proportion of work done by the respective stages. These proportions vary with actual blades, but give some idea of the value of the lower stages.

89. Practical Notes on Velocity-stage Turbines.—The velocity coefficients for velocity-stage turbines vary considerable, making the diagram efficiency calculated from (30) rather uncertain. With the

low value of q sometimes found by experiment, but little work would be done by the lower stages, with even the possibility that the extra work of dragging one or more idle wheels be imposed upon the turbine, especially at light loads with the steam throttled. It is therefore seldom that more than three velocity stages are employed in stationary turbines, although four are sometimes used for the high-pressure stage (see Par. 91) of marine turbines, and as many as five for the high-pressure stage of the reversing turbine, in which economy of space is more important than economy of steam for the relatively short periods it is in use.

It is evidently important to consider all points which tend toward efficiency. The radial length at entrance of blades and guides may be slightly greater than the exit length of the preceding guides, blades or nozzles, to avoid choking; and the ratio of exit to entrance length of each row of blades and guides may be found from (17) or (19).

In some turbines, even though the blades are not symmetrical, the blade length is the same at entrance and exit, probably for constructional reasons. Martin, commenting upon a Curtis marine turbine with four velocity stages in the first pressure stage says: "The bucket angle of the first row at entrance is, it will be seen, 28 degrees, while at discharge it is 22 degrees. From this it follows that the space between the buckets is narrower at discharge than at entrance, but this is in accord with the fact that the stream of fluid as it passes through the bucket tends to spread laterally, and consequently becomes thinner. It will further be observed that each successive bucket is longer than its predecessor. This is necessary, because in each bucket the fluid, besides thinning, as already mentioned, also loses velocity, and thus a greater steamway is required at each successive set of buckets."

As the turbine referred to had shrouded blades, the spreading and thinning of the stream could occur only if the passages did not run full at entrance. Concerning the necessary increase in area due to decreasing velocity, this is provided for by the increasing angles; the areas are proportional to the sines of the angles, and between two rows of blades the product of the sine of the angle and the velocity is the same, so that neglecting friction in the space between the rows, and spilling, the exit length of one row and entrance length of the next could be the same. Had the blades in question been computed by (19), with an increase of entrance over the exit of the preceding row as already mentioned, then if in each case both blade edges had been made the same length as the longer, a similar result would have been obtained; any difference would probably be due to the arbitrary increase of entrance allowed.

In some cases, after the exit length of the last blade has been

found, straight lines from its extremities, drawn to the extremes of nozzle opening, give rather arbitrarily the dimensions of all blades and guides.

Should the thickness factor of the blades not be the same, allowance may be made, but this is usually negligible.

In Par. 97, Fig. 163 shows the arrangement of blades for two velocity stages of the first pressure-stage of the turbine designed as an example of formula application. Formula (19) was used to determine blade lengths, neglecting the drying due to friction, and the increase of entrance height over the preceding exit height; then the form was modified as explained. The ratio of maximum height of the last blade to the nozzle diameter is 1.71.

A similar layout for a Curtis turbine shown by Martin gives a ratio of 1.73; another, however, in which each row has a constant height, has a ratio of but 1.2.

It seems probable that Formula (19) is practical, with modifications suggested and illustrated in Figs. 164 and 165. The drying effect may be taken into account if desired, although such drying is discredited by Jude, as previously stated.

In spacing blades, the pitch is sometimes made the same for all rows, but is often increased as the entrance and exit angles are increased, which seems reasonable. Methods of laying out blades are given in Chap. XXXI.

Axial clearance between nozzles and blades has less influence upon economy than clearance between blades and guides, and experiment has shown that the loss due to the latter increases rapidly with increase of clearance. Axial clearance should therefore be kept as small as reliability of operation will permit.

As stated in Par. 86, the reduction of exit angle of blades is a matter of judgment but is customary, and the exit angle of guides should be reproduced in order that the following blades may not be too flat: a comparison of Figs. 139 and 140 will make this clear. Prof. Peabody says that a conservative and convenient arrangement is to make the exit guide angle equal to the preceding blade angle; or, $\alpha_2 = \theta_1$, $\alpha_3 = \theta_2$, etc.

The entrance angles of all moving blades should, according to Martin, be made from 5 to 10 degrees greater than θ, as stated in Par 86.

Values of e_D found from practice are given by Martin as follows:

2-stage. . . . . $e_D = 0.72$
3-stage. . . . . $e_D = 0.65$
4-stage. . . . . $e_D = 0.52$

He states that for maximum efficiency the ratio of V/S for a 4-stage turbine should be about 11. These values may be used as a check. Velocity coefficients are given in Par. 84.

90. Pressure-stage Impulse Turbine.—The equation connecting flow with heat drop is given by (14), Par. 84, and is:

$$V = 223.7\sqrt{(1 - y)\ (C_1 - C_2)}$$

Taking $C_1 - C_2$ as the total adiabatic heat drop from steam pipe to condenser, we may divide it into a number of parts as explained in Par. 10, Chap. IV. This will cause a series of pressure drops until the exhaust pressure is reached, the pressure in the successive compartments of the turbine being maintained by properly proportioning the nozzle areas connecting them, so that the inflow and outflow for each compartment in a given time is the same, and equilibrium is not secured until a certain definite pressure, predetermined for a given weight of steam per second, is attained in each stage.

For simplicity, let n_P be the number of equal pressure stages; that is, the heat drops being equal. We then have for the velocity of the jets flowing into each stage, assuming y to be the same in each stage and neglecting certain influences to be mentioned later:

$$V = 223.7\sqrt{(1 - y)\frac{C_1 - C_2}{n_P}} \tag{31}$$

If the pitch circle of the wheels is the same in all stages, and the nozzle and blade angles the same, the velocity diagram will be the same for each stage, and from (31) it may be seen that the blade velocity is in inverse proportion to the square root of the number of pressure stages.

In the nozzle example of Par. 84 for a noncondensing turbine, the velocity V from the nozzle is 2750. Assume Fig. 133 as the velocity diagram, in which S/V is $\frac{1}{3}$. Then S, the blade velocity is 2750/3, or 916 ft. per sec. Now assume the same total heat drop for a turbine with four pressure stages. The blade speed is then:

$$S = \frac{916}{\sqrt{n_P}} = \frac{916}{2} = 458.$$

If the wheels of the two turbines are the same diameter, the r.p.m. of the 4-stage turbine is one-half that of the single stage. Pressure-stage turbines may then be used when lower speeds are desired. The velocity diagram may be drawn as for a simple turbine and used for a pressure-stage turbine with any number of stages by changing the scale.

Intermediate pressures may be found by means of entropy table or chart. Taking the same 4-stage turbine but neglecting friction and

assuming adiabatic expansion, if we find the heat content C for each stage, the corresponding pressure may be read from table or chart.

During expansion in the first set of nozzles, a quantity of heat equal to $(C_1 - C_2)/n_P$ was used, leaving the heat content in the first stage:

$$C_A = C_1 - \frac{C_1 - C_2}{n_P}$$

in the second stage:

$$C_B = C_A - \frac{C_1 - C_2}{n_P} = C_1 - 2\frac{C_1 - C_2}{n_P}$$

in the third stage:

$$C_C = C_B - \frac{C_1 - C_2}{n_P} = C_1 - 3\frac{C_1 - C_2}{n_P}$$

and in the fourth stage:

$$C_D = C_2$$

This may be applied to any number of stages. Proceeding with the example: At 164.8 lb. absolute and entropy 1.56, $C_1 = 1193.3$. At 14.7 lb. absolute and the same entropy, $C_2 = 1018$. Then as $(C_1 - C_2)/4 = 43.825$:

First stage	$C_A = 1193.3 - 43.825 = 1149.475$,	and $P = 95.715$
Second stage	$C_B = 1193.3 - 87.650 = 1105.650$,	and $P = 53.568$
Third stage	$C_C = 1193.3 - 131.475 = 1061.825$,	and $P = 28.758$
Fourth stage	$C_D = C_2 = 1018$	and $P = 14.700$

Pressures were found by interpolation, using Peabody's entropy table. The heat quantity 43.825 B.t.u. is then available in each stage, and may be illustrated by the entropy diagram, Fig. 141, which is divided into four equal parts by the pressures given. To make the problem a little plainer, a diagram of a 4-stage turbine is given in Fig. 142, showing the pressure distribution.

Effect of Heat Factor.—The example and Fig. 141 assume a perfect turbine, in which the heat factor (see Par. 41, Chap. VIII) is unity. Due to the diagram efficiency being less than unity, to disc friction and other disturbing influences, the heat factor is less than unity, and a part of the kinetic energy of the jet is converted into heat, increasing the dryness factor, volume and entropy. This change takes place during the entire pressure drop, and while we do not know the exact form of the curve, we know the overall heat factor from tests and may thus find the entropy of exhaust pressure, and assuming the same percentage of loss in each stage, may fix several points.

Assume the total heat factor F to be 0.6; then, neglecting radiation loss $0.4 \times 175.3 = 70.12$ B.t.u. are returned to the steam at exhaust pressure

as heat, making the heat content at 14.7 lb., 1088.12 B.t.u. Interpolating between entropies 1.66 and 1.67;

1091.9	1088.12	1.67	1.6600
1085.1	1085.10	1.66	0.0044

6.8 is to 3.02 as 0.01 is to 0.0044. Entropy is 1.6644

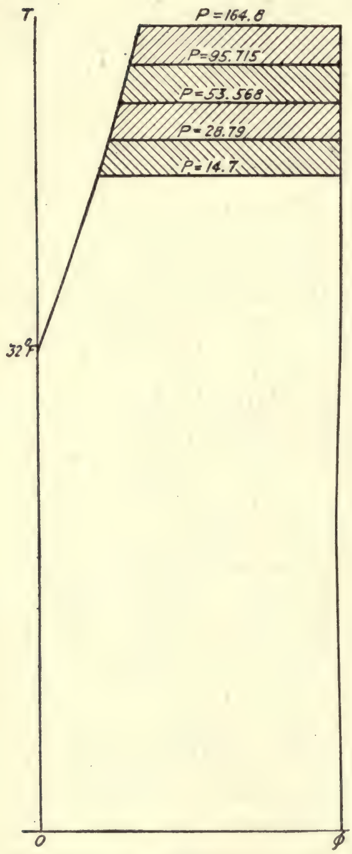


Fig. 141.

Interpolating between entropies is usually unnecessary for practical work, especially with few pressure stages. Even with the nearest entropy 1.66, the increase of entropy is 0.1 between 164.8 and 14.7 lb. Fig. 143 shows the increase of entropy, the dotted lines being the assumed curves of entropy change due to nozzle friction, and the full curves at the right the general curve of entropy increase. The distance from the lower end of the dotted curves to the full curve represents the increase of entropy due to the change of

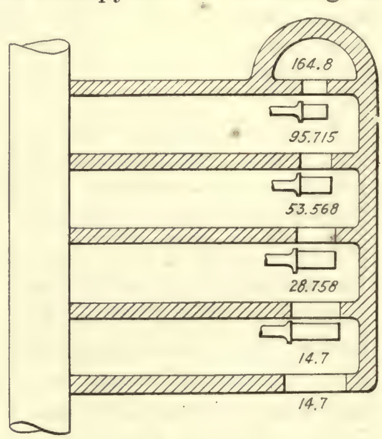


Fig. 142.

the kinetic energy of residual velocity into heat at constant pressure, and this is the larger part of the change. The diagram is exaggerated for illustration.

The shaded areas represent the available adiabatic heat drop for each stage, and it is obvious that if these areas are to be equal, the intermediate pressure must be lower than for Fig. 141. It must be remembered

that the work done is no more than before, but assuming an equal division, the work per stage is:

$$\frac{F(C_1 - C_2)}{n_P} \tag{32}$$

This is a larger fraction of one shaded area of Fig. 141 than of one area of Fig. 143, the former being equal to $(C_1 - C_2)/n_P$, and the fraction the total heat factor F; for the latter, the area being greater than $(C_1 - C_2)/n_P$, the fraction is less than F and is known as the *heat factor per stage*, F_S. This is also sometimes known as the *hydraulic efficiency,* and the ratio F/F_S as the *reheat factor*, F_R.

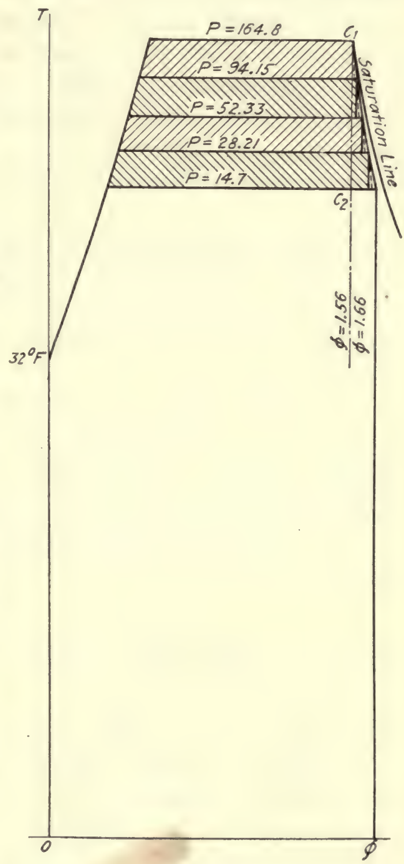


Fig. 143.

For a single-stage turbine F equals F_S. The fact that F/F_S is greater than unity for two stages and increases (slowly) as the number of stages increases must not lead to the conclusion that increasing the number of stages increases the efficiency. The over-all heat factor is based upon ideal performance, viz., adiabatic expansion from the maximum to the minimum temperature, of the heat available at maximum temperature, and not upon an uncertain lot of rejuvenated heat collected along the downward path. The increased available heat of the multi-stage turbine is produced by the failure of the preceding stages to utilize it, being degraded by friction and received again at lower temperatures, and its use must be accompanied by decreased efficiency.

That F_S is less than F does not seem inconsistent; it assumes that the ratio of the losses attending the operation of one turbine wheel to the work done in one stage of a multi-stage turbine, is greater than the ratio of loss due to the wheel of a single-stage turbine of equal power to the entire work of the turbine. This could be true if the wheel loss of the simple

turbine were any less than the sum of the losses of the wheels of the multi-stage turbine, which may reasonably be assumed to be the case.

Any actual superiority of multi-stage over simple turbines is due to size of unit and other features accompanying the design of the larger units.

The determination of F_S by calculation, from the results of laboratory experiments to determine the value of the factors upon which it depends, is not as reliable as the determination of F, the over-all heat factor, by turbine tests. If F_S were known, we could, by trial and error, with the aid of the intermediate pressures found by the assumption of constant entropy, find the actual intermediate pressures which would give equal work, or for any division of work. Or we may assume F_S and check our results by comparison with F, which must after all be only an assumed value.

Peabody's Direct Method.—The simplest method of determining the *reheat factor* F_R for saturated steam, and of applying it to determine intermediate pressures for any heat distribution is given by Prof. Peabody in The Steam Turbine, and it will be given here with some modifications.

The entropy table, $T\phi$ diagram, or Molier's diagram shows that between two given temperatures the difference of heat content is greater at a greater entropy. An example is given in Table 39, the values being taken from Peabody's entropy table. An inspection of the entropy table also shows that at a certain entropy the heat content increases at a nearly uniform rate for considerable intervals—of 20 or even 40 degrees; or

TABLE 39

Entropy 1.52		Entropy 1.59	
Temperature	Heat content	Temperature	Heat content
228	1010.2	228	1037.7
181	953.0	181	978.6
	57.2		59.1

$\Delta C/\Delta t$ is practically constant for this interval and is more accurate for the wider range.

We may then find the rate at which the available heat increases with the entropy as steam expands between two pressures in a turbine with a certain heat factor F. The calculation may be made at some intermediate point between C_1 and C_2, where the heat content is C_A, all being taken at

the same entropy ϕ_A. It is preferable for the sake of accuracy to have:

$$C_A = \frac{C_1 + C_2}{2}$$

as nearly as possible without interpolation, but its exact value must be used in the following formulas.

The increase of entropy in expanding from P_1 to P_A will be due to the addition of the heat quantity:

$$(1 - F)\ (C_1 - C_A)$$

The heat content C_B at entropy ϕ_B (at constant pressure P_A) will be:

$$C_B = C_A + (1 - F)\ (C_1 - C_A) \tag{33}$$

from which ϕ_B may be found.

Let ΔC_A be the change of heat content for the temperature interval ΔT at constant entropy ϕ_A, and ΔC_B the change for the same interval at entropy ϕ_B; then the ratio of heat change is:

$$\frac{\dfrac{\Delta C_B}{\Delta T}}{\dfrac{\Delta C_A}{\Delta T}} = \frac{\Delta C_B}{\Delta C_A}$$

The temperature interval should be equally divided on either side of T_A.

The rate of increase of available heat between C_1 and C_2 over that due to adiabatic expansion, computed for heat content C_A is:

$$\frac{\dfrac{\Delta C_B}{\Delta C_A} - 1}{\dfrac{C_1 - C_A}{C_1 - C_2}} = \frac{C_1 - C_2}{C_1 - C_A}\left(\frac{\Delta C_B}{\Delta C_A} - 1\right) \tag{34}$$

If this gives the same value when C_A is taken at any temperature between C_1 and C_2, the variation is in a straight line for equal increments of C; as this is very nearly true for saturated steam, the fraction of increase of available heat between C_1 and C_2 will be one-half the value given by (34). Then the ratio of the available heat to that available due to a single adiabatic expansion from P_1 to P_2 is:

$$R = 1 + \frac{1}{2}\left(\frac{C_1 - C_2}{C_1 - C_A}\right)\left(\frac{\Delta C_B}{\Delta C_A} - 1\right) \tag{35}$$

This is only true for an infinite number of pressure stages, for which R is equal to the reheat factor F_R; but it may be used for constructing a diagram which is generally applicable.

To make allowance for this increase of available heat during expansion, the adiabatic heat assignments, whether equal or in any desired proportion, must decrease from C_1 to C_2 at the same rate that the available heat increases, but their sum must not change. This is best illustrated by the diagram of Fig. 144. The vertical height represents $C_1 - C_2$, and ΔC the desired heat assignment per stage, being a fraction of $C_1 - C_2$ which is the adiabatically available heat from pressure P_1 to P_2.

It is obvious from (34) and (35) that the difference between the horizontal dimensions R and $2 - R$ is equal to the rate of change of available heat, and that their average is unity. The dimensions F_D, to the same

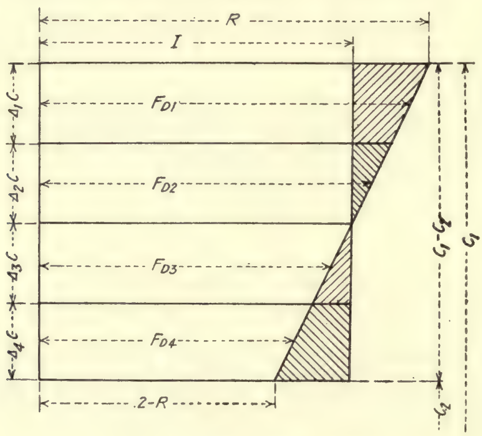


FIG. 144.

scale as R, are the distribution factors, each one bisecting the distance ΔC; multiplying ΔC by corresponding values of F_D gives the heat quantities which, beginning with the first stage, should be successively subtracted at constant entropy (corresponding to C_1), in order to determine the heat content at each stage, in the same manner that equal heat quantities $(C_1 - C_2)/n_P$ were subtracted to determine the pressures for Fig. 143.

An inspection of Fig. 144 will show that no matter how the heat is distributed, the sum of the products of F_D and ΔC is equal to the sum of ΔC. For equal divisions, F_{D1}, being the ratio of the available heat to that which would be available without heat degradation, is also equal to the ratio F/F_S which is the reheat factor previously mentioned. It is obvious that F_{D1} $(= F_R)$ increases with the number of stages, its limit being the distribution ratio R for an infinite number of stages, and this value, found by another method, is used by Martin as the reheat factor.

As,

$$F_R = \frac{F}{F_S} \tag{35a}$$

it is obvious that F_S decreases as the number of stages increases, and this may be seen in Fig. 143. It then follows that F_R increases and F_S decreases as ΔC becomes smaller, which indicates that when the stages are unequal, F_R and F_S do not have the same value for each stage. From Fig. 144, as $F_{D1} = F_R$, it may be seen that for equal stages:

$$F_R = 1 + \left(1 - \frac{\Delta C}{C_1 - C_2}\right)(R - 1) \tag{36}$$

It may then be assumed that for any work division, the reheat factor for the heat drop ΔC is given by (36).

As an example of application, the pressures of Fig. 143 will now be determined with the same data as for Fig. 141 and a heat factor of 0.6.

Then when $P_1 = 164.8$ and $\phi = 1.56$, $C_1 = 1193.3$; and when $P_2 = 14.7$ and $\phi = 1.56$, $C_2 = 1018.0$. The mean heat content is:

$$C_A = \frac{1193.3 + 1018}{2} = 1105.65 \text{ B.t.u.}$$

At entropy 1.56 the nearest value in Peabody's entropy table is at 53.6 lb. and 285 degrees, for which $C = 1105.2$ and this will be taken as C_A, at entropy ϕ_A (= 1.56). From (33):

$$C_B = 1105.2 + [0.4 \times (1193.3 - 1105.2)] = 1140.44 \text{ B.t.u.}$$

At temperature 285 degrees, the entropy ϕ_B at which this value is found is between 1.60 and 1.91; interpolation gives $\phi_B = 1.607$.

At $\phi_A = 1.56$, taking ΔT at 40 degrees, 20 degrees on either side of 285 degrees:

$$\Delta C_A = 1127.7 - 1082.1 = 45.6$$

At $\phi_B = 1.61$, for 20 degrees either side of 285 degrees.

$$\Delta C_B = 1163.88 - 1116.36 = 47.52$$

Then:

$$\frac{\Delta C_B}{\Delta C_A} = \frac{47.52}{45.60} = 1.041$$

The ratio R is given by (35), and is, when C_A is the mean between C_1 and C_2:

$$R = \frac{\Delta C_B}{\Delta C_A} = 1.041 \text{ or } 1.04 \text{ nearly.}$$

For practical application a portion of Fig. 144 may be omitted; then

Fig. 145 is the diagram for four equal stages with the data given and determined.

The quantity ΔC is the same for each stage; or:

$$\Delta C = \frac{C_1 - C_2}{n_P} = \frac{1193.3 - 1018}{4} = 43.825$$

TABLE 40

1 Stage	2 $F_D \cdot \Delta C$	3 $C_1 - \Sigma(F_D \cdot \Delta C)$ at $\phi = 1.56$	4 P (abs.)	5 $C_1 - \Sigma(F \cdot \Delta C)$	6 ϕ	7 Actual heat drop	8 v
Steam	.....	1993.30		1193.30		1193.30	
chest		45.15	164.80	26.30	1.560	1148.15	
						45.15	
		1148.15		1167.00		1167.00	
1	45.15	44.25	94.15	26.30	1.584	1121.85	4.590
						45.15	
		1103.90		1140.70		1140.70	
2	44.25	43.40	52.33	26.30	1.609	1095.26	7.852
						45.44	
		1060.50		1114.40		1114.40	
3	43.40	42.50	28.21	26.30	1.636	1069.17	14.306
						45.23	
4	42.50	1018.00	14.70	1088.10	1.665	......	25.070

Table 40 gives the steps in determining intermediate pressures, and a check calculation showing actual heat drops per stage. In column 5, $F \cdot \Delta C$ subtracted from the heat content of each stage gives the amount of heat available for adiabatic expansion in the following stage. The entropy, column 6, is that corresponding to the pressure (column 4) of the same line.

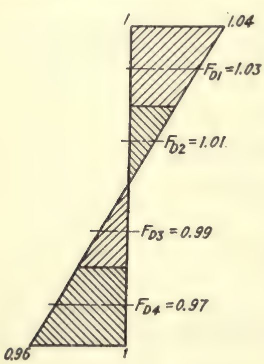


FIG. 145.

The subtrahend in column 7 is found by following down the entropy given in the same line of the table, to the pressure given in the line below.

The quantities in the table were taken from Peabody's entropy table by interpolation and the work done on a slide rule. The heat drop occurs in the nozzles connecting the compartments, and which really belong to the stage related to the compartment into which they discharge.

An alternative method which possesses some advantages, makes use of the reheat factor instead of the distribution factor. This is given in Table 41. In column 2, $F_R \cdot \Delta C$, for stage $n + 1$, is subtracted from the heat content; the resulting heat content is placed in the line below, and at the original entropy the corresponding pressure is found—sometimes by interpolation. The unused heat of this stage $(1 - F_S)F_R \cdot \Delta C$ $(= F_R \cdot \Delta C - F \cdot \Delta C)$ is now added, and at the pressure just found the new entropy is taken. $F_R \cdot \Delta C$ of the following stage is now subtracted and the resulting heat content down this new entropy line locates the next pressure, and so on.

TABLE 41

1 Stage	2 $C_n + (1 - F_S)F_R \cdot \Delta_n C$ $- F_R \cdot \Delta_{n+1} C$	3 P	4 ϕ
Steam chest	1193.30 45.15	164.80	1.5600
1	1148.15 18.06 ——— 1166.21 45.15	94.15	1.5840
2	1121.06 18.06 ——— 1139.12 45.15	52.24	1.6087
3	1093.97 18.06 ——— 1112.03 45.15	27.83	1.6343
4	1066.88	14.478	......

The specific volume for any pressure used for determining nozzle areas, must be found at the entropy of the stage above, as the entropy of each stage shows the increase due to heat degradation in that stage, and this occurs after the steam leaves the nozzles, with the exception of a slight amount due to nozzle friction. This applies to both methods.

The check of this latter method is the lowest pressure, and had $F_R \cdot \Delta C$.

been more accurately taken as 45.14, the resulting pressure in Table 41 would have been more nearly 14.7.

The value of R is found as before from (35), and F_R easily calculated from (36). The necessity of determining many values of F_D for multistage turbines is obviated and much labor saved.

Approximate values of R are given in Table 42 for various pressure ranges and heat factors, for saturated steam, taken from Peabody.

TABLE 42

Heat factor	150 lb. gage to 28 in. vacuum	150 lb. gage to atmosphere	Atmosphere to 28 in. vacuum
0.55	1.09	1.050	1.040
0.60	1.08	1.045	1.035
0.65	1.07	1.035	1.030
0.70	1.06	1.030	1.025
0.75	1.05	1.025	1.020

Superheated Steam.—The direct method of pressure distribution just given will not give as accurate results when applied to superheated steam; an inspection of a $T\phi$ Diagram shows this. Assuming the nozzles to be designed for saturated steam, and the same heat factor in each case, the nozzles, from (11), will accommodate less superheated steam, but from (3), less superheated steam is required for the same power.

The entropy diagram for superheated steam is given in Fig. 146. The dotted lines, as before, show assumed entropy changes due to nozzle friction, while the shaded areas are available adiabatic heat drops per stage. With less initial superheat, the lower stages would be like those of Fig. 143.

For any pressures, heat drops may be determined as by the method given for columns 5 to 7 of Table 40. This assumes that the nozzle areas are determined for these pressures, heat drops and specific volumes, and that the actual heat factor is the value used in the calculations.

Velocity Due to Reheating.—Early in the present paragraph it was assumed in Formula (31) that the heat available for producing velocity is $(C_1 - C_2)/n_P$. Later it was shown that heat drop along the new entropy due to reheating is the available heat; and while a smaller percentage of this is converted into work (as discussed under Effect of Heat Factor), there seems to be no reason why it should not be available for producing velocity if we assume that all of the energy of residual velocity is used for reheating. With this assumption $(C_1 - C_2)/n_P$ should be multiplied

by F_D, or, more generally, each part of $C_1 - C_2$, or ΔC, for any distribution, should be multiplied by the reheat factor F_R for that stage, the quantity $F_R \cdot \Delta C$ being the heat drop along the entropy found for a given stage. The quantity ΔC is the desired fraction of the total heat drop before being multiplied by the distribution factor F_D.

For saturated steam F_R may be found from (36) for any work division. The formula for velocity then becomes:

$$V = 223.7\sqrt{(1 - y)F_R \cdot \Delta C} \qquad (37)$$

The sum of the products $F_R \cdot \Delta C$ is greater than $C_1 - C_2$, but the unused heat in the stages of a multi-stage turbine is available to cause flow into the following stages, while in the single-stage turbine the surplus energy due to residual velocity is discharged at the lower pressure limit and is no longer available. The actual work done, however, is the product of $C_1 - C_2$ and F, so that apparently the velocity increase is not available for increased work; in view of this it may perhaps be permissible to ignore F_R in drawing velocity diagrams and in area calculations, or to make some allowance for it in choosing the friction factor y.

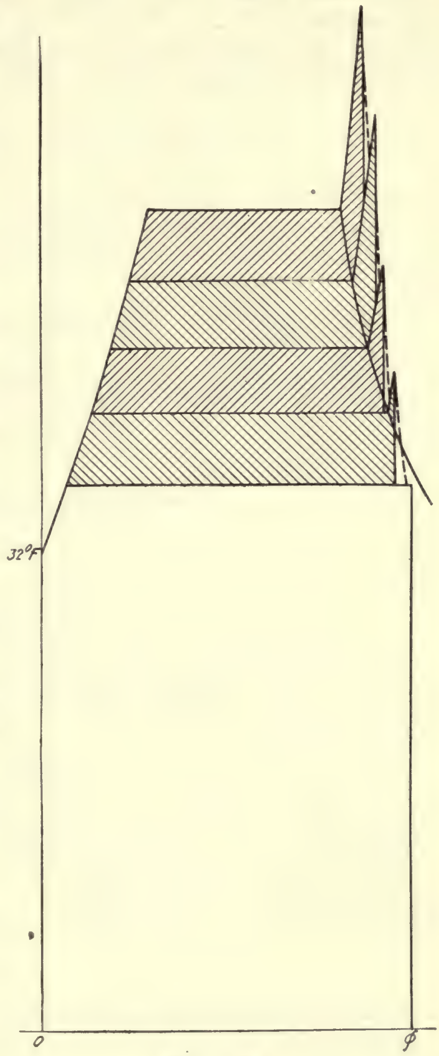


Fig. 146.

General Dimensions.—Equating (11) and (13) gives:

$$kmD^2 \sin \alpha = 144w\frac{v}{V} \qquad (38)$$

from which:

$$D = 6.77\sqrt{\frac{wv}{kmV \sin \alpha}} \qquad (39)$$

where D is the diameter in inches at pitch circle of blades, guides or nozzles. In determining conditions, or analyzing, it may also be desirable to solve for other quantities in (38).

With impulse turbines the value of m for maximum ratio of blade length to diameter of pitch circle will give the minimum wheel diameter for the last stage; this is found by multiplying this ratio by the ratio d_1/d_2 found from (19). Then V and v are for the discharge from the last nozzles. V may depend upon S, which has been previously fixed, or, *vice versa*, their relation being given by the velocity diagram. Then N, the r.p.m., is given by:

$$\frac{\pi DN}{12 \times 60} = S \qquad \text{or,} \qquad N = 229\,\frac{S}{D}$$

Some limiting values of m were given in Par. 86.

According to Martin, in a pressure-stage turbine with single velocity stages, with a 28-in. vacuum, D^2 should not be less that $0.57 \times$ output in kw.

The relation between diameter, speed, number of stages and efficiency is mentioned in Par. 95.

Multiplying (38) by S and substituting the value of w in terms of (3) gives:

$$D = 19.5\sqrt[3]{\frac{Hv\dfrac{S}{V}}{kmNF(C_1 - C_2)\sin\alpha}}$$

$$= 21.5\sqrt[3]{\frac{kwv\dfrac{S}{V}}{kme_M\,NF(C_1 - C_2)\sin\alpha}} \tag{40}$$

The value of v (or x, from which v may be computed) may be found at the pressure within the stage considered, and at the value of heat content equal to:

$$C_X = Cn + (1 - F)\,(C_1 - Cn) - (1 - F_S)F_R\cdot\Delta nC \tag{40a}$$

The subscript n refers to any stage; Cn is the heat content at the pressure considered, at the same entropy as C_1, and ΔnC is the heat drop for the stage, along the same entropy. It is accurate enough for most purposes, especially with reaction turbines, to take F instead of F_S and omit F_R; then

$$C_X = Cn + (1 - F)\;(C_1 - Cn - \Delta nC) \tag{40b}$$

Formula (40) is convenient, as the effect of the various factors may be readily seen.

Number of Equal Stages.—As calculations to determine the number of stages seldom results in a whole number, the influence of reheating will be neglected and the simpler formulas used Two factors influence the number of pressure stages: first, the form of nozzles, and second, the

peripheral speed. If convergent nozzles are to be used, the heat drop ΔC in any stage must be no greater than that due to a pressure drop to 0.58 of the pressure in the preceding stage; then:

$$n_P \overline{\gtrless} \frac{C_1 - C_2}{\Delta C} \tag{41}$$

With less pressure stages and a greater heat drop, divergent (convergent-divergent) nozzles must be used.

If S is the limiting factor, V may be found after the velocity diagram is designed; then solving for n_P in (31) gives:

$$n_P \overline{\gtrless} \frac{50{,}000}{V^2}(1 - y)(C_1 - C_2)$$

The actual value of ΔC, or of V and S may then be determined if n_P is taken at the nearest whole number which will not exceed the limits imposed.

Unequal wheel diameters are sometimes employed to avoid extreme blade lengths. This leads to unequal stages and sometimes to the reduction of the number of stages. If the same form of velocity diagram is used in each stage, V is proportional to S, which is proportional in turn to D; then if nozzle friction is assumed the same, the heat drop is proportional to D, and for any stage, with these assumptions:

$$\frac{\Delta C}{C_1 - C_2} \quad \frac{D^2}{\Sigma D^2} \tag{43}$$

D may be determined by (39) for the last stage by assuming V, and the design worked along stage by stage toward high-pressure end, reducing wheel diameters at discretion and checking by (38) for blade lengths. Assuming D and S, V may be calculated tentatively, and heat drop ΔC may be found from (37), neglecting F_R; then from (36), F_R may be found, from which V, S and D may be corrected, using (37) for calculating V. The corrected relation bewteen heat drop and pitch diameter would then be, for similar velocity diagrams:

$$\frac{F_R(1 - y)\Delta C}{\Sigma[F_R(1 - y)\Delta C]} = \frac{D^2}{\Sigma D^2}$$

but the error is slight.

If convergent nozzles are to be used, divide the absolute condenser pressure by 0.58 to obtain the absolute pressure in the preceding stage; the heat drop ΔC between these pressures will be that for the last stage, and V may be found from (37) instead of assumed. This will determine S, which may be greater or less than desired and a compromise must be made.

Another method is to start from the high-pressure end with the assumption of equal stages and equal diameters. If toward the low-pressure end the blades are too long, the wheel diameter may be increased, V increasing in proportion, until m in (38) is not too large; if the form of velocity diagram is not changed, D/V is constant and m varies inversely as D^3. A greater heat drop will accompany an increase in diameter and the remaining stages must reduce the heat content to exactly C_2. The values of ΔC should check with (43) for similar velocity diagrams; they may then be laid off on Fig. 144 and the design completed.

In some of the modern large multi-stage turbines the wheels increase in diameter from the first stage to the last. There is a single velocity stage in each pressure stage, with full peripheral admission. The work per stage necessarily increases toward the low-pressure end of the turbine.

Conservation of Residual Velocity.—The higher stages of most pressure-stage turbines have partial admission; that is, steam is admitted through but a portion of the periphery. The reason for this is, that otherwise extremely short blades are required to keep the nozzle area to the required amount. The blades are then kept an appreciable distance from the next set of nozzles in order to allow the steam to spread out and fill the nozzles. With admission around the entire periphery, equal wheel diameters, and clearance on the discharge side reduced to about one-half the blade width, a good percentage of the residual velocity is available as initial velocity of entrance to the nozzles. For any stage thus situated the available energy is:

$$\begin{aligned}\left(\frac{V}{223.7}\right)^2 &= \left[z\left(\frac{V_R}{223.7}\right)^2 + F_R\cdot\Delta C\right]\left(1-y\right)\\ &= \left(1-y\right)\left[1+\frac{z}{F_R\cdot\Delta C}\left(\frac{V_R}{223.7}\right)^2\right]F_R\cdot\Delta C\\ &= (1-y)\,(1+\mu)\,F_R\cdot\Delta C \qquad (44)\end{aligned}$$

where z is the fraction of residual energy available as kinetic energy, and V_R is the residual velocity of the preceding stage.

The value of F should be nearly $1 + \mu$ times its value without conservation of residual energy and this slightly reduces F_R, making it difficult to arrive at exact values. The fraction of increase:

$$\mu = z\left(\frac{V_R}{223.7}\right) \div F_R\cdot\Delta C \qquad (45)$$

may be determined approximately by taking:

$$\Delta C = \frac{C_1 - C_2}{n_P}$$

or dividing unequally along the adiabatic as previously described, as V_R^2 will have the same ratio to $F_R \cdot \Delta C$ as when the correction is made.

In order to have V the same in each stage the available energy must be the same. For the stages in which there is no conservation—the stages with partial admission—let:

$$\left(\frac{V}{223.7}\right)^2 = \left(1 - y\right) F'_R \cdot \Delta' C$$

This must equal the value given by (44); as the stages are so nearly equal, F'_R may be taken as equal to F_R, and y may be assumed the same for each stage. Then:

$$\Delta' C = (1 + \mu)\, \Delta C \tag{46}$$

If n_P and n'_P denote the number of stages with and without conservation respectively, it is obvious that:

$$\begin{aligned} C_1 - C_2 &= n'_P \Delta' C + n_P \Delta C \\ &= \Delta C[n'_P(1 + \mu) + n_P] \end{aligned}$$

Then:

$$\Delta C = \frac{C_1 - C_2}{n'_P(1 + \mu) + n_P} \tag{47}$$

Taking $\Delta' C$ from (46), a diagram similar to Fig. 144 may be constructed, and the proper assignments per stage made.

For the value of z in (44), Peabody gives 0.9; Martin gives 0.4 to 0.5 when the clearance between the wheels and diaphragms is reduced to a minimum, and the entrances to the guide blades are properly formed to receive the discharge from the preceding wheel. When the stages are not nearly equal, or part of them contain velocity stages, the calculation becomes involved; trial and error must be resorted to and check calculations made.

The foregoing discussion will give some idea of the conservation of the energy of discharge; aside from increasing the heat factor between 5 and 10 per cent., it is probably permissible to arrange blading to take advantage of conservation but neglect it in calculation.

91. Pressure-velocity-stage impulse turbines have two or more velocity stages in one or more of the pressure stages. The simplest case is the stationary Curtis turbine built for small powers by the General Electric Co. There are usually two pressure stages, with two, and sometimes three velocity stages in each. In this turbine the rim velocity varies inversely as the number of velocity stages and as the square root of the number of pressure stages. Due to fewer pressure stages the heat

drop per stage is so large that diverging nozzles are required. The velocity diagram for each stage is similar to Fig. 140 for two velocity stages, from which V may be measured or calculated for a desired value of S. Then ΔC for each stage may be calculated from (37), and the required number of pressure stages from (41); the design may then proceed as for any pressure-stage turbine with equal pressure stages.

A not uncommon arrangement is to place two velocity stages in the first pressure stage only, reducing the pressure and the number of stages. The velocity diagrams may be designed and the values of V obtained. Sometimes S may be the same for the one-stage and two-stage diagrams, but often the two-stage wheels are made smaller—though sometimes larger—in diameter; in any case V is greater for the first stage and the heat drop ΔC will be greater. In general:

$$\frac{\Delta nC}{C_1 - C_2} = \frac{Vn^2}{\Sigma V^2} \tag{48}$$

the subscript n denoting any stage. ΣV^2 may be solved for, and subtracting successively $V^2{}_1$, $V^2{}_2$, etc., the number of stages may be determined. The distribution factors may be neglected for the present, the pressures, specific volumes and areas being approximate; then the blade lengths may be checked by (38) as the calculating proceeds toward the low-pressure end, and an increase in diameter may be desirable. It may be necessary to increase the diameter of the last one or two wheels to have ΣV^2 entirely accounted for. This is equivalent to the second method under unequal wheel diameters in the preceding paragraph, and is more general. As previously stated, in finding ΔC from (37), F_R may first be neglected, but from the value so found, F_R may be obtained from (36) and ΔC recalculated by (37) with sufficient accuracy.

As with Formula (43), (48) is more correctly stated:

$$\frac{F_R(1 - y)\Delta nC]}{\Sigma[F_R(1 - y)\Delta nC} = \frac{Vn^2}{\Sigma V^2}.$$

The method just outlined is applicable to any arrangement, such for instance, as the Curtis marine turbine referred to in Par. 12, Chap. IV. When the approximate calculation is complete and the number and pitch diameter of the wheels determined, the values of ΔC may be laid off on a diagram such as Fig. 144 and the nozzle and blade dimensions checked. In the preliminary calculations, nozzle exits only need be determined.

92. Trajectory and Lead.—The actual path of the steam in its passage through the moving vanes of a turbine is known as the *trajectory*. The relative velocity of steam V_N in impulse blading is constant if we neglect the effect of friction, which for simplicity will be done in the present discussion.

Let the concave surface of a blade in Fig. 147 be divided into a number of parts and lettered a, b, c, etc. When the steam of velocity V_N has passed over the distance ab, the blade has moved the distance bb' due to its velocity S; then:

$$bb' = ab \frac{S}{V_N}$$

$$cc' = (ab + bc) \frac{S}{V_N}$$

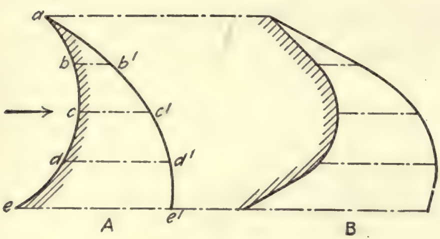


Fig. 147.

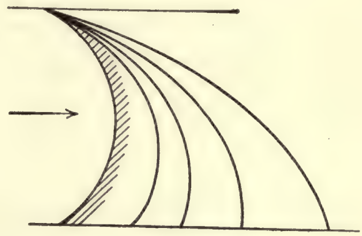


Fig. 148.

and so on until:

$$ee' = ae \frac{S}{V_N} \tag{49}$$

is the distance the blade travels in the time it takes a particle of steam to pass over its surface. The distances ab, ae, etc., must be measured along the curve; a flexible scale may be used for this, or a close approximation may be made by small divisions on a large scale drawing.

Trajectories are shown in Fig. 148 for a number of ratios of S to V.

The convex side of the blade is different in form and gives another path; by tracing the trajectories of both sides as in Fig. 149, some idea may be obtained of the path of the stream as it passes between the blades. An example of a simple wheel in which $\alpha = 20$ and $S = N \cos \alpha/2$, is given in Fig. 149, and for a 2-velocity-stage wheel in which $S = V \cos \alpha/5$ in Fig. 150.

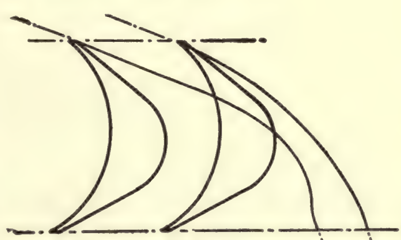


Fig. 149.

Lead.—In pressure stages with partial admission in which there is considerable space between the wheel and the next set of nozzles, the residual energy of the jet is dissipated and there is little or no initial velocity of entrance, so it makes little difference where the succeeding noz-

zles are located; but if the clearance is small, so that part of this velocity might be utilized, the nozzles should be placed so as to receive this residual steam with as little loss as possible. This may be done by finding the distance passed over as the steam passes through the blades as just described, and placing the next set of nozzles so as to receive this discharge; as this set usually subtends a larger arc than those preceding, but little difficulty is experienced. Where there are two or more velocity stages with partial admission, it is absolutely necessary for the guides to

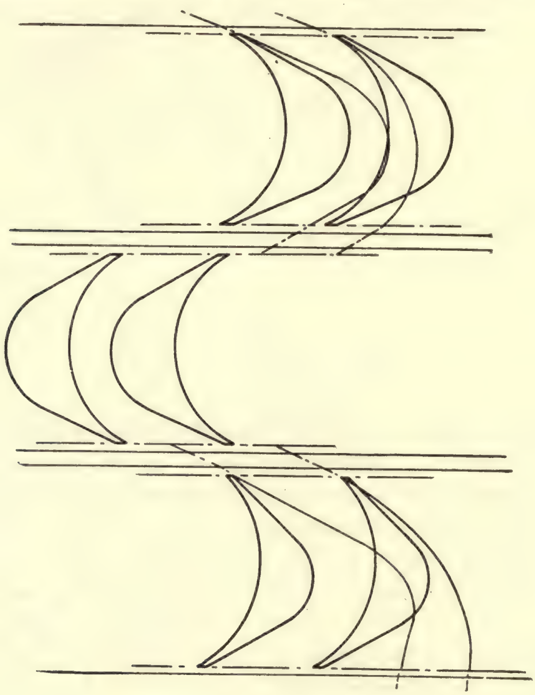


Fig. 150.

be so placed that all of the steam discharged from one wheel will be directed to the next; it is well in this case to provide one or two extra guides at each end of the row. For marine turbines run at variable speeds, enough additional guides must be provided for the extremes of speed.

In using formulas for finding the trajectory, the average velocity would probably be more correct. This may be found by assuming the friction loss uniform throughout the passage. Between entrance and exit the fraction of loss is $1 - q$; at the point b it is:

$$\frac{ab}{ae}(1 - q)$$

in which ae is the total length of the passage measured along the surface of the blade. The velocity coefficient between a and b is then:

$$1 - \frac{ab}{ae}(1 - q)$$

and the velocity at b is:

$$V_N\left[1 - \frac{ab}{ae}(1 - q)\right].$$

The average velocity between a and b is:

$$\frac{V_N + V_N\left[1 - \frac{ab}{ae}(1 - q)\right]}{2} = V_N\left(1 - \frac{ab}{ae}\frac{1 - q}{2}\right)$$

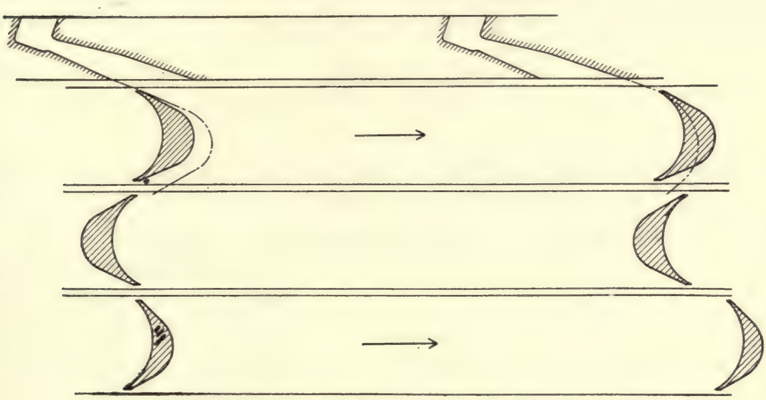


Fig. 151.

Then:

$$bb' = ab \cdot \frac{S}{V_N\left(1 - \frac{ab}{ae}\cdot\frac{1 - q}{2}\right)} \tag{50}$$

The distance cc may be found by using ac instead of ab, and in like manner other points on the curve may be found. At exit, where ab is changed to ae:

$$ee' = ae\frac{S}{V_N}\left(\frac{2}{1 + q}\right) \tag{50a}$$

Formula (50a) may be used for finding the lead. Fig. 151 shows the arrangement of nozzles and guide blades for two velocity stages, the dotted lines showing the theoretical limits of the steam path. The lines from nozzles to blades follow the direction of the stream from the nozzles, while from moving blades to guides the direction is tangent to the trajectory curves at exit.

Due to variable velocity of steam in reaction blading, it is not practicable to find the trajectory; and as the passages form convergent nozzles in which expansion occurs, and there is full admission, it is not important.

It is desirable to draw trajectory curves for both front and back surfaces of the passage for all moving blades. If the curves are not smooth, and free from too sudden changes, the blade form should be modified if the best efficiency is desired.

If friction is neglected, (50) may be written:

$$\frac{bb'}{ab} = \frac{S}{V_N}.$$

Then it is obvious from Fig. 133 that for the convex side of the blade the trajectory along the straight portion at entrance will be parallel to the jet from the nozzle. If friction is considered, (50) may be written:

$$\frac{bb'}{ab} = \frac{S}{V_N\left(1 - \frac{ab}{ae}\cdot\frac{1-q}{2}\right)}.$$

Then in order that the trajectory at entrance may be parallel to the jet so as not to have a retarding influence on the back of the blade, bb' must be greater, making the entrance angle greater than θ as shown in Fig. 135.

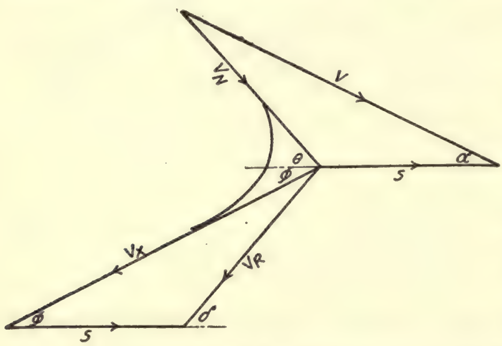


FIG. 152.

93. The Reaction Turbine. The essential feature of the reaction turbine is the expansion of steam in the moving blades. The guides correspond to the nozzles between stages in a pressure-stage impulse turbine and present no new feature. Although the principle is simple, the large number of stages makes a detailed analysis difficult due to the resulting accumulation of error if correct coefficients are not used. It has been stated that the design of reaction turbines is based upon empirical rules fixed by experience, to a larger extent than for other turbines. Be that as it may, considerable deviation from the conditions indicated by theory is practised in the interest of simplified construction, with apparently no deleterious effect upon efficiency. The same general method will be used as in the study of the impulse turbine, with certain modifications found desirable for this type.

Reaction Blading.—The velocity diagram is similar to that of the impulse turbine as shown in Fig. 152. V_X is always greater than V_N due to expansion between the blades caused by heat drop ΔC. The effect of friction may not well be shown on the diagram.

If Fig. 152 is for the first stage, V is due to the heat drop only, but if for any following stage it is due to the heat drop and also the residual velocity V_R from the preceding row of blades. Likewise the exit velocity V_X from the moving blades is due to the relative velocity V_N at entrance, and the expansion due to the heat drop ΔC.

As already stated in Par. 84, the angle of discharge of both blades and guides depends upon the pitch and setting of the blades and is not obvious from the drawing, but that has no special bearing upon our calculations, as the blades are assumed to be set to secure the angles desired, and shown upon the velocity diagram.

The forces acting upon the blades are determined as in Par. 86 for the impulse turbine, and are:

$$f_W = \frac{w}{g}(V \cos \alpha + V_X \cos \phi - S) \tag{51}$$

and:

$$f_F = \frac{w}{g}(V \sin \alpha - V_X \sin \phi) \tag{52}$$

For any but the first set of guides—which are really the first set of nozzles—let $\Delta_G C$ be the heat drop, and let $\Delta_B C$ be the heat drop for the following row of blades including reheating; then equating energy:

$$\left(\frac{V_X}{223.7}\right)^2 = i \cdot \Delta_B C + j\left(\frac{V_N}{223.7}\right)^2$$

and:

$$\left(\frac{V}{223.7}\right)^2 = i \cdot \Delta_G C + j\left(\frac{V_R}{223.7}\right)^2$$

where i and j are coefficients making allowance for losses.

It is convenient to consider one row each of guides and blades as a stage; then for one stage, the available heat is:

$$F_R \cdot \Delta C = \Delta_B C + \Delta_G C = \frac{V^2 + V_X^2 - j(V_N^2 + V_R^2)}{50{,}000\, i} \tag{53}$$

It is usual to assume that $V_X = V$, and $V_R = V_N$; it then follows that $\phi = \alpha$, and $\delta = \theta$. Then (53) becomes:

$$F_R \cdot \Delta C = \frac{V^2 - jV_N^2}{25{,}000\, i} \tag{54}$$

which is the heat drop per stage (for one row each of guides and blades).

Also (51) and (52) become:

$$f_W = \frac{w}{g}(2V \cos \alpha - S)$$

$$= \frac{w}{g} V \left(2 \cos \alpha - \frac{S}{V}\right) \tag{55}$$

and:

$$f_F = O$$

From Fig. 152:

$$V_N^2 = V^2 \sin^2 \alpha + (V \cos \alpha - S)^2$$

$$= V^2 \left[1 - \frac{S}{V}\left(2 \cos \alpha - \frac{S}{V}\right)\right] \tag{55a}$$

Then the energy supply in foot-pounds per second received by each stage is:

$$E_S = 778wF_R \cdot \Delta C = \frac{wV^2}{gi}\left[1 - j\left[1 - \frac{S}{V}\left(2 \cos \alpha - \frac{S}{V}\right)\right]\right] \tag{56}$$

As mentioned in connection with impulse turbines, F_R may be neglected in preliminary calculations, and it is not altogether clear whether values of i and j given by Martin were based upon $F_R \cdot \Delta C$ or ΔC; if the latter, F_R may be neglected.

The work done in foot-pounds per second, per stage is:

$$E_W = f_W S = \frac{wSV}{g}\left(2 \cos \alpha - \frac{S}{V}\right) \tag{57}$$

The diagram efficiency is then:

$$e_D = \frac{E_W}{E_S} = \frac{i \frac{S}{V}\left(2 \cos \alpha - \frac{S}{V}\right)}{1 - j\left[1 - \frac{S}{V}\left(2 \cos \alpha - \frac{S}{V}\right)\right]} \tag{58}$$

More general values of f_W and ΔC given by (51) and (53) may be used in (57) and (56) for determining e_D, but (58) gives a good idea of the relative effects of different values of α and S/V.

Martin gives $i = 0.9$ and $j = 0.52$, which were found by analyzing the performance of a large high-pressure marine turbine, but he adopts the values:

$$i = 0.89 \text{ and } j = 0.5.$$

As many reaction turbines have open-ended blades and guides, there is tip leakage, and only a portion of the steam does useful work. Fig. 153 is a diagram showing the arrangement of blades and guides, d being

the blade length and c the tip clearance. The flow through passages of this kind presents a very complicated problem and the fraction of the steam passed through which may be assumed to be effectively applied is not obvious; but Martin gives it as:

$$\frac{d - c}{d + \lambda c}$$

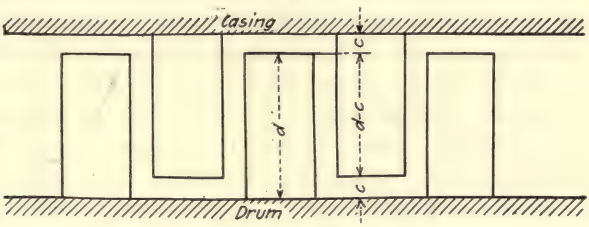


FIG. 153.

where

$$\lambda = \frac{1}{\sin \alpha} - 1.$$

He gives for normal blades, $\lambda = 2.15$ ($\alpha = 18° - 30'$); for semi-wing blades, $\lambda = 1.1$ ($\alpha = 28° 30'$); and for wing blades $\lambda = 0.6$ ($\alpha = 38° 30'$). The efficiency of the blading is then:

$$e_B = e_D \frac{d - c}{d + \lambda c} \tag{59}$$

Neglecting leakage from shaft glands and other packings, the heat actually converted into work is:

$$\Sigma e_B F_R \cdot \Delta C = F(C_1 - C_2).$$

If e_B and F_R are assumed constant, it is apparent that:

$$F = e_B F_R. \tag{60}$$

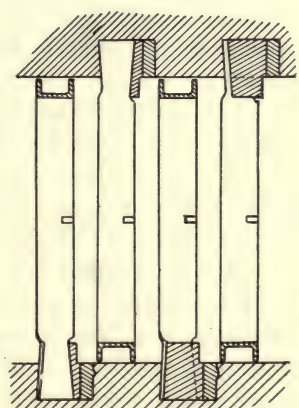


FIG. 154.—Shrouded blades.

The average value of $e_B F_R$ may be taken, and (60) used to obtain an approximate value of the total heat factor. The loss by leakage is about 4 or 5 per cent., and this may be deducted from the value found by (60). As $F_R = F/F_S$, (60) reduces to $F_S = e_B$, which is nearly true for the reaction turbine, as e_B, the blade efficiency accounts for all losses except gland leakage.

The blading of some reaction turbines is shrouded, as shown in Fig. 154, with the idea of preventing tip leakage. Jude says that if shrouded blades or guides have their roots raised above the level of the drum or casing, the remedy is worse than the evil, unless such construction allows a working clearance of less than one-half of that used in the ordinary construction.

Work Division.—As explained in Chap. IV, the reaction turbine consists usually of several *cylinders,* and upon each cylinder are several *groups,* or *barrels,* each of which are made up of several rows of moving blades or stages. The blades of a group are all of the same length, although theoretically they should increase gradually to allow for the increased volume. The pressure drop in each row is so small that the ratio of final to initial pressure is always greater than 0.58; therefore the passages should be converging, as explained in Par. 84, and even though the theoretical increase in blade length were effected in practice, the length of each blade or guide would be constant.

Fig. 28, Chap. IV, shows a drum divided into cylinders and barrels. It is obvious that at entrance to the first stages of each cylinder, and in some cases of the different groups, there will be no initial velocity, and a treatment similar to that in the preceding paragraph for conservation of residual velocity may be applied, allowing a greater heat assignment to these stages. In view of the large number of stages, the difference is insignificant and will be neglected.

The blades of a group are usually identical in form and setting, but sometimes the blades are "gaged" so as to keep the velocity of discharge constant throughout the group. The gaging consists in setting the blades for a larger discharge angle; this gives an increased area as shown by (12), thus fulfilling the theoretical requirement which would also be obtained by increasing blade length with a constant discharge angle. In the present discussion angles will be assumed constant, or an average value will be assumed for α, and calculations for height of blades will be at the center of the group, or for the average height if the blades were assumed to progress in height as called for theoretically.

The values of α and S/V are selected to give a good diagram efficiency, and the values of S for the largest and smallest cylinders are usually determined. The heat drop per stage may be found for any group from (54), neglecting F_R. Let subscripts 1, 2, etc., denote the different groups, beginning with the high-pressure end, the groups being arranged on the different cylinders. Then if n is the number of stages in a group:

$$n_1 \cdot \Delta_1 C + n_2 \cdot \Delta_2 C + n_3 \cdot \Delta_3 C \ \ldots . = C_1 - C_2 \qquad (61)$$

The groups may be arranged tentatively and calculations started from either end.

If there are three cylinders, which is common in practice, the work may be equally divided between them, or approximately so; Martin says that the high and intermediate cylinders each do ¼ of the work, and the low-pressure cylinder the rest. Any division of work may be assumed and the heat quantity $\Sigma n \cdot \Delta C$ for each cylinder may be laid off on a diagram such as Fig. 144. Then the groups may be arranged for each cylinder. As the increase of volume is slower in the upper stages,

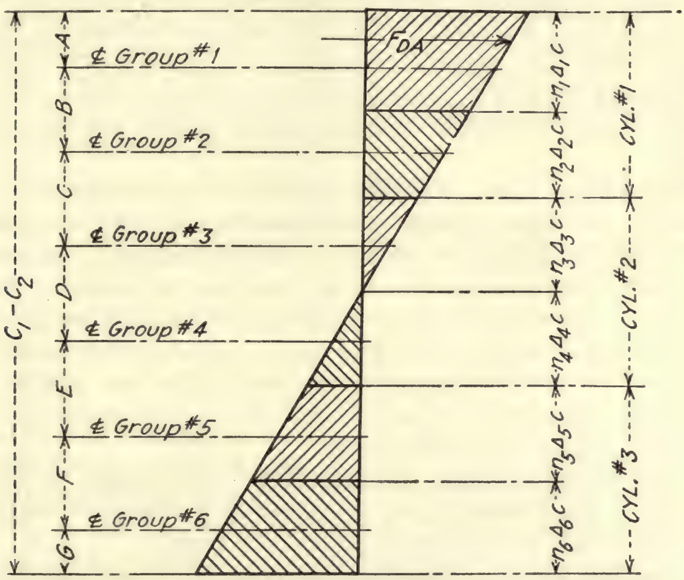


Fig. 155.

the number of rows per group would naturally be greater at the high-pressure end, decreasing toward the low-pressure end, but this is apparently not always so in practice.

When the adiabatic heat drop $C_1 - C_2$ has been arranged, the apportionment diagram may be something like Fig. 155, which is assumed to give equal work for each cylinder. Two barrels per cylinder are shown, but there are usually more.

It was stated that blade heights would be determined at the center of each group, or, at half the heat drop. It is possible that some other heat drop might be preferable, but when this is determined, it becomes necessary to find the pressures at these points in order to find the specific

volumes to be used in determining blade heights. Assuming the center of the group—although any other point would offer no difficulty—the first heat drop is:

$$A = \frac{n_1 \cdot \Delta_1 C}{2}$$

Then $C_1 - F_{DA}A$ gives the heat content at the center of the first group at the original entropy, and the pressure P_1 may be found. F_D is taken at the center of the heat drop as explained in Par. 90. $F_{DB}B, F_{DC}C, F_{DD}D$, etc., may be subtracted in succession and the pressures at the center of each group determined. Then the new entropies and specific volumes may be found as described in Par. 90, and the necessary area found from (11), using the value of V already determined. Blade height d may be found from (12), taking $k = 1$, or m may be found from (13), and this should be within practical limits, which will be considered presently.

Disc-and-drum Type.—Various combinations of impulse and reaction turbines are built, and as discs are generally used for the former and drums for the latter, the above name is sometimes used. In some there is an impulse pressure stage containing two velocity stages, followed by a large number of reaction stages, sometimes divided into two parts, forming a double-flow turbine. The Tosi marine turbine described by Martin has six impulse pressure stages, the first containing four velocity stages, the remainder three; this is followed by a reaction drum containing fourteen stages—or rows of moving blades.

From what has preceded, no detailed method of design need be given for this type; the procedure may be similar to the design of the pressure-stage turbine with velocity stages in the first stage.

94. Practical Notes on Reaction Turbines.—Few cases arise in turbine design in which conditions may be fixed by the application of theoretical formulas; many preliminary calculations and alterations must be made before the final design is complete. However, empirical rules are formulated during the progress of an art which greatly facilitate design, and a few suggestions, derived from various sources will be given.

From (58) it may be found that the highest diagram efficiency is obtained when S/V is about 0.9, when the values of i and j given by Martin are used. Such a high value is not usually practicable, but the value of e_D changes but little when S/V is as small as 0.4. From 0.37 to 0.52 are values used for marine turbines, and Peabody gives 0.6 as a common ratio of S to V for electrical work.

The peripheral speed of marine turbines varies from 110 to 210 ft. per sec. for the low-pressure end, and from 70 to 130 ft. for the high-pressure.

For electrical work, the low-pressure speed is from 200 to 360 ft. per sec. for the low-pressure and 100 to 135 ft. for the high-pressure end.

As already stated, blades must not be less than $\frac{1}{25}$ of the drum diameter for land turbines, but are sometimes as small as $\frac{1}{75}$ of the diameter for marine turbines, at a sacrifice of efficiency. They should not be longer than $\frac{1}{5}$ of the diameter of the pitch circle for the best results, this applying at the low-pressure end. Speakman's rule is that the blade length shall not be less than 0.03 nor greater than 0.15 of the pitch diameter.

As with impulse turbines, the square of the mean diameter of the blading at the low-pressure end should not be less than 0.57 times the output in kw. for a 28-in. vacuum according to Martin, and for very high vacuum is sometimes made equal to the output in kw. For a double-flow turbine it may be one-half of these figures.

Assuming a 28-in. vacuum for a single-flow turbine, the diameter of pitch circle for the low-pressure end is:

$$D \geqq 0.755\sqrt{kw} \tag{62}$$

which may be determined from (39) by substitution.

This may be compared with (40). It may usually be assumed that the only variables in (40) are v and m; letting H and L denote high pressure and low pressure respectively,

$$\frac{D_L}{D_H} = \frac{S_L}{S_H} = \sqrt[3]{\frac{m_H}{m_L} \cdot \frac{v_L}{v_H}} \tag{63}$$

In practice this ratio ranges from 2 to 3. To keep m_H and m_L within the prescribed limits, v_L will be less than the specific volume corresponding to the vacuum when the ratio D_L/D_H is that sometimes found in practice. If v_L is taken midway of the portion G, Fig. 155, the results will more nearly correspond; in this case the exit area is smaller than theoretically called for which may be offset by the use of wing blades or semi-wing blades, which give a greater exit area, and the result may be checked by (38). The pitch diameter of the intermediate cylinder may be made a mean proportional between the high- and low-pressure cylinders, or:

$$D_1 = \sqrt{D_H D_L} \tag{64}$$

If there are four cylinders, D_A and D_B may denote the first and second intermediate respectively; then:

$$D_A = \sqrt[3]{D_L D_H^2}$$

and,

$$D_B = \sqrt[3]{D_H D_L^2}.$$

There are no general rules for proportioning cylinders, dividing the work, or determining the number of barrels; suggestions are given by

different authorities, but they are not in agreement. It is probable that a rather wide range of conditions will give good results if certain necessary fundamentals are adhered to.

The loss due to tip clearance (see Fig. 153) may be considerable, and it is obvious that this is greater when the ratio c/d is large. Formulas for allowable tip clearances are as follows:

$$\left.\begin{array}{l} \text{Peabody} \ldots\ldots c = 0.00066D + 0.01 \\ \text{Martin.} \ldots\ldots c = 0.001D_D + 0.005d \end{array}\right\} \qquad (65)$$

where c is in inches; D_D is drum diameter, also in inches, and D the diameter of pitch circle as given before.

In some turbines the diameter of the pitch circle is constant for each cylinder, but it is more usual to have the drum diameter constant, with a different pitch diameter for each barrel. A compromise may be made in which both drum and pitch diameter change, but each in a lesser degree. For a constant drum diameter, blade height and pitch diameter are variables and must be determined by trial and error for great accuracy; however, this seems to be neglected by some authorities, the pitch circle being assumed constant. The heat drop is proportional to V^2, which is in turn proportional to D^2. Having determined blade lengths on the assumption of a constant pitch diameter D, if n is the calculated number of stages in the barrel, the number of stages may sometimes be reduced if there are a large number; if D_A is the actual diameter of the pitch circle of a given barrel and n_A the new number of blades; then:

$$n_A D_A{}^2 = nD^2$$

or:

$$n_A = \frac{nD^2}{D_A{}^2} \qquad (66)$$

The corrected blade length d_A, which with actual pitch diameter D_A will give the same passage areas as the length d and the assumed diameter D is:

$$d_A = \sqrt{Dd + \frac{D_D{}^2}{4}} - \frac{D}{2}.$$

The drum diameter D_D (which for shrouded blades is the diameter at inner ends of blades) may be found for any barrel on a cylinder, D and d being the actual pitch diameter and blade length for this barrel. The other barrels may be corrected. It is obvious that:

$$D_A = D_D + d_A.$$

It would seem that the blade lengths just calculated should be the effective length $d - c$ of Fig. 153. Then the drum diameter D_C used in cal-

culation would be the actual drum diameter plus $2c$. This would give the worst condition, the steam leaking over the tips of the guides not really being wholly ineffectual.

In applying (40) and (40*b*), it may be seen that there is some difficulty in providing ample passage for the steam in the lower stages if the usual peripheral velocity limit is not exceeded, even if wing blades having the maximum practical angle are employed. This is especially true for high rotative speeds. To overcome this, the last two or three rows may be increased in pitch diameter and placed upon a solid disc, which will better withstand the high centrifugal forces.

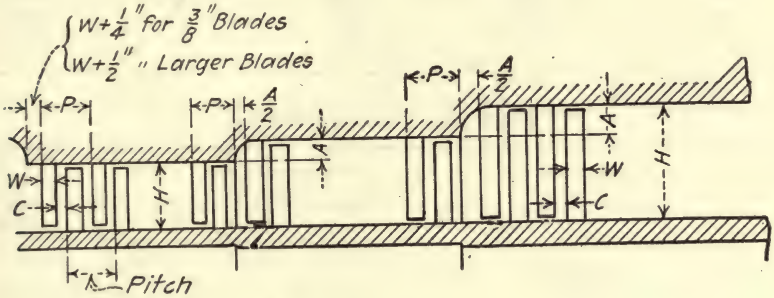


Fig. 156.

Peabody gives dimensions and proportions of blades and accessories as recommended by Speakman, and these are shown in Fig. 156 and Table 43; they may be used as a guide. Tables 44 and 45 are also from the same source.

Peabody states that at the time of Speakman's paper, the maximum speeds of Parsons turbines for electrical work was 170 ft. per sec. for high-pressure blades, and 375 ft. for low-pressure.

Marine turbine rotative speeds are governed by the propeller speed when direct-connected, a compromise being necessary between the most economical speed of turbine and propeller. The compromise is unnecessary when driving is through gears or electricity. The relative merits of gear and electric drive is a contested subject, especially as applied to battleships.

Table 43

H	1″	2″	3″	4″	6″	8″	10″	12″	15″	18″	21″	24″	30″
W	3/8″	3/8″	3/8″	1/2″	1/2″	5/8″	5/8″	3/4″	3/4″	1″	1″	1 1/8″	1 1/4″
P	1 1/8″	1 1/8″	1 1/4″	1 5/8″	1 3/4″	2 1/8″	2 1/4″	2 1/2″	3 5/8″	2 1/8″	3 1/4″	3 5/8″	4″
C	3/16″	3/16″	1/4″	5/16″	3/8″	7/16″	1/2″	1/2″	9/16″	9/16″	5/8″	11/16″	3/4″

TABLE 44.—(ELECTRICAL WORK)

Normal output *kw.*	Peripheral speed, ft. per sec.		Number of stages	R.p.m.
	First expansion	Last expansion		
5000	135	330	70	750
3500	138	280	75	1200
2500	125	300	84	1360
1500	125	360	72	1500
1000	125	250	80	1800
750	125	260	77	2000
500	120	285	60	3000
250	100	210	72	3000
75	100	200	48	4000

TABLE 45.—(MARINE WORK)

Type of vessel	Peripheral speed, ft. per sec.		S/V	Number of shafts
	H.p.	L.p.		
High-speed mail steamers	70–80	110–130	0.45–0.50	4
Intermediate steamers	80–90	110–135	0.47–0.50	3 or 4
Channel steamers	90–105	120–150	0.37–0.47	3
Battleships and large cruisers	85–100	115–135	0.48–0.52	4
Small cruisers	105–120	130–135	0.47–0.50	3 or 4
Torpedo craft	110–130	160–210	0.47–0.51	3 or 4

95. Factors Influencing Turbine Operation.—The heat factor has been used in the preceding discussion of turbine design, and while it is more reliable to use values determined by tests than to rely upon laboratory determinations of the different factors upon which it depends, it is well to briefly notice some of the latter. The main sources of loss are:

(*a*) Nozzle friction.
(*b*) Blade friction.
(*c*) Disc friction.
(*d*) Fan effect of blades.
(*e*) Spilling and tip leakage.
(*f*) Leakage past glands and diaphragm packing.
(*g*) Degradation of heat due to residual velocity.
(*h*) Radiation.

These quantities are discussed in more or less detail in the more elaborate treatises, and a study of them is helpful to a more complete

knowledge of turbine operation. Some of them are included in the diagram efficiency e_D already discussed, and still more in the blade efficiency e_B For the reaction turbine, Formula (60) gives a value of F including most of these factors, and it is clear that for this type some of the items do not apply.

If in each case, one minus the fraction of loss be the efficiency e, the heat factor is the product of all these efficiencies; or:

$$F = e_A e_B e_C e_D, \text{ etc.}$$

According to Martin, the heat factor depends upon a certain coefficient which is based upon construction and speed, and is:

$$\gamma = n_B \left(\frac{D}{10}\right)^2 \left(\frac{N}{100}\right)^2 \tag{67}$$

in which n_B is the number of rows of moving blades. The relation between this coefficient and the heat factor is given by a curve in Martin's book (The Design and Construction of Steam Turbines), but may be approximately expressed by a simple formula, thus:

$$\text{If } \gamma = \text{from } 30{,}000 \text{ to } 120{,}000, \quad F = 0.2\left(\frac{\gamma}{100{,}000}\right) + 0.45 \tag{68}$$

$$\text{If } \gamma = \text{from } 120{,}000 \text{ to } 300{,}000, \quad F = 0.04\left(\frac{\gamma}{100{,}000}\right) + 0.65 \tag{69}$$

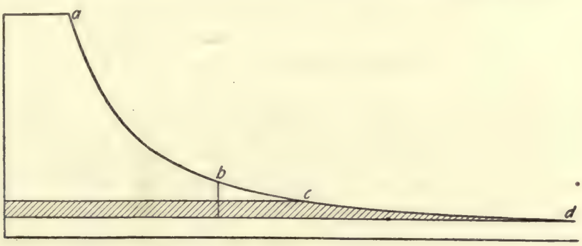


Fig. 157.

These apply especially to reaction turbines, but may be used for other turbines of good design. It is assumed that all turbines having the same value of γ have the same heat factor; this may be affected by superheat or other factors (correction curves are given by Martin), and must be used with judgment, but it will serve as a check upon other calculations or assumptions.

The Condenser.—A high vacuum is of much greater advantage to the steam turbine than to the steam engine; this may be explained by Fig. 157, which is a pressure-volume diagram for the Rankine cycle. First assume that the back-pressure line is the upper boundary of the shaded

area. For the turbine, expansion is complete and is from a to c. Complete expansion is not practicable for the steam engine, as explained in former chapters, so ab is the expansion line. Now assume an increase of vacuum, bringing the back-pressure line down to the bottom of the shaded area. The gain for the engine is the shaded area up to b, while for the turbine with complete expansion to d, the gain is equal to the entire shaded area.

As about double the amount of cooling water is required to increase the vacuum from 26 to 28 in. of mercury, the cost may offset the gain of power for the steam engine in some cases; the gain of power being so much greater for the turbine, the higher vacuum is of decided financial advantage, in spite of the more highly efficient condensing apparatus required.

Steam passages may readily be determined from (11) when suitable steam velocities are decided upon. This reduces to:

$$a = \frac{vWH}{25V} \tag{70}$$

With an initial gage pressure of 150 lb. per sq. in., a vacuum of 28 in. and a heat factor of 0.6, values of a, found by substituting the value of W from (3), are nearly as follows:

$$\text{For steam inlet} \ldots\ldots a_S = 1.5\frac{H}{V_S} \tag{71}$$

$$\text{For exhaust outlet} \ldots\ldots a_E = 150\frac{H}{V_E} \tag{72}$$

To allow for overload and sudden fluctuation of load, V_S may be made 75 ft. per sec. This is some lower than that allowed for the steam inlet of a reciprocating engine, but it is computed upon a different basis. The heat drop per lb. of pressure is vastly greater at the vacuums usually employed than at initial pressure; the velocity of flow is proportional to the square root of the heat drop; therefore a slight drop in pressure will cause a high velocity at the exhaust pressure, so V_E may be much higher than V_S. Assuming V_E as 300 ft. per sec., (71) and (72) become:

$$a_S = \frac{H}{50} \tag{73}$$

and:

$$a_E = \frac{H}{2} \tag{74}$$

Martin gives two rules for the area of the exhaust outlet, the first being 1 sq. in. for each 25 lb. of steam passed per hour; and the second, 3 sq. ft. per 1000 horsepower developed. The first gives $V_E = 300$

for the assumptions already made, and the second gives $V_E = 350$, nearly. It is good practice to make the exhaust passage as large as construction will permit, exceeding the area given by (74) if possible.

96. Governing, Rating and Overload.—The speed control of steam turbines is usually accomplished by throttling, although in some cases the flow to the nozzles is controlled by a number of valves, a part of which are wide open and the remainder closed, depending upon load requirements. The latter method may be said to correspond to the automatic cut-off of the steam engine. In both methods, large overloads are provided for by admitting high-pressure steam directly to some lower stage; this is also under governor control.

There is no definite standard for the rating of steam turbines. They are sometimes rated so that about 20 per cent. overload may be carried without opening the by-pass to a lower stage, and sometimes the rated load is the maximum load obtainable without the by-pass. In the latter case it is probable that some allowance is still made, and it is always well to have the nozzle area for the first stage a little in excess when the area is controlled by a number of valves.

In proportioning the nozzle areas for the second and lower stages of a pressure-stage impulse turbine, or the passages between blades and guides of a reaction turbine, the weight of steam must be known. This necessitates the selection of a load for which these areas will be calculated. This matters less with the reaction turbine on account of the small pressure drop per stage, but a decrease from the load for which the nozzles of a pressure-stage impulse turbine is designed, reduces the pressure in all but the last stage, which may result in the last wheel running nearly or entirely idle. To prevent this, the last stage nozzles may be reduced some in area. It is obviously not wise to design any but the first-stage nozzles for the maximum load (not considering the by-pass), and this only when part of the nozzles are cut out by valves at lighter loads, either by hand or by the action of the governor.

From (11):

$$\frac{V}{v} = \frac{144}{a}w.$$

or, for constant area as in most turbines, the ratio V/v varies directly with the weight of steam passed through. The determination of intermediate pressures for given nozzle areas would be necessary in order to find V, but this would be exceedingly difficult and of little practical value. It is apparent that both V and v change with change of load, usually in opposite directions, and for practically constant rotor speed, the velocity diagram would be altered. Referring to Fig. 133, the reduction of V

at light loads, with a constant or if anything, slightly increased value of S, would result in a larger value of the angle θ. For this reason if for no other, the practice of making the entrance angle greater than θ as shown in Fig. 135, seems advisable.

In view of the foregoing, too much refinement in pressure distribution may be ill-advised, and the more refined method of Par. 90 may perhaps be superfluous. It must also be remembered that all such calculations depend upon the heat factor, which is not known with accuracy.

It seems rational to design turbines for dry saturated steam; this will provide ample capacity when superheated steam is used. For high degrees of superheat, however, it may be advisable to make allowance in nozzle or guide blade design. Heat distribution may be determined by either the simple or more refined method of Par. 90, checked as in columns 5 to 7 of Table 40, and any desired alterations made.

As with the steam engine, the increase in economy due to superheating is greater than that theoretically indicated, while the gain due to increased vacuum is less. Very liberal exhaust openings increase the gain due to high vacuum and should be provided when possible.

Examples of the application of the foregoing principles to design are given in the following paragraphs. These are given to direct the use of the equations, and it is not claimed that the results coincide with actual practice in turbine design. In what has preceded, an attempt has been made to bring out most of the important principles relative to the power of a turbine in a general way. More refinement is possible, but it is doubtful if this is necessary or even desirable. The practicability of some of the refinements already noted must be left to the designer.

It is probable that in certain phases of design considerable latitude is allowable, but there is no doubt about the truth of the following statement from the bulletin of the General Electric Co.: "In order to obtain the best possible efficiency it is necessary that the details of design shall be in accordance with determinations resulting from a large amount of experimental research, and the experience gained in a very extended manufacture."

97. Application of Formulas. *Impulse Turbine.*—The design of a pressure-stage turbine with two velocity stages in the first stage involves practically all of the principles of the impulse turbine, so this will be taken as an example.

Problem: Design a 750-kw. turbine to carry a steam pressure of 150 lb. per sq. in. gage and a vacuum of 28 in. of mercury. The speed is to be 3600 r.p.m. and the steam initially dry saturated. Let the maximum velocity at the pitch line of blades be 500 ft. per sec. and the mechanical

efficiency of turbine and generator combined be assumed as 90 per cent.

The design of the velocity diagrams is first required, and the velocity coefficient q for the first stage will be taken as 0.85 (see Par. 84). The nozzle angle α will be taken as 20 degrees and the ratio S/V for the first stage as:

$$\frac{\cos \alpha}{5} = 0.1879.$$

The diagram, Fig. 158, was drawn to scale and the angles chosen rather arbitrarily, and as results were fairly good, no changes were made. Trigonometric functions were found by measuring the diagram and calculations were by slide rule. The functions needed for drawing blades and for calculation are as follows:

$$\frac{S}{V} = \frac{\cos \alpha_1}{5} = \frac{0.9397}{5} = 0.1879, \quad \cos \theta_1 = \cos \alpha_2 = 0.915, \quad \theta_1 = \alpha_2 = 23° - 48'.$$

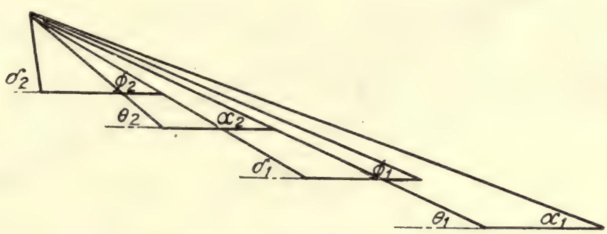


Fig. 158.

According to Fig. 135, θ_1 may be made 28 degrees in the blade, but this does not change the velocity diagram.

$\cos \phi_1 = 0.924$ $\qquad \phi_1 = 22° - 29'$
$\cos \phi_2 = \cos \delta_1 = 0.868$ $\qquad \phi_2 = \delta_1 = 29° - 46'$
$\cos \theta_2 = 0.76$ $\qquad \theta_2 = 40° - 32.$ This may be made 45 degrees in the blade. As velocities have not been definitely decided upon, measurements in inches may be used; then (24) gives:

$$V_{W1} = 5.95$$

and

$$V_{W2} = 1.0905.$$

The ratio of work is:

$$\frac{V_{W1}}{V_{W2}} = \frac{5.95}{1.0905} = 5.45$$

instead of 3, as given in Par. 88 for frictionless flow with symmetrical blades.

From (30), the diagram efficiency is:

$$e_D = \frac{2 \times 0.8 \times 7.855}{4.26} = 0.69.$$

Fig. 158 is such a diagram as Fig. 140, but drawn with the apexes of all triangles at the same point, a convenient construction for design.

The velocity diagram for all other stages is shown in Fig. 159, drawn in the same way, in which q is taken as 0.75 (Rateau's value). Other quantities are:

$$\alpha = 20, \qquad \frac{S}{V} = 0.5 \cos \alpha = 0.47, \text{ and } K \text{ in (27) is } 0.44.$$

$$\cos \phi = 0.885 \qquad \phi = 27° - 45', \qquad \cos \theta = 0.806, \quad \theta = 36° - 18'.$$

This last may be made 40 degrees in the blades according to Fig. 135. Then using inches as before, from (27):

$$e_D = 0.44[(0.75 \times 1.096) + 1] = 0.783.$$

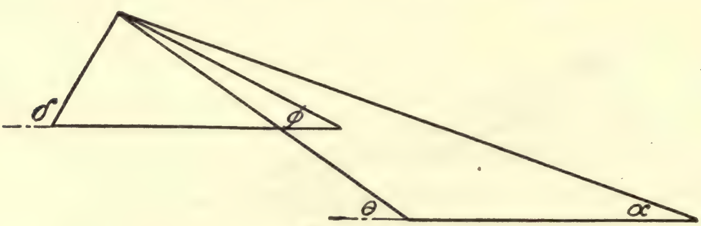


FIG. 159.

From Par. 95, assuming the loss by nozzle and disc friction, gland leakage and radiation combined to be 20 per cent., the heat factor for the first stage is:

$$F = 0.80 \times 0.69 = 0.552.$$

For the other stages:

$$F = 0.80 \times 0.783 = 0.626.$$

Assume tentatively that the blade pitch line of all wheels travels 500 ft. per sec.; then for the first stage:

$$V = \frac{S}{0.1879} = 2660.$$

From (37), neglecting F_R and y, the heat drop for this stage is:

$$\Delta C = \frac{2660^2}{(223.7)^2} = 141.$$

Peabody's entropy table will be used for heat quantities, volumes, etc., in these calculations; the nearest tabular value of absolute pressure corresponding to 150 lb. gage is 164.8, and to 28-in. vacuum, 1.005, and these

will be used for P_1 and P_2. The heat content C_1 is 1193.3, and C_2 is 871.1 B.t.u. The available heat at constant entropy, which for initially dry saturated steam is 1.56, is:

$$C_1 - C_2 = 322.2 \text{ B.t.u.}$$

The heat left for the remaining stages is:

$$322.2 - 141 = 181.2.$$

For these stages:

$$V = \frac{S}{0.47} = 1063.$$

From (37), neglecting F_R and y as before, the heat drop per stage is:

$$\Delta C = \frac{1063^2}{(223.7)^2} = 22.7.$$

The number of these stages after the first stage will be:

$$n = \frac{181.2}{22.7} = 8.$$

As F_R and y were neglected, the velocity may exceed the limiting velocity of 500 ft. per sec. if the tentative values are adopted. Let us now arbitrarily take the number of single wheel stages to be 10, making 11 stages, and 12 rows of moving blades. Assume a heat drop per stage along constant entropy of 20 B.t.u. The heat drop for the first stage is then:

$$\Delta_C = 322.2 - 200 = 122.2.$$

Assume the heat factor to be a compromise between those found from the efficiencies of the two diagrams, as follows:

$$F = \left(\frac{122.2}{322.2} \times 0.552\right) + \left(\frac{200}{322.2} \times 0.626\right) = 0.6.$$

This is perhaps rather low, but conservative. From Table 41, $R = 1.08$. The reheat factor for the first stage, from (36), is:

$$F_R = 1 + \left[\left(1 - \frac{122.2}{322.2}\right) \times 0.08\right] = 1.05.$$

For the other stages:

$$F_R = 1 + \left[\left(1 - \frac{20}{322.2}\right) \times 0.08\right] = 1.075.$$

Taking the friction factor for all nozzles, $y = 0.1$, (37) gives for the first stage:

$$V = 223.7\sqrt{0.9 \times 1.05 \times 122.2} = 2405.$$

Also:

$$S = 0.1879 \times 2405 = 452 \quad \text{and} \quad D = \frac{60 \times 12 \times 452}{3600} = 28.75 \text{ in.}$$

For the other stages:

$$V = 223.7\sqrt{0.9 \times 1.075 \times 20} = 983.$$

Also:

$$S = 0.47 \times 983 = 462.5 \quad \text{and} \quad D = \frac{60 \times 12 \times 462.5}{3600} = 29.375 \text{ in.}$$

A check calculation may now be made by means of (40) and (40a); from the latter, at the condenser pressure, 1.005 lb.:

$$C_x = 871.1 + (0.4 \times 322.2) - (0.442 \times 1.075 \times 20) = 990.48.$$

Interpolating in Peabody's entropy table for the corresponding volume gives:

$$v = 295.$$

Then, assuming m as 0.2, and k as 0.9, (40) gives:

$$D = 21.5\sqrt[3]{\frac{750 \times 295 \times 0.47}{0.9 \times 0.2 \times 0.9 \times 3600 \times 0.6 \times 322.2 \times 0.342}} = 29.95 \text{ in.}$$

Properly, m should be multiplied by the ratio d_1/d_2 as explained in connection with equations (38) and (39), but for the sake of variety in the problem this will be neglected at this place and a correction made for the last stage. It may then be assumed that the diameters already found are correct.

In finding the nozzle areas, the steam weight must be known. Calculations will be made for rated load at maximum steam pressure, a condition probably not obtaining in practice except when governing is effected by cutting out valves, which will be assumed to be this case; then enough extra nozzles must be provided for the first stage to take care of the overload. For governing by throttling, the nozzles of the higher stages should be designed for overload at maximum steam pressure.

From (5), the turbine horsepower is:

$$H = \frac{1.34 \times 750}{0.9} = 1118.$$

From (3), the water rate is:

$$W = \frac{2545}{0.6 \times 322.2} = 13.15 \text{ lb.}$$

and:

$$w = \frac{WH}{60 \times 60} = \frac{13.15 \times 1118}{3600} = 4.08.$$

From (11), the total nozzle area is:

$$a = 144 \times 4.08 \frac{v}{V} = 603 \frac{v}{V}.$$

For the first stage, $V = 2405$, and $a = 0.2503v$.

For the other stages, $V = 983$, and $a = 0.614v$.

TABLE 46

1 Stage no.	2 $C_n + (1 - F_S)F_R \cdot \Delta_n C -$ $F_R \cdot \Delta_{n+1} C$	3 P	4 ϕ	5 v	6 u
Steam chest	1193.30 128.31	164.800	1.56000	2.750	
1	1064.99 54.99 1119.98 21.50	30.125	1.63737	12.260	7.53
2	1098.48 9.50 1107.98 21.50	22.315	1.65112	16.999	10.42
3	1086.48 9.50 1095.98 21.50	16.375	1.66507	22.570	13.82
4	1074.48 9.50 1083.98 21.50	11.923	1.67924	30.207	18.55
5	1062.48 9.50 1071.98 21.50	8.633	1.69391	40.513	24.90
6	1050.48 9.50 1059.98 21.50	6.178	1.70899	55.077	33.80
7	1038.48 9.50 1047.98 21.50	4.374	1.72447	75.805	46.50
8	1026.48 9.50 1035.98 21.50	3.069	1.74020	104.945	64.30
9	1014.48 9.50 1023.98 21.50	2.138	1.75629	146.515	90.00
10	1002.48 9.50 1011.98 21.50	1.470	1.77141	207.032	127.20
11	990.48 9.50 999.98	1.016	At P = 1.005 1.78943	291.930	175.00

The method of pressure distribution of Table 41, Par. 90 is used, the results being given in Table 46. Considerable accuracy is required due to the small pressure drops in the later stages, and cross interpolation is resorted to in the use of the entropy table. Careful use of the slide rule is permissible in making interpolations. As stated in Par. 90, the check on this system is the last pressure, which in this case is 1.016 instead of 1.005, a difference which has no practical bearing.

Total nozzle areas are given in Col. 6. This is the exit area at rated load for the first-stage nozzles. In all but the first stage the pressure is greater than 58 per cent. of that in the stage above it, so that convergent nozzles are used. In the first stage divergent nozzles are required.

The pressure in the throat of the first-stage nozzle is:

$$164.8 \times 0.58 = 95.5.$$

The heat content at entropy 1.56 and this pressure is 1149.29, and the corresponding volume 4.44. The heat drop to the throat is 44.01. Assuming the friction factor for flow to the throat as 0.02, the velocity at the throat is:

$$V = 223.7\sqrt{0.98 \times 44.01} = 1470 \text{ ft. per sec.}$$

There is, of course, no heat factor at this point. The throat area for rated load is then:

$$a = 603\frac{v}{V} = \frac{603 \times 4.44}{1470} = 1.823 \text{ sq. in.}$$

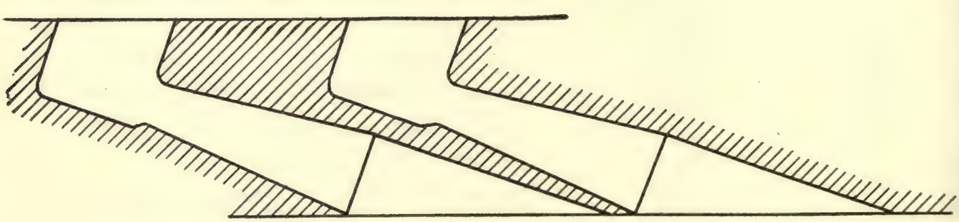


Fig. 160.

The first stage nozzles are shown in Fig. 160: these are machined out of bronze and bolted to the diaphragm as shown in Fig. 460, Chap. XXXIII. Fig. 161 shows second- and third-stage nozzles machined in the diaphragm, while Fig. 162 shows cast-in nozzles for the remainder of the stages. The first-stage nozzles have partial admission and are located on one side of the diaphragm. The second- and third-stage nozzles have partial admission but extend over a larger portion of the periphery; they are equally divided between the two halves of the diaphragm. The cast-in nozzles of the remaining stages have full admission.

The number of nozzles have been assumed, keeping in mind the blade length and are given in Table 47. As computed in Table 46 there would

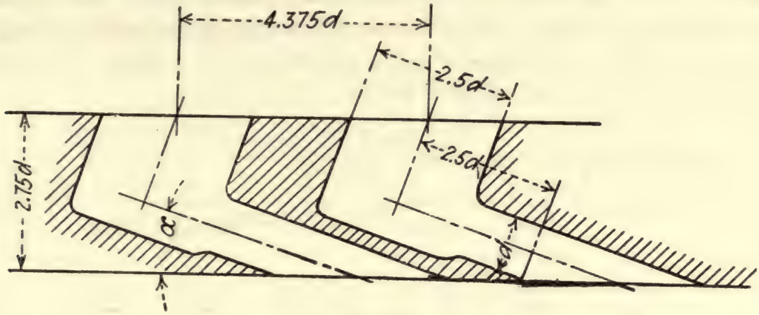


FIG. 161.

be 15 first-stage nozzles of the diameter given; 20 per cent. overload is provided for making 18 first-stage nozzles, and this number appears in Table 47.

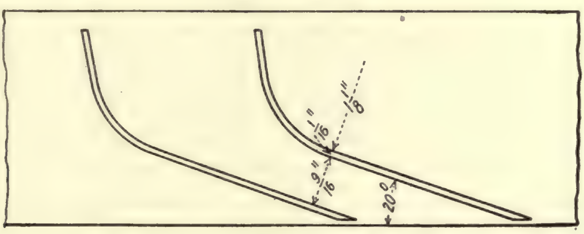


FIG. 162.

TABLE 47

Stage	Number of nozzles	d, in.	Fraction of circumference
1–throat	18	0.394	0.497
1–exit	18	0.800	0.497
2	20	0.832	0.846
3	20	0.940	Full
4	56	0.650	Full
5	56	0.880	Full
6	56	1.200	Full
7	56	1.640	Full
8	56	2.275	Full
9	56	3.180	Full
10	56	4.150	Full
11	56	6.320	Full

For the conditions assumed, the blading data are given in Table 48. For the first stage, the entrance height of the first row of blades is made about 6 per cent. greater than the diameter of nozzle exit. The entrance to the guides and second row of blades equals the preceding exit heights, which are found in each case by (19), neglecting the drying. The ratios sin A/sin B are given in Table 48.

TABLE 48

Stage	Row	No.	Angle		d, in.		$\frac{\sin A}{\sin B}$
			Entrance	Exit	Entrance	Exit	
	1st	167	28°–0′	22°–29′	0.850	0.900	1.055
1	Fixed	84	29°–46′	23°–48′	1.050	1.110	1.230
	2d	156	45°–0′	29°–46′	1.260	1.450	1.305
2	......	162	40°–0′	27°–45′	0.920	1.060	1.270
3	......	162	40°–0′	27°–45′	1.030	1.200	1.270
4	......	162	40°–0′	27°–45′	0.715	0.825	1.270
5	......	162	40°–0′	27°–45′	0.970	1.120	1.270
6	......	162	40°–0′	27°–45′	1.320	1.520	1.270
7	......	162	40°–0′	27°–45′	1.700	2.080	1.270
8	......	162	40°–0′	27°–45′	2.500	2.875	1.270
9	......	162	40°–0′	27°–45′	3.500	4.050	1.270
10	......	162	40°–0′	27°–45′	4.570	5.280	1.270
11	......	162	40°–0′	27°–45′	7.000	8.050	1.270

After drawing the blading in this way as shown in Fig. 163 by the full lines, straight lines are drawn from nozzle extremities to the last blade

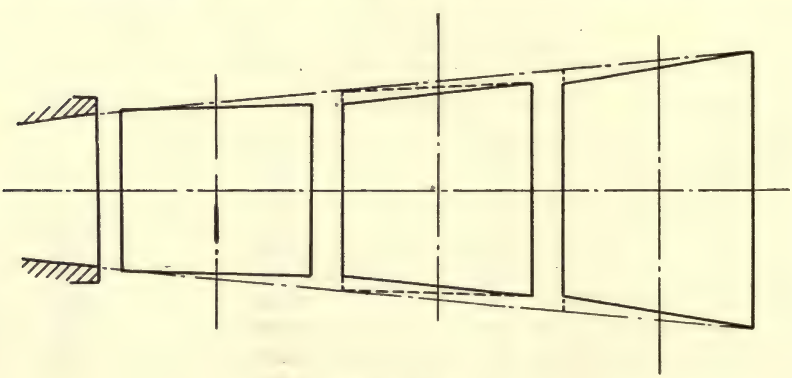


FIG. 163.

extremities as mentioned in the latter part of Par. 88. Then the entrance height of guides and blades are brought up to the height given by the

dotted straight line. This change is arbitrary and is shown in heavy dotted lines. The completed arrangement showing shrouding, etc., is shown in Fig. 164. The sections of blading with their trajectory curves are shown in Figs. 149 and 150.

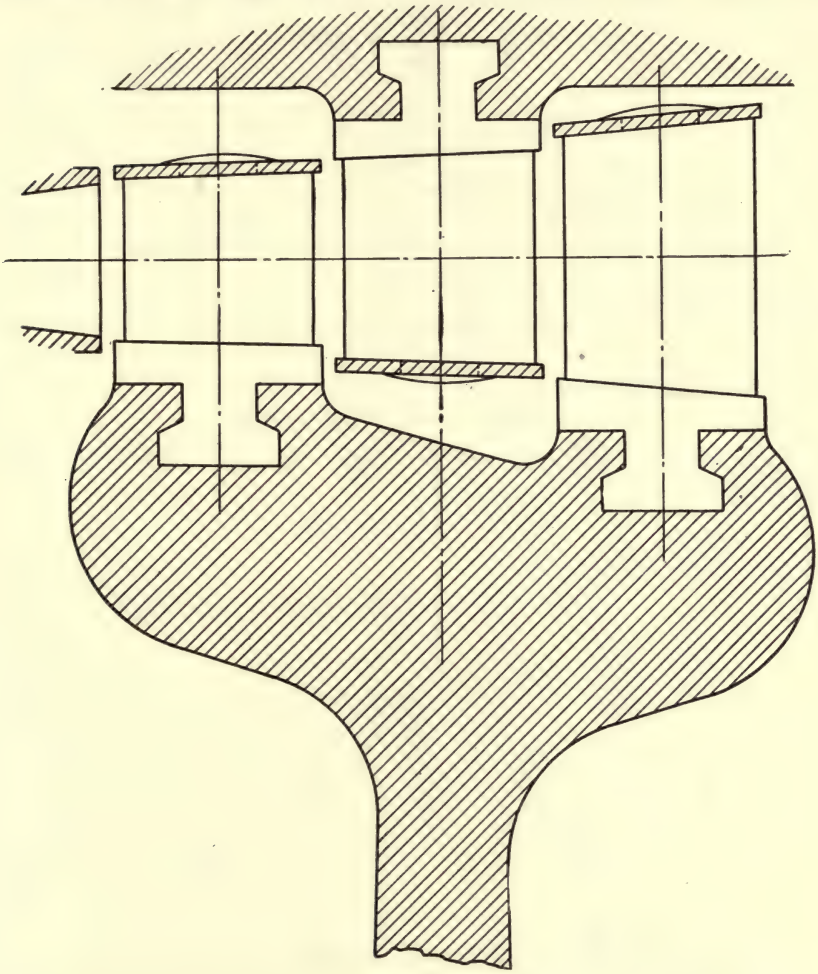


Fig. 164.

Returning to the last stage, the blade length given in Table 48 is greater than $D/5$, which has been assumed as a limit. Correcting m as mentioned in connection with (19), gives:

$$m = \frac{0.2}{1.27} = 0.157.$$

Substituting this in (39) gives:

$$D = 6.77\sqrt{\frac{4.08 \times 295}{0.9 \times 0.157 \times 983 \times 0.342}} = 34 \text{ in.}$$

This assumes the same heat drop and nozzle angle as before. Then:

$$S = \frac{\pi DN}{720} = 534$$

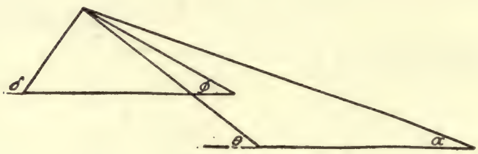


FIG. 165.

and

$$\frac{S}{V} = 0.543.$$

The nozzle height is now:

$$d = mD = 0.157 \times 34 = 5.32$$

and this checks with the area in Table 46. The maximum blade height is:

$$d_2 = 1.27d = 6.75 \text{ in.}$$

The minimum may be made:

$$d_1 = 1.12 \times 5.32 = 6 \text{ in.}$$

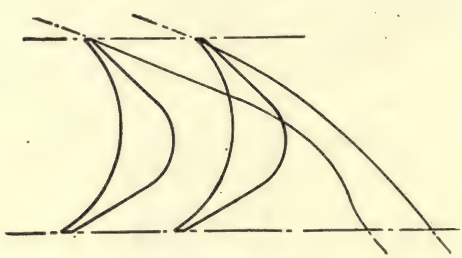


FIG. 166.

The blade height is now ⅕ the pitch diameter.

The change of S/V modifies the form of the velocity diagram and the blade section. The velocity diagram is shown in Fig. 165 and the sections with trajectory in Fig. 166. The calculations are:

$$\cos\theta = 0.778$$

and

$$\theta = 38°\text{–}55'.$$

Make the angle 45 degrees.

$$\sin 38°\text{–}55' = 0.628.$$

For the same ratio as the other stages:

$$\sin \phi = \frac{0.628}{1.27} = 0.494$$

and

$$\phi = 29°\text{–}37'$$
$$\cos \phi = 0.869.$$

Then from (27), $e_D = 0.79$;

which is greater than for the other stages due to the greater ratio of $\cos \phi/\cos \theta$.

The number of blades on the last wheel will be 176, and their form is almost identical as for those on the second wheel of the first stage; the

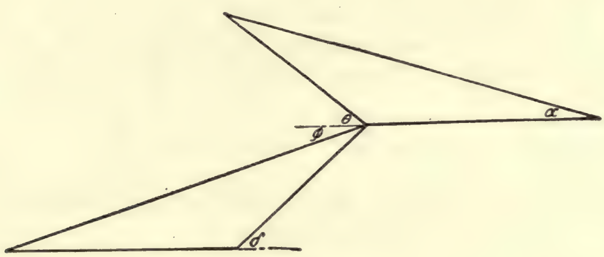


FIG. 167.

trajectory, however, is quite different, due to the difference in the ratio of relative velocity of steam to blade speed.

Reaction Turbine.—Using the same power, rotative speed and pressures as for the impulse turbine, the dimensions of a reaction turbine will be determined. The total heat factor will be the same and the steam assumed to be dry saturated initially.

Let it further be assumed that:

$$\alpha = 18 \text{ degrees}, \quad S/V = 0.6, \quad i = 0.89, \quad j = 0.5 \quad e_M = 0.9$$

and let the velocity diagram be as in Fig. 167.

The diagram efficiency from (58) is 0.78. As preliminary assumptions let $S_H = 150$ and $S_L = 360$, the subscripts H, I and L denoting high-, intermediate- and low-pressure cylinders respectively. Then:

$$V_H = \frac{150}{0.6} = 250$$

and

$$V_L = \frac{360}{0.6} = 600$$

From (54) and (55*a*), neglecting F_R, the heat drop per stage is:

$$\Delta_H C = 1.25$$

and

$$\Delta_L C = 7.2.$$

Taking $F_S = F$ and neglecting F_R, the heat content at exit from the last row of guides is, from (40*b*), 997.1 and $v = 298$. Assuming $k = 1$ for all stages, and $m = 0.2$ for the last stage, (40) gives, taking $\alpha = 45$ for wing blades:

$$D_L = 24.7 \text{ in.}$$

Then $S_L = 387.5$, which is excessive, for the assumptions made.

With $S_H = 150$, $D_H = 9.5$ in.

As $w = 4.08$, the heat drop $\Delta_H C = 1.25$, and $v_H = 2.8$. Then (38) gives: $m = 0.075$. As this is large, assume D_H to be 10 in. Then restricting S_L to 360, $D_L = 23$ in. From (64), $D_I = 15.25$ in. Then:

$S_H = 157$	$S_I = 239$	$S_L = 361$
$V_H = 278$	$V_I = 398$	$V_L = 602$

Neglecting wing blades on the low-pressure cylinder, the heat drop per row of moving blades (or per stage) is:

$$\Delta_H C = 3.08, \qquad \Delta_I C = 6.34, \qquad \Delta_L C = 14.5.$$

It now remains to distribute the work among the three cylinders. This is rather arbitrary, but we will take three barrels per cylinder and designate the barrels by subscripts 1, 2, 3, etc.

By taking $D_H = 10.75$ and re-arranging the data, the following results are obtained, in which n_H, n_I, and n_L denote the number of rows of moving blades, or the number of stages on the respective cylinders, and n_1, n_2, etc. the stages per barrel.

$D_H = 10.75''$	$D_I = 15.25''$	$D_L = 23''$
$S_H = 168.6$	$S_I = 239$	$S_L = 361$
$V_H = 281$	$V_I = 389$	$V_L = 602$
$n_H = 20$	$n_I = 18$	$n_L = 10$
$\Delta_H C = 3.16$	$\Delta_I C = 6.33$	$\Delta_L C = 14.5$
$n_H \cdot \Delta_H C = 63.2$	$n_I \cdot \Delta_I C = 114$	$n_L \cdot \Delta_L C = 145$

The number of rows per barrel and the heat drop per barrel are as follows:

Cyl. No. 1	$n_1 = 7$	$\Delta_1 C = 22.12$
	$n_2 = 7$	$\Delta_2 C = 22.12$
	$n_3 = 6$	$\Delta_3 C = 18.96$
Cyl. No. 2	$n_4 = 6$	$\Delta_4 C = 38.00$
	$n_5 = 6$	$\Delta_5 C = 38.00$
	$n_6 = 6$	$\Delta_6 C = 38.00$
Cyl. No. 3	$n_7 = 4$	$\Delta_7 C = 58.00$
	$n_8 = 4$	$\Delta_8 C = 58.00$
	$n_9 = 2$	$\Delta_9 C = 29.00$
	48	322.20

These may be arranged upon a distribution diagram similar to Fig. 155, and this is done in Fig. 168. Then heat drops to the center of each barrel are taken and denoted by A, B, C, etc. The distribution factors are then scaled from the diagram and Table 49 computed after the manner of Table 40. Nozzle areas and blade lengths are added to the table. The latter, computed for the center of the barrel, applies to all blades and guides for that barrel. It will be found that blade lengths for the two last barrels are excessive, and to accommodate the steam they must either be placed upon a larger wheel, or wing blades (or possibly semi-wing) must be provided. If $m = 0.2$, the last two barrels will have blades $4\frac{5}{8}$ in. long. Then from (13), for the eighth barrel:

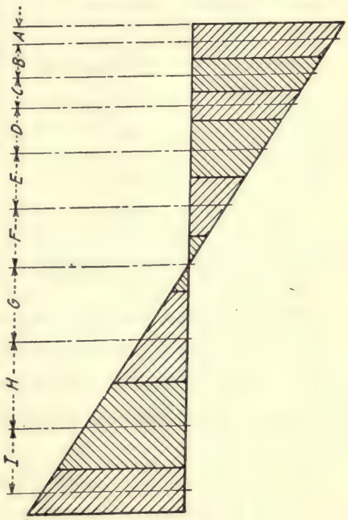


FIG. 168.

$$\sin \alpha = 0.326$$

and

$$\alpha = 19 \text{ degrees.}$$

This is but slightly greater than the other blade angles. For the last stage:

$$\sin \alpha = 0.682$$

and

$$\alpha = 43 \text{ degrees}$$

which is for a wing blade. As this was computed from the center of the last heat drop, it would not give full area at condenser pressure, but as there are but two rows on the last barrel, the difference may be neglected.

It must be remembered that the same discrepancy is found in every barrel, the blades not being lengthened as theory indicates that they should.

TABLE 49

1 Stage	2 $F_D \cdot \Delta C$	3 $C_1 - \Sigma(F_D \cdot \Delta C)$ At $\phi = 1.56$	4 P (Absolute)	5 $C - \Sigma(F \cdot \Delta C)$	6 ϕ	7 v	8 a	9 α, in.
Steam chest	.....	1193.30	164.800	1193.30	1.56	....	....	....
		11.90		6.64				
A	11.90	1181.40	143.500	1186.66	1.57	3.109	6.11	0.55
		23.60		13.28				
B	23.60	1157.80	105.800	1173.38	1.58	4.098	8.04	0.72
		21.75		12.33				
C	21.75	1136.05	81.100	1161.05	1.59	5.218	10.90	1.00
		29.80		17.10				
D	29.80	1106.25	54.100	1143.95	1.61	7.508	11.40	0.71
		39.10		22.80				
E	39.10	1067.15	31.420	1121.15	1.64	12.310	18.70	1.27
		38.38		22.80				
F	38.38	1028.77	17.520	1098.35	1.66	21.090	32.00	2.16
		48.00		22.80				
G	48.00	980.77	8.020	1069.55	1.70	42.670	41.50	1.86
		55.75		34.80				
H	55.75	923.02	2.812	1034.75	1.75	111.700	109.00	4.96*
		40.75		26.10				
I	40.75	882.27	1.271	1008.65	1.78	235.000	228.00	10.20*

* Make 4.625 in., the last stage being wing blades.

Blade widths for the various lengths may be selected from Table 43.

In neither of these examples is it claimed that conditions are met in the best manner. The heat factor chosen is lower than should be found in such machines, but it is probable that it is more nearly the usual value for units of this size.

For each example, the steam inlet from (73) is 21.3 sq. in. and the diameter 5.22 in. The next larger standard pipe size is 6 in. The exhaust opening is from (74), 559 sq. in., which if circular would be 26.7 in., or in a standard pipe size, 28 in. A certain Curtis turbine of the same power has an exhaust connection 30 in. in diameter.

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PART V—MECHANICS

CHAPTER XVI

THE SLIDER CRANK

98. Introduction.—The slider-crank mechanism is the transmission machinery of the reciprocating engine. With certain assumptions it permits of a very rigid mathematical analysis, involving trigonometry and the trigonometric functions of the calculus, subjects with which many good designers are not at all intimate. No designer of reciprocating engines is well equipped who cannot deal in a fairly thorough manner with problems involving the slider crank, so in this chapter an attempt is made to give a treatment which will cover essential features in as simple manner as possible.

For a thorough study of the movement of the connecting rod and the forces resulting therefrom, the reader is referred to the excellent work of Prof. W. E. Dalby, The Balancing of Engines, in which derivations of formulas for acceleration used in this book are given.

It is believed that the treatment will be of greater practical use if the convention respecting signs of trigonometric functions is ignored. The formulas will therefore give absolute values and the proper signs for plotting, found by inspection, will be designated for the different quadrants, usually on a diagram employed, and in tables.

An attempt to account for friction is apt to involve more error than its neglect, therefore it is ignored.

To simplify expression, all dimensions are in feet in this chapter.

Notation.

w = weight in pounds of section of connecting rod. Also weight in general.

W = weight in pounds of any part or collection of parts.

P = force due to steam or gas pressure. To this may be added—if desired—inertia of the reciprocating parts and sometimes, for approximate work, the inertia of the connecting rod; for vertical engines the weight of the reciprocating parts is added on the down stroke and subtracted on the return stroke.

G = force in pounds due to gravity, exerted by connecting rod.

C = force exerted by gravity of crank and pin; this may include counterbalance, in which case it may have a negative value.

F = force due to acceleration, in pounds.

T = total turning effort in pounds at crank circle, due to combined forces.

L = length of connecting rod in feet.

L_G = distance in feet from center of crosshead pin to center of gravity of connecting rod.

L_P = same to center of percussion of rod.

l = any measurement in feet.

R = radius of crank circle in feet.

$n = L/R$.

r_C = distance in feet from center of shaft to center of gravity of crank, pin, or counterbalance.

r = radius of gyration in feet, of connecting rod about crosshead pin center.

I = moment of inertia of rod in pound-feet about center of crosshead pin.

x = any measurement in feet parallel to line of stroke; also specific measurements in this direction as given on diagrams.

y = same normal to line of stroke.

e = efficiency.

V = velocity in feet per second.

ω = angular velocity in radians per second.

N = r.p.m.

S = piston speed in feet per minute.

a = acceleration in feet per second per second.

g = acceleration due to gravity (= 32.16).

π = 3.1416.

The following subscripts are generally used:

x and y refer to quantities in direction of line of stroke and normal to it.

A pertains to cylinder end of engine.

B pertains to shaft end of engine.

H refers to head end of stroke.

C refers to crank end of stroke.

T signifies the turning effect of a force.

N refers to the normal component.

n refers to any number.

L pertains to force along connecting rod.

R pertains to force along crank.
W refers to effect of weight only.
P pertains to force along piston path.

99. Properties of the Connecting Rod.—If a finished rod is available, the center of gravity may be found by weighing each end. Referring to Fig. 169:

$$W_1 L_G = W_2(L - L_G).$$

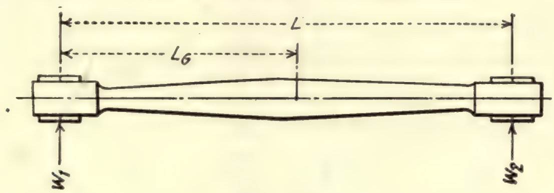


FIG. 169.

From which:

$$L_G = \frac{W_2 L}{W_1 + W_2} \tag{1}$$

The center of percussion may be found by hanging the rod upon a knife edge at the center of the crosshead pin bearing as shown in Fig. 170; support a plumb line from a point near by. Adjust the plumb bob until it swings in unison with the rod, and the length of the line to the center of the bob measures the distance from the center of the crosshead pin to the center of percussion of the rod.

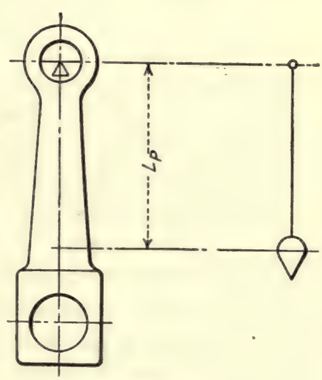


FIG. 170.

The radius of gyration is a mean proportional between these two distances or:

$$r = \sqrt{L_G L_P} \tag{2}$$

If no rod is available the weight must be estimated from the drawing. The rod may be divided into sections as shown in Fig. 171. The weight of each section is estimated and designated by a subscript. From any line, as AB, measure the distance to the center of gravity of each section; then the following equation holds:

$$W(l_1 + L_G) = w_1 l_1 + w_2 l_2 + w_3 l_3 \text{ etc.}$$

From which:

$$L_G = \frac{\Sigma wl}{W} - l_1 \tag{3}$$

The stub ends may be divided into as many sections as desirable, or

l may be taken to the approximate center of the complete stub. For certain forms of rod the center of the body may be found more easily, but the method given provides data in a usable form for what follows.

By a similar approximate method the moment of inertia may be determined directly. Taking moments about the center of the crosshead pin instead of the line AB gives:

$$I = w_1l_1^2 + w_2l_2^2 + w_3l_3^2 \text{ etc.} = \Sigma wl^2 \tag{4}$$

As $I = Wr^2$,

$$r = \sqrt{\frac{W}{I}} \tag{5}$$

The center of percussion is, from (2):

$$L_P = \frac{r^2}{L_G} \tag{6}$$

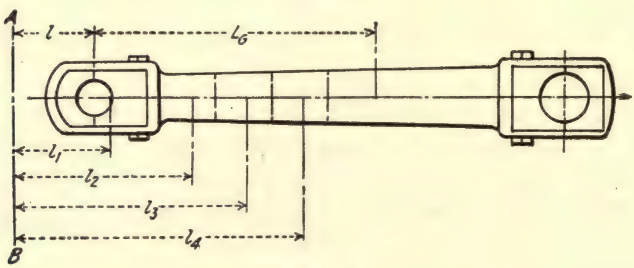


Fig. 171.

Theoretically, the greater the number of sections into which the rod is divided, the greater the accuracy, but this may be carried too far for accurate practical work; it will probably suffice if the length of the section is about equal to the mean depth or diameter.

For the crosshead stub, the portion on either side of the pin center must be taken separately and not considered as the entire stub weight concentrated at the center of gravity; parts on both sides add to the inertia and do not neutralize each other as in the case of gravity. Practically the inertia effect is relatively small near the crosshead pin and too much refinement is uncalled for. For this reason it is not necessary to add the moment of inertia of each section to wl^2, although this is theoretically correct.

Turned taper rods are often used and in some cases the maximum diameter is at the center. The rod may then be divided into parts composed of the stub ends and one or two frustums of a cone. The volume, center of gravity, and moment of inertia of the frustum of a cone may be found by the following formulas, in which d_1, d_2 and d_0 are diameters

in ft. at the small and large ends and at the center of gravity respectively, w_C the weight per cu. ft. and v the volume in cu. ft. Let l_1 be the distance from the small end to the center of gravity, l_2 the same to the large end, and l the sum of the two as shown in Fig. 172. The volume is:

$$v = \frac{\pi l}{12}[(d_1 + d_2)^2 - d_1 d_2].$$

The weight is:

$$W = w_C v.$$

The distance from the small end to the center of gravity is:

$$l_1 = \frac{l}{4}\left[\frac{(d_1 + d_2)^2 + 2d_2^2}{(d_1 + d_2)^2 - d_1 d_2}\right].$$

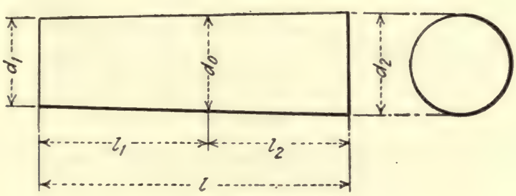


Fig. 172.

The moment of inertia about the center of gravity is:

$$I = \frac{\pi w_C}{120}[l_1^3[d_0^2 + 3d_1(2d_1 + d_0)] + l_2^3[d_0^2 + 3d_2(2d_2 + d_0)]]$$

in which:

$$d_0 = d_1 + \frac{l_1}{l}(d_2 - d_1).$$

If l_G is the distance from the center of gravity to the center of the crosshead pin, the moment of inertia about the latter is:

$$I_A = I + Wl_G^2.$$

If the rod has the largest diameter at or near the center, forming two frustums, they may both be treated in this manner. The product of the weight of the crank-end stub and the distance of its center of gravity from the crosshead pin center, added to the values of I for the two parts of the rod (the two frustums), and the moment of inertia of the crosshead stub—which is relatively small—give the total moment of inertia for this form of rod more easily and with greater accuracy than the method of Formula (4).

100. Piston Velocity.—In Fig. 173, the instant center of the connecting rod relative to the engine frame is at c. If V_T, the tangential velocity of the crank pin, is known, the velocity of the piston and cross-

head at any instant may be found. The linear velocity at any point in a rigid revolving system is directly proportional to its distance from the center; then:

$$\frac{V}{V_T} = \frac{l_1}{l_2}$$

or:

$$V = V_T \frac{l_1}{l_2} \tag{7}$$

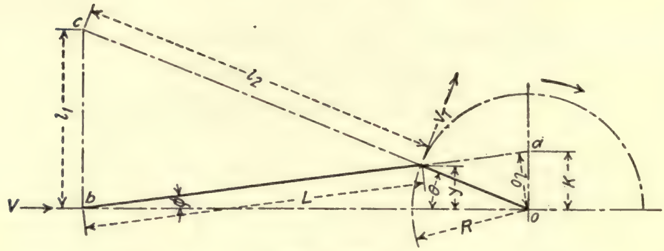


FIG. 173.

In Fig. 173, *ceb* and *oea* are similar triangles; then:

$$\frac{l_1}{l_2} = \frac{K}{R}.$$

Substituting in (7) gives:

$$V = V_T \cdot \frac{K}{R}.$$

If the scale of velocity is such that V_T is represented by R, then V is given

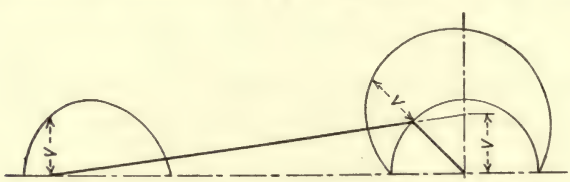


FIG. 174.

by K, which may be measured from a diagram and plotted on the piston path or crank circle, the latter being sometimes rectified. Fig. 174 shows the velocity plotted on the crosshead path and crank circle.

From an inspection of Fig. 173 it may be seen that:

$$K = R \sin \theta \pm R \cos \theta \tan \phi.$$

The plus sign applies to the half of the crank circle toward the cylinder and the minus sign to the opposite half. Then:

$$\frac{K}{R} = \sin \theta \pm \cos \theta \tan \phi.$$

Also:

$$\tan \phi = \frac{\sin \phi}{\cos \phi}.$$

From Fig. 173:

$$L \sin \phi = R \sin \theta.$$

From which:

$$\sin \phi = \frac{R}{L} \sin \theta.$$

Let

$$\frac{L}{R} = n;$$

then:

$$\sin \phi = \frac{\sin \theta}{n}$$

and:

$$\cos \phi = \sqrt{1 - \sin^2 \phi} = \sqrt{1 - \left(\frac{\sin \theta}{n}\right)^2}.$$

Then:

$$\tan \phi = \frac{\sin \theta}{n\sqrt{1 - \left(\frac{\sin \theta}{n}\right)^2}}$$

and:

$$\frac{K}{R} = \left[1 \pm \frac{\cos \theta}{n\sqrt{1 - \left(\frac{\sin \theta}{n}\right)^2}}\right] \sin \theta \tag{8}$$

Then:

$$V = V_T \cdot \frac{K}{R}$$

$$= V_T (\sin \theta \pm \cos \theta \tan \phi)$$

$$= V_T \left[1 \pm \frac{\cos \theta}{n\sqrt{1 - \left(\frac{\sin \theta}{n}\right)^2}}\right] \sin \theta \tag{9}$$

101. Forces Due to Pressure in Cylinder.—The forces acting on the slider-crank mechanism may also be determined by the method of instant centers and by the use of Fig. 175, in which P is the total unbalanced pressure on the piston (see notation).

The *tangential force* (also known as *turning effort* and *crank effort*) due to P, denoted by P_T, may be found by taking moments about instant center c; or:

$$Pl_1 = P_T l_2.$$

From which,

$$P_T = P\frac{l_1}{l_2}.$$

By similar triangles as before:

$$\frac{l_1}{l_2} = \frac{K}{R}.$$

Then:

$$P_T = P\cdot\frac{K}{R}$$

$$= P(\sin\theta \pm \cos\theta\tan\phi)$$

$$= P\left[1 \pm \frac{\cos\theta}{n\sqrt{1-\left(\frac{\sin\theta}{n}\right)^2}}\right]\sin\theta \tag{10}$$

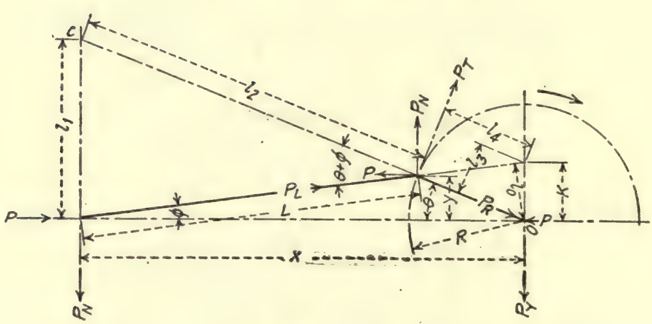


FIG. 175.

P_T is a positive turning effort when the pressure P acts in the same direction as the movement of the piston; if in the opposite direction it is negative.

The force due to P transmitted *along the connecting rod,* denoted by P_L, is found by taking moments about shaft center o and substituting (10). Then:

$$P_L l_o = P_T R = PK$$

or:

$$P_L = P\frac{K}{l_o}.$$

By similar triangles:

$$\frac{K}{l_o} = \frac{L}{\sqrt{L^2-y^2}} = \frac{n}{\sqrt{n^2-\left(\frac{y}{R}\right)^2}} = \frac{1}{\sqrt{1-\left(\frac{\sin\theta}{n}\right)^2}}$$

Then:

$$P_L = P\frac{K}{l_o} = \frac{P}{\sqrt{1-\left(\frac{\sin\theta}{n}\right)^2}} \tag{11}$$

P_L is the maximum force due to P producing bending moment on crank pin and shaft at any time, and equals P at dead center.

The *normal force on guide* due to P is denoted by P_N, and is also found by taking moments about shaft center o and substituting (10). Then:

$$P_N x = P_T R = PK$$

or:

$$P_N = P\cdot\frac{K}{x}\cdot$$

By similar triangles:

$$\frac{K}{x} = \frac{y}{\sqrt{L^2-y^2}} = \frac{\frac{y}{R}}{\sqrt{n^2-\left(\frac{y}{R}\right)^2}} = \frac{\sin\theta}{n\sqrt{1-\left(\frac{\sin\theta}{n}\right)^2}}.$$

Then:

$$P_N = P\cdot\frac{K}{x} = \frac{P\sin\theta}{n\sqrt{1-\left(\frac{\sin\theta}{n}\right)^2}} \tag{12}$$

The direction of action of P_N is shown in Fig. 204 and Table 56.

Force P is transmitted as an equal and parallel force to the crank pin, which in turn is transmitted to the main bearing. Two couples then hold the rod in equilibrium and may be equated as follows:

$$P_N L\cos\phi = PL\sin\phi$$

or:

$$P_N = P\tan\phi.$$

It is obvious from Fig. 175 that $K/x = \tan\phi$, so that the value of P_N is the same as that given by (12).

The *radial force on the crank* due to P is denoted by P_R, and is found by taking moments about instant center a; or:

$$P_R l_3 = P_T l_4$$

and:

$$P_R = P_T\frac{l_4}{l_3} = P_T\cot(\theta+\phi).$$

But:

$$P_T = P_L \frac{l_0}{R} = P_L \sin(\theta + \phi).$$

Then:

$$\begin{aligned} P_R &= P_T \frac{l_4}{l_3} \\ &= P_L \cos(\theta + \phi) \\ &= \frac{P \cos(\theta + \phi)}{\sqrt{1 - \left(\frac{\sin\theta}{n}\right)^2}} \\ &= P \left[\cos\theta - \frac{\sin^2\theta}{n\sqrt{1 - \left(\frac{\sin\theta}{n}\right)^2}} \right] \end{aligned}$$

This is zero when $\theta + \phi$ equals 90—when crank and connecting rod are at right angles.

Efficiency of the Slider Crank.—From (9) and (10) it is clear that:

$$P_T V_T = PV \tag{14}$$

It is shown in mechanics that PV is the rate of work done at any instant, and since (14) holds good for every position occupied by the mechanism, the work done at the crank pin equals that done in the engine cylinder if friction is neglected; or the efficiency is:

$$e = \frac{P_T V_T}{PV} = 1 \tag{15}$$

When it is considered that in high-grade steam engines the entire loss by friction is sometimes less than 5 per cent., the futility of trying to replace the slider crank with mechanisms for which great saving is claimed is apparent.

102. Forces Due to Gravity. *Horizontal Engine.*—The connecting rod of a horizontal engine has a negative effect upon the turning effort when the crank is in the half of the crank circle toward the cylinder, and positive when on the opposite side.

TABLE 50

Force	Formula	Quadrant			
		1	2	3	4
G_T	(16)	−	+	+	−
C_T	(18)	−	+	+	−

Let G_B be the reaction of the rod of weight W at the crank pin. The turning effort may be found by taking moments about shaft center o; then:

$$G_T R = G_B R \cos \theta$$

or:

$$G_T = G_B \cos \theta = \frac{L_G}{L} W \cos \theta \tag{16}$$

The force G_B also acts as an equal and parallel force at the center of the main bearing. The force due to W acting on the guide is:

$$G_A = W - G_B \tag{17}$$

Let C be the weight of the unbalanced part of the crank, or the weight of the crank pin; or if a counterbalance is used it may be the unbalanced

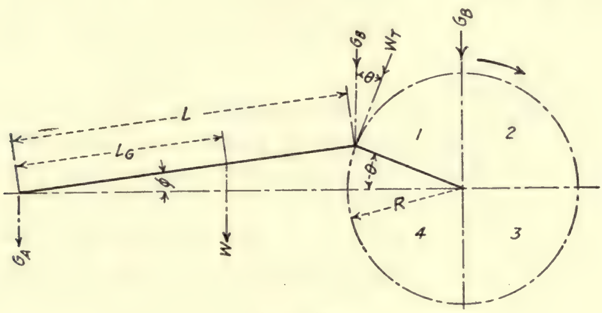


FIG. 176.

part of this weight. Let r_C be the distance from the center of the shaft to the center of gravity of the weight C. The algebraic sum of these weights referred to the crank pin center may be expressed by:

$$\Sigma \left(\frac{r_C}{R} \cdot C \right)$$

Then the turning effort due to these is:

$$C_T = \Sigma \left(\frac{r_C}{R} C \right) \cos \theta \tag{18}$$

The signs for G_T and C_T for the different quadrants in Fig. 176 are given in Table 50 under the corresponding quadrant numbers. The plus sign indicates a positive turning effort—that the force acts in the direction of motion; while the minus sign is for a negative effect. If the overbalance due to C is on the crank side the signs are as given in Table 50; if opposite the crank the signs for C_T must be reversed.

Aside from friction the weight of other parts does not affect the turning effort.

Vertical Engine.—The weight of the piston, piston rod and cross-head obviously affect the turning effort, but as it acts in the same line with the pressure it may be accounted for in drawing the indicator diagram.

The effect of the connecting rod is to increase the turning effort on the down stroke and decrease it on the up stroke; its weight is sometimes added to that of the strictly reciprocating parts but it is more correctly treated by itself and this will be done, leaving the approximate method to the judgment of the designer.

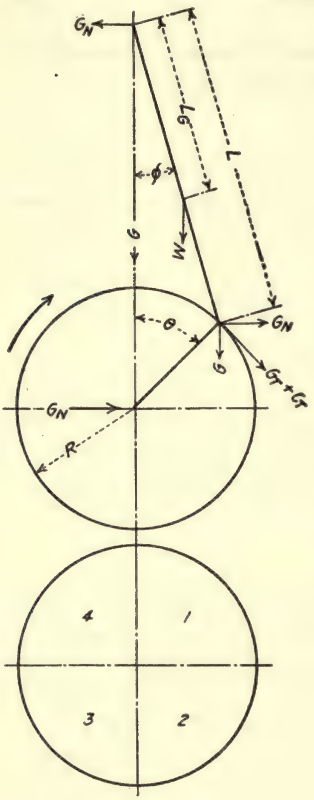


FIG. 177.

The rod exerts a downward force G and a lateral force G_N as shown in Fig. 177. It must be assumed that the entire weight is supported by the crank pin; then:

$$G = W.$$

The weight of the rod acting at the center of gravity and resisted by an equal force at the crank pin forms a couple equal to:

$$G(L - L_G) \sin \phi.$$

This is balanced by the couple:

$$G_NL \cos \phi$$

or:

$$G_NL \cos \phi = W(L - L_G) \sin \phi.$$

From which:

$$G_N = W(1 - \frac{L_G}{L}) \frac{\sin \phi}{\cos \phi}$$

$$= W\left(1 - \frac{L_G}{L}\right) \frac{\sin \theta}{n\sqrt{1 - \left(\frac{\sin \theta}{n}\right)^2}} \qquad (19)$$

Taking moments about crank center o for forces G, G_N and C, the corresponding turning efforts are found. The equations are:

$$G_{XT}R = GR \sin \theta$$

$$G_{NT}R = G_NR \cos \theta$$

$$C_TR = \Sigma \left(\frac{r_c}{R} \cdot C\right) R \sin \theta \cdot$$

Then by cancellation and substitution the turning efforts are:

$$G_{XT} = W \sin \theta \tag{20}$$

$$G_{NT} = G_N \cos \theta = W\left(1 - \frac{L_G}{L}\right) \frac{\sin 2\theta}{2n\sqrt{1 - \left(\frac{\sin \theta}{n}\right)^2}} \tag{21}$$

$$C_T = \Sigma\left(\frac{r_C}{R}C\right) \sin \theta \tag{22}$$

The total turning effort due to the gravity of the rod is:

$$G_T = G_{XT} + G_{NT} \tag{23}$$

TABLE 51

Force	Formula	Quadrant			
		1	2	3	4
G_{XT}	(20)	+	+	−	−
G_{NT}	(21)	+	−	+	−
C_T	(22)	+	+	−	−

The signs for the quantities G_{XT}, G_{NT} and C_T for crank positions in the different quadrants of Fig. 177 are given in Table 51, the plus sign for forces acting in the direction of turning as before; the sign for C_T assumes the over-balance on the pin side of the crank. The forces G and G_N also act at the center of the main bearing.

If the engine center line is inclined from the vertical δ degrees as shown in Fig. 178, modification of the reactions must be made as follows:

$$G_B = W - W\left(1 - \frac{L_G}{L}\right) \sin \delta$$

$$= W\left[1 - \left(1 - \frac{l}{L}\right) \sin \delta\right] \tag{24}$$

$$G_N = G\left(1 - \frac{L_G}{L}\right) \tan \phi \cos \delta \tag{25}$$

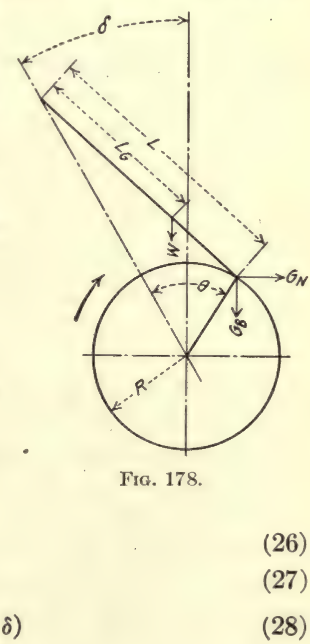


FIG. 178.

The effect on the turning effort is:

$$G_T = G_B \sin (\theta - \delta) \tag{26}$$

$$G_{NT} = G_N \cos (\theta - \delta) \tag{27}$$

$$C_T = \Sigma\left(\frac{r_C}{R}C\right) \sin (\theta - \delta) \tag{28}$$

The signs for the different portions of the crank pin path depend upon the angle δ and may be determined by inspection for a particular case. Formulas (24) to (28) reduce to those already given for a horizontal or vertical engine.

103. Forces Due to Acceleration.—Practically all the inertia forces of the slider-crank mechanism may be accounted for by considering the acceleration of any point in the connecting rod in the direction x, parallel to the crosshead path, and the direction y normal to the crosshead path. In the formulas, which will be given without derivation, ω is the angular velocity in radians per second, and may be expressed in terms of r.p.m. thus:

$$\omega = \frac{2\pi N}{60} = \frac{\pi N}{30} \tag{29}$$

The acceleration in ft. per sec. per sec. in the direction x, Fig. 179 is:

$$a_X = R\omega^2\left[\pm\cos\theta + \left(1 - \frac{l}{L}\right)\left(\frac{\pm\cos 2\theta + \left(\frac{\sin^2\theta}{n}\right)^2}{n\left[1 - \left(\frac{\sin\theta}{n}\right)^2\right]^{3/2}}\right)\right] \tag{30}$$

and in the direction y:

$$a_Y = \frac{l}{L} R\omega^2 \sin\theta \tag{31}$$

As force is the product of mass and acceleration, the general formula is:

$$F = \frac{W}{g} a \tag{32}$$

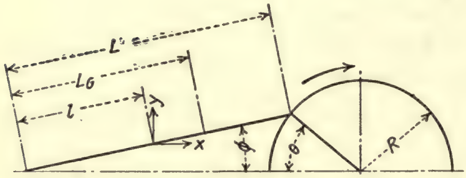


Fig 179.

in which a may be either a_X or a_Y, and if W is the total weight, a must be the acceleration of the mass center. Formulas (30) and (31) give acceleration in any point in the rod; and for any mass assumed as concentrated at that point, (32) gives the force. An approximation to (30) is given by (48), the latter being much more simple, and for most practical applications accurate enough.

Hereafter F_Y will denote the inertia of the rod normal to the line of stroke, F_{YA} its effect at the crosshead and F_{YB} at the crank. F_X will denote the inertia of the rod along the line of stroke, referred to its mass center.

When l is zero, a_X is the acceleration of the piston and other purely reciprocating parts. The inertia of these parts will be denoted by F_P.

The force exerted by the inertia of a mass upon the engine parts in a

horizontal direction is toward the cylinder when the crank is on this side of the crank circle, and in the opposite direction when the crank is on the side away from the cylinder; an exception is when the crank is in the vicinity of 90 degrees from the engine center line, and this may be seen by plotting (30) as in Fig. 180, in which a_H and a_C are accelerations at head and crank ends of the stroke respectively.

The signs in Formula (30) are used simply to obtain numerical results, and for this purpose may be taken as plus when angles θ and 2θ are in the half of the crank circle toward the cylinder. This has a negative effect upon the turning effort on the stroke from head to crank end when the numerical result has a positive sign, and a positive result when the sign is minus. On the return stroke, however, the effect is just the reverse. This may be shown by Fig. 181, which gives the acceleration diagrams for the two strokes, the positive sign denoting that the inertia of the moving part would assist the turning effort and the negative sign that it would have a retarding effect.

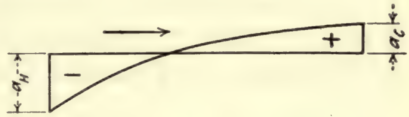


FIG. 180.

The Reciprocating Parts.—If l equals L in (30), a_X is the acceleration of the crosshead and piston. At the dead centers (30) becomes:

$$a_X = R\omega^2\left(\pm 1 - \frac{1}{n}\right).$$

The minus sign gives the numerical value for the head end and the plus sign for the crank end. Then the inertia of the reciprocating parts for head and crank ends may be computed, and other points found graphically by Klein's method shown in Fig. 182.

From (32):

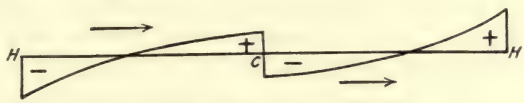


FIG. 181.

$$F_H = \frac{W}{g} R\omega^2\left(-1 - \frac{1}{n}\right)$$

$$= \frac{WRN^2}{2930}\left(-1 - \frac{1}{n}\right) \tag{33}$$

and:

$$F_C = \frac{W}{g} R\omega^2\left(+1 - \frac{1}{n}\right)$$

$$= \frac{WRN^2}{2930}\left(1 - \frac{1}{n}\right) \tag{34}$$

Fig. 182 is an exact representation of (30) and proof is given in Dalby's Balancing of Engines previously referred to. The lower part of the construction is added for the convenience of plotting to any scale. After deciding upon a force scale, F_H and F_C may be laid off on the diagrams as shown; the lines *ab* and *cd* are located by making the construction at the dead center—only one of these being necessary.

The Connecting Rod.—If in (30), l equals L_G, the distance to the mass center, the effect of acceleration of the rod parallel to the line of stroke may be found. If we assume n_0 an imaginary value of n, and substitute in (30):

$$n_0 = \frac{n}{1 - \dfrac{L_G}{L}}$$

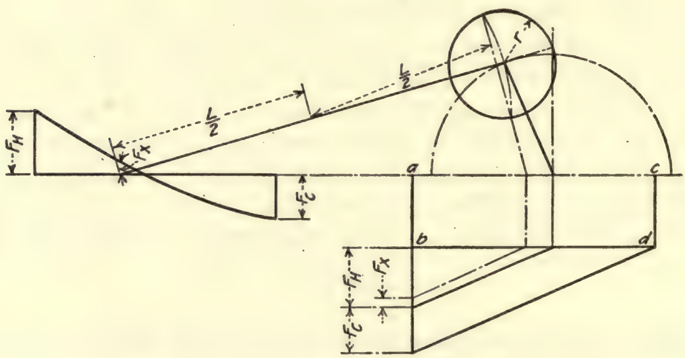


FIG. 182.

then using this value in (33) and (34), values of F_X for the rod may be found graphically.

The inertia of the reciprocating parts may be combined with the indicator diagram of total pressure before determining turning effort, and often the connecting rod is added to the reciprocating parts to simplify calculation, probably with sufficient accuracy in most cases; but to be more exact the rod requires a separate treatment, and F_X, the inertia of the rod assumed concentrated at the mass center may be treated as W in connection with the gravity effect of the rod for vertical engines. Then for the component parallel to line of stroke, comparing with (20):

$$F_{XT} = F_X \sin\theta \tag{35}$$

For the component normal to line of stroke, comparing with (21):

$$F_{XNT} = F_X\left(1 - \frac{L_G}{L}\right)\frac{\sin 2\theta}{2n\sqrt{1 - \left(\dfrac{\sin\theta}{n}\right)^2}} \tag{36}$$

Fig. 183 shows the effect of rod inertia; it may also be used as a diagram for the effect of inertia of the reciprocating parts, taking F_P only, if it is desired to treat it separate from the indicator diagram. The total effect of F_X on the turning effort is:

$$F_{XT} + F_{XNT} \tag{37}$$

TABLE 52

Force	Formula	Sector of circle					
		1	2	3	4	5	6
F_{XT}	(35)	−	+	+	−	−	+
F_{XNT}	(36)	−	+	−	+	−	+

The signs of F_{XT} and F_{XNT} are given in Table 52 for different sectors of the circle as shown in Fig. 183. A change occurs when the inertia

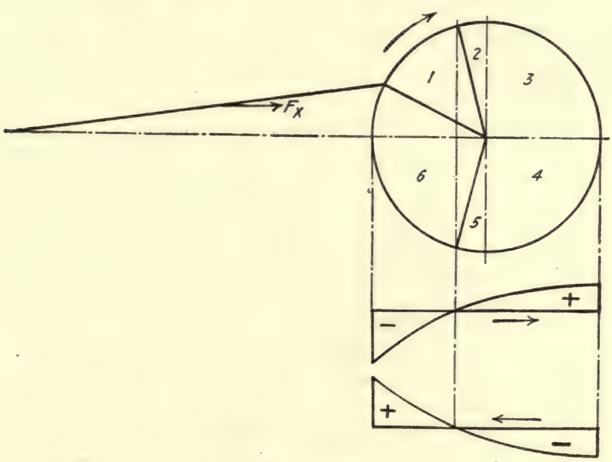


FIG. 183.

curve crosses the line in the case of F_{XNT}, and again when the crank is at right angles to the center line of the engine.

The effect of angular acceleration is best treated by finding the moment of inertia as described in Par. 99. To derive the formulas, assume the rod divided into a number of parts of weight w, each a distance l ft. from the center of the crosshead pin. The force due to acceleration of the rod normal to the line of stroke acts away from the center line of engine, and

its effect on turning effort is shown in Fig. 184. The force exerted by each section of the rod is:

$$F_Y = \frac{w}{g} a_Y = \frac{w}{g} \cdot \frac{l}{L} \cdot R\omega^2 \sin \theta \tag{38}$$

The reaction of this force on the crank pin in a direction normal to the line of stroke is:

$$F_{YN} = \frac{l}{L} F_Y = \frac{w}{g} \left(\frac{l}{L}\right)^2 R\omega^2 \sin \theta.$$

The total reaction is:

$$F_{YB} = \Sigma F_{YN} = \frac{\omega^2}{gnL} \cdot \Sigma w l^2 \cdot \sin \theta$$

$$= \frac{\omega^2}{gnL} \cdot I \sin \theta \tag{39}$$

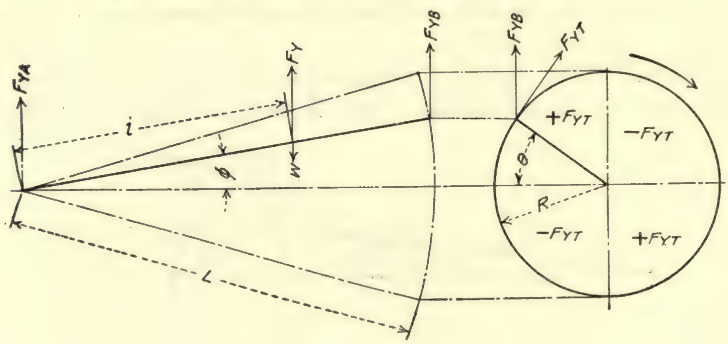


Fig. 184.

The effect upon the turning effort is:

$$F_{YT} = F_{YB} \cos \theta$$

$$= \frac{\omega^2}{2gnL} \cdot I \sin 2\theta$$

$$= \frac{N^2}{5865nL} \cdot I \sin 2\theta \tag{40}$$

The signs for F_{YT} for the different quadrants are given in Fig. 184.

The total turning effect of the rod inertia is:

$$F_T = F_{XT} + F_{XNT} + F_{YT} \tag{41}$$

The reaction on the guide due to F_Y is found by taking moments about the center of percussion, the location of which is given by (6). If F_{YA} is this reaction:

$$F_{YA}L_P = F_{YB}(L - L_P).$$

From which:

$$F_{YA} = F_{YB}\left(\frac{L}{L_P} - 1\right) \tag{42}$$

The reaction on the guide due to F_X is found as for W in (19); or:

$$F_{XN} = F_X\left(1 - \frac{L_G}{L}\right)\frac{\sin\theta}{n\sqrt{1 - \left(\frac{\sin\theta}{n}\right)^2}} \tag{43}$$

The total guide pressure due to acceleration of the rod is:

$$F_A = F_{YA} + F_{XN} \tag{44}$$

The forces F_{XN}, F_X and F_{YB} acting at the crank pin, also act as equal and parallel forces at the center of the shaft.

104. Combined Indicator and Inertia Diagrams.—As the pressure on the piston and inertia of the reciprocating parts are both applied to the crosshead pin, they may be combined and their turning effort considered together. It is first necessary to construct a stroke diagram giving the unbalanced steam or gas pressure; that is, the difference between the pressure on the two sides of the piston. This is found by plotting the distance between the forward pressure of one diagram and the back pressure of a diagram for the other side of the piston taken at the same time.

Steam engine diagrams will be first considered. A pair of indicator diagrams is shown in Fig. 185, and it is best that they should be diagrams of total pressure as this allows for the piston rod. The shaded area represents the stroke-diagram for the head end, giving the pressures acting on the piston during the stroke from head to crank end of cylinder. The pressure is zero at a and becomes negative after that. For convenience this may be plotted from a straight line, and taking pressures above the plotting line as driving the crank in the direction of motion (with the plus sign), and below the line as tending to retard the motion of the crank (with the minus sign), Fig. 186 is plotted, with inertia diagram shown below, and finally the combined diagrams.

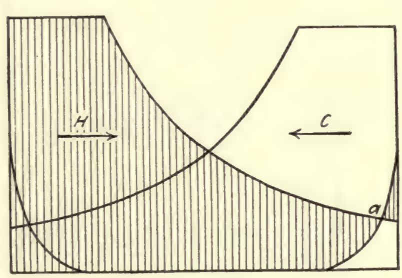


FIG. 185.

A convenient method of plotting the combined diagrams is to invert the inertia diagram on the stroke diagram as shown in dotted lines, thus

performing graphical addition and subtraction; then the measurements may be transferred to a diagram plotted from a straight line, or may be taken directly to plot a crank effort diagram. This applies especially to steam engine diagrams; for the internal-combustion engine confusion may be avoided by following the method shown by the full lines, adding the pressures algebraically.

If the engine is vertical the weight of the reciprocating parts should be added to the down-stroke diagram and subtracted from the up-stroke

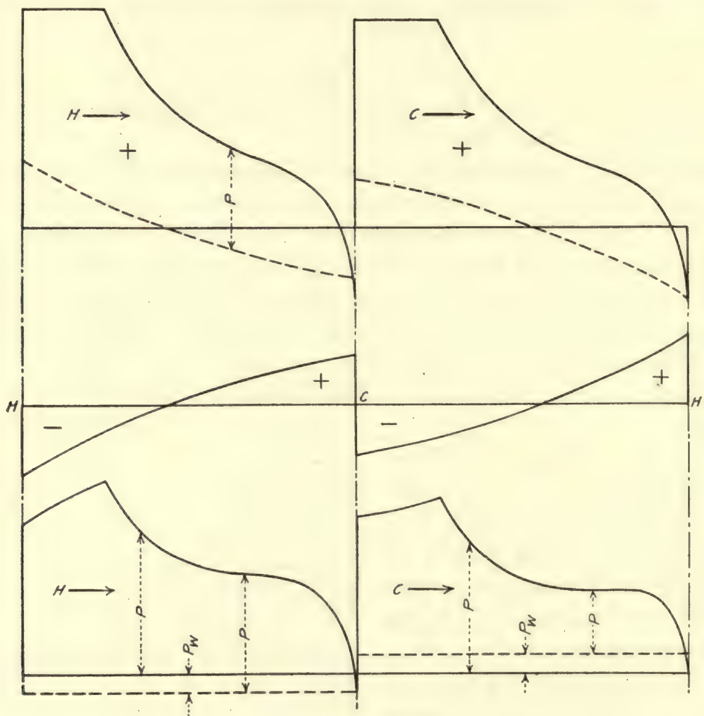


FIG. 186.

diagram as shown in dotted lines; Fig. 186 shows the addition made to the head-end diagram, which would be the case if the cylinder were above the crank—the most common arrangement. If the center line of the engine leans δ degrees from the vertical as in Fig. 178:

$$P_W = W \cos \delta \tag{45}$$

This quantity is added to the head end if the cylinder is higher than the shaft, and subtracted from the crank end.

Internal-combustion Engine Diagrams.—The conventional stroke diagram for a 4-cycle constant-volume engine is shown in Fig. 187 with inertia diagram and combined diagram below. The diagram is for an automobile engine running 1500 r.p.m. The connecting rod is not included in the inertia diagram, so that the combined diagram shows only the forces applied at the wrist pin. The inertia of the rod must be included in the forces acting at the crank, but this is best given a separate treatment. Data for the diagrams are given in Table 55. Accurate results cannot be obtained by including the rod with the reciprocating parts, especially if the center of gravity is near the crank pin, which is true of this type. In modern well-designed engines, suction and exhaust pressures differ but little and both are assumed as atmospheric in this chapter.

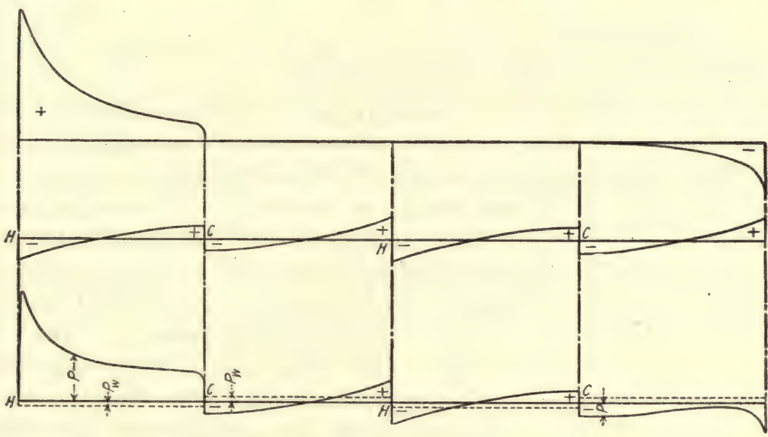


FIG. 187.

If the engine is not horizontal, P_W (shown dotted) must be taken into account as in Fig. 186 and Formula (45).

In finding values of P for different crank positions, the spacing around the crank circle should be equal; the corresponding piston positions are therefore not equally spaced and the spaces are greater at the head end than at the crank end of the stroke. These points in the piston path may be found as in Fig. 188, then marked on a piece of tracing cloth and transferred to diagrams such as Figs. 186 and 187 with a pricker point, placing the tracing cloth end-for-end for the return strokes in order that letters H and C may fall on the same letters on the diagram.

Figs. 186 and 187 show that pressure is constantly exerted against the bearings, absorbing work by friction. By plotting the forces of such

diagrams upon a rectified crank circle the mean pressure upon crank pin and shaft may be obtained for the cycle. An approximation may be made by including the connecting rod with the reciprocating parts. Then the mean pressure from such diagrams is a measure of the work absorbed by friction; this is not the mean effective pressure or the algebraic sum of the pressures, but the mean of the actual pressure against the bearings during the cycle, regardless of signs. For the main bearing a resultant should be taken, including weight of wheel, shaft, etc., pull of belt or reaction due to gears, or any other forces acting on the bearing.

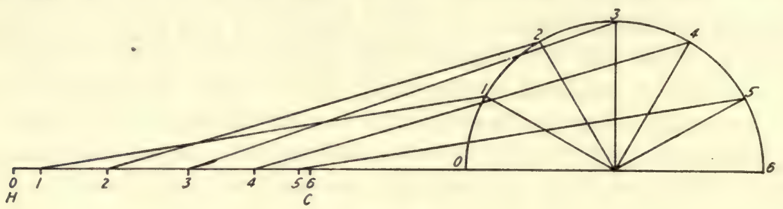


FIG. 188.

105. Reversal of Thrust and Effect of Compression.—It has been generally accepted that compression is necessary to smoothness of operation of a steam engine, but in few instances is it probable that the necessary compression has been fixed by calculation, or if so, the inertia of the reciprocating parts has probably been neglected. An examination of this is best made by inverting the crank side of the upper diagram of Fig. 186. The pressures *above* the plotting line are those acting *toward the crank,* while those *below* the line act *toward the head end.* This is shown in Fig. 189. Neglecting inertia, reversal of thrust occurs when the pressure line crosses the plotting line, which, for the diagrams shown gives a gradual reversal, occurring before the end of the stroke. When inertia is considered it occurs where the pressure line crosses the inertia curve; this curve may be considered the virtual plotting line for the remaining diagrams of this paragraph.

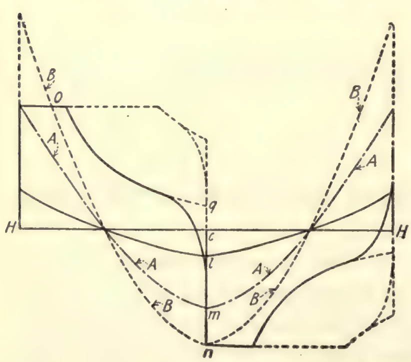


FIG. 189.

The effect of inertia is to cause reversal to occur later in the stroke, but in Fig. 189 it is still gradual; if the diagrams were plotted on a recti-

fied crank circle—a more correct way of determining the relative time of reversal—it would appear still more gradual.

With higher speed assume the inertia curve to be the dotted line *A*. In any case reversal occurs where the inertia curve crosses the pressure curve, therefore for curve *A* the reversal is at initial pressure for the head end and a little less at the crank end. The reversal at head end is gradual, and at crank end the reversal force is the pressure represented by *mn*, and while it appears to occur abruptly, it is probable that the crank travels over an appreciable angle while the pressure is being applied and there would be little or no tendency to "pound," even with some play in the pin bearings. However, if the inertia is less and the curve does not cut the pressure curve (due to compression) an appreciable distance before the dead center, a pound may be heard if the pin bearings are not properly adjusted.

With inertia curve *B*, reversal at head end occurs at *o*, after the stroke has begun, but is gradual and would probably cause no pound; at crank end it occurs at dead center but is gradual.

If there were no compression the pressure curve would be as shown dotted; then for the inertia curve shown in full lines the force due to reversal would change at dead center from *ql* to *ln*; for curve *A* it is from *qm* to *mn*, and for curve *B* from *qn* to zero, the reversal force being gradually applied. It is obvious that if reversal does not occur before the end of the stroke, the force *ql*, *qm* or *qn* has no influence upon smoothness of running, but that this depends upon the magnitude of the force starting the piston on its return stroke, such as *ln* or *mn*; if this is large a "knock" or "pound" may be the result, and it is clearly an advantage when inertia is small, to have the pressure curve cross the inertia curve before the end of the stroke. A serious obstacle to this is that without a high compression, the terminal pressure at large loads is higher than the compression pressure and the pressure line does not cross the plotting line; this is especially true with single-eccentric, non-releasing gears, which reduce the compression as the terminal pressure is increased. The uniflow engine has some advantage in this respect, but with heavy loads reversal may occur but little, if any before the end of the stroke.

A study of the diagrams makes clear the fact that increasing the weight of the reciprocating parts has sometimes aided in smoothness of operation; it may also account for the non-analytical discussions in technical papers from time to time by operating engineers, as to the necessity of compression, with fervent adherents to both sides of the question, which may only be satisfactorily explained by the use of reversal diagrams.

As the piston speed $S = 4RN$, Formulas (33) and (34) may be written:

$$F = \frac{WSN}{11{,}730}\left(\pm 1 - \frac{1}{n}\right) \qquad (46)$$

This shows the influence of weight, piston speed and rotative speed upon inertia. To obtain the reversal pressure at the crosshead pin, the reciprocating parts proper must be considered alone; for the effect at the crank pin, it will be approximately correct to add the weight of the connecting rod to W, but more accurate to determine the inertia of the rod separately as explained in Par. 103.

When an engine is running idle, as a locomotive in "coasting," reversal curves are due to inertia only, and are shown in Fig. 190. Reversal occurs near mid-stroke and is very gradual. Indicator diagrams for a single-acting engine are also shown in Fig. 190 with dotted lines. As the pressure line does not cross the inertia curve at the crank end of the stroke, there is no reversal of thrust at this place; on the return stroke reversal

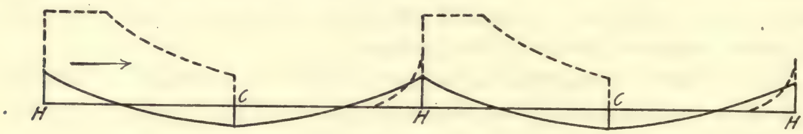


Fig. 190.

occurs when the compression curve crosses the inertia curve, and unless inertia is nearly equal to or greater than initial pressure, there is advantage in having the compression pressure greater than the inertia. It should be noted that the reversal which would have occurred at the crank end of the stroke in a double-acting engine, is deferred until the inertia curve crosses the plotting line on the return stroke a short time before the head-end reversal.

It may be seen from Fig. 189 that if compression pressure is high relative to inertia, the effect of inertia is more largely applied at the cylinder end of the frame; whereas with high inertia pressure and small compression the retarding effect comes on the bearing.

In Fig. 189, if the cut-off is long, as shown dotted, the turning effort will be as uniform as possible if the inertia effect is negligible; the shorter the cut-off the less uniform it becomes. If the inertia effect is increased for the short cut-off the effective pressure becomes more uniform throughout the stroke. A locomotive may thus be started with a maximum cut-off, and by bringing back the reverse lever gradually as the engine gains speed, the maximum uniformity may be obtained at all times. Failure to do this results in discomfort to passengers and increased wear and tear on the mechanism.

A steam automobile will take a hill more smoothly when running slow, if the cut-off be lengthened and the pressure throttled.

A single-acting, constant-volume internal-combustion engine diagram is shown in Fig. 191. For the full lines there is no reversal except where the inertia curve crosses the plotting line during the exhaust and suction strokes. Should inertia at the head end of the stroke be high enough to cross the compression line as shown dotted, reversal occurs gradually

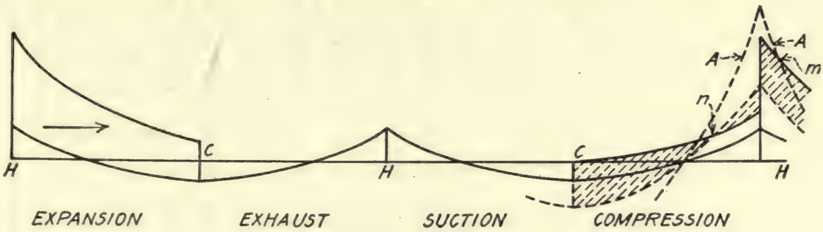


Fig. 191.

where the lines cross at n, and again at the end of the stroke. The magnitude of the reversing force depends upon the difference between inertia and the maximum gas pressure; if inertia is higher as in curve A, Fig. 191, reversal occurs after the expansion stroke has begun, and is gradual.

A reversal diagram of a 4-cycle, double-acting internal-combustion engine is shown in Fig. 192, and with a different order of firing in Fig. 193. In Fig. 192, for the values assumed, reversal occurs at the beginning of the head-end and crank-end expansion strokes, and again, gradu-

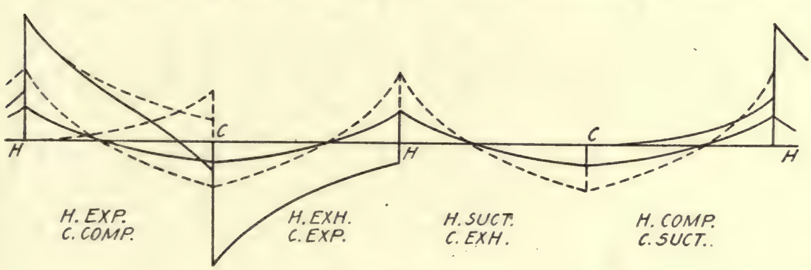


Fig. 192.

ally, during the head-end suction stroke. In Fi5. 193, abrupt reversal occurs only at the beginning of the head-end expansion stroke, and gradual reversal during the head-end exhaust stroke. Increasing the inertia to the dotted lines improves conditions for Fig. 192 by making the reversal gradual near the end of head-end compression, and reducing the reversal pressure at the beginning of crank-end expansion.

TABLE 53

No.	n	1	2	3	4	5	6	7	8	9	10	11	12
θ		15	30	45	60	75	90	105	120	135	150	165	180
Sin θ		0.26	0.50	0.71	0.87	0.97	1	0.97	0.87	0.71	0.50	0.26	0.01
Cos θ		0.97	0.87	0.71	0.50	0.26	0	0.26	0.50	0.71	0.87	0.97	1
Sin 2θ		0.50	0.87	1	0.87	0.50	0	0.50	0.87	1	0.87	0.50	0
Cos 2θ		0.87	0.50	0	0.50	0.87	1	0.87	0.50	0	0.50	0.87	1
$\sqrt{1-\left(\frac{\sin\theta}{n}\right)^2}$	4	1.00	0.92	0.98	0.98	0.97	0.97	0.97	0.98	0.98	0.99	1.00	1
	5	1.00	1.00	0.99	0.99	0.98	0.98	0.98	0.99	0.99	1.00	1.00	1
	6	1.00	1.00	0.99	0.99	0.99	0.99	0.99	0.99	0.99	1.00	1.00	1
$\frac{K}{R}=\left[1\pm\frac{\cos\theta}{n\sqrt{1-\left(\frac{\sin\theta}{n}\right)^2}}\right]\sin\theta$	4	0.32	0.61	0.83	0.98	1.03	1	0.90	0.76	0.58	0.39	0.20	0
	5	0.31	0.59	0.81	0.96	1.02	1	0.92	0.78	0.60	0.41	0.21	0
	6	0.30	0.57	0.79	0.94	1.01	1	0.92	0.79	0.62	0.43	0.22	0
$\frac{\sin\theta}{n\sqrt{1-\left(\frac{\sin\theta}{n}\right)^2}}$	4	0.065	0.126	0.179	0.222	0.249	0.258	0.249	0.222	0.179	0.126	0.065	0
	5	0.052	0.101	0.143	0.166	0.196	0.204	0.196	0.166	0.147	0.101	0.052	0
	6	0.043	0.084	0.119	0.146	0.164	0.169	0.164	0.146	0.119	0.084	0.043	0
$\pm\cos\theta\pm\frac{\cos 2\theta}{n}$	4	1.21	1.08	0.71	0.38	0.04	−0.25	−0.48	−0.63	−0.71	−0.74	−0.75	−0.75
	5	1.15	1.04	0.71	0.40	0.08	−0.20	−0.43	−0.60	−0.71	−0.77	−0.79	−0.80
	6	1.13	1.01	0.71	0.42	0.12	−0.17	−0.40	−0.58	−0.71	−0.78	−0.82	−0.83

No.	13	14	15	16	17	18	19	20	21	22	23	24
θ	195	210	225	240	255	270	285	300	315	330	345	360
Sin θ	0.26	0.50	0.71	0.87	0.97	1	0.97	0.87	0.71	0.50	0.26	0
Cos θ	0.97	0.87	0.71	0.50	0.26	0	0.26	0.50	0.71	0.87	0.97	1
Sin 2θ	0.50	0.87	1	0.87	0.50	0	0.50	0.87	1	0.87	0.50	0
Cos 2θ	0.87	0.50	0	0.50	0.87	1	0.87	0.50	0	0.50	0.87	1
$\sqrt{1-\left(\frac{\sin\theta}{n}\right)^2}$	1.00	0.99	0.98	0.98	0.97	0.97	0.97	0.98	0.98	0.99	1.00	1
	1.00	1.00	0.99	0.99	0.98	0.98	0.98	0.99	0.99	1.00	1.00	1
	1.00	1.00	0.99	0.99	0.99	0.99	0.99	0.99	0.99	1.00	1.00	1
$\frac{K}{R}=\left[1 \pm \frac{\cos\theta}{n\sqrt{1-\left(\frac{\sin\theta}{n}\right)^2}}\right]\sin\theta$	0.20	0.39	0.58	0.76	0.90	1	1.03	0.98	0.83	0.61	0.32	0
	0.21	0.41	0.60	0.78	0.92	1	1.02	0.96	0.81	0.59	0.31	0
	0.22	0.43	0.62	0.79	0.92	1	1.01	0.94	0.19	0.57	0.30	0
$\frac{\sin\theta}{n\sqrt{1-\left(\frac{\sin\theta}{n}\right)^2}}$	0.065	0.126	0.179	0.222	0.249	0.258	0.249	0.222	0.179	0.126	0.065	0
	0.052	0.101	0.143	0.166	0.196	0.204	0.196	0.166	0.143	0.101	0.052	0
	0.043	0.084	0.119	0.146	0.164	0.169	0.164	0.146	0.119	0.084	0.043	0
$\pm\cos\theta \pm \frac{\cos 2\theta}{n}$	−0.75	−0.74	−0.71	−0.63	−0.48	−0.25	0.04	0.38	0.71	1.08	1.21	1.25
	−0.79	−0.77	−0.71	−0.60	−0.43	−0.20	0.86	0.40	0.71	1.04	1.15	1.20
	−0.82	−0.78	−0.71	−0.58	−0.46	−0.17	0.12	0.42	0.71	1.01	1.13	1.17

In Fig. 193, while the reversal pressure at the beginning of head-end expansion is reduced by increased inertia, an abrupt reversal is caused at the beginning of crank-end expansion. This would indicate that the smoother order of firing may only be predetermined by the use of such diagrams.

106. Total Turning Effort T may be found by combining the results of (10), (16) or (23), (18) or (22), and (41); or:

$$T = P_T + G_T + C_T + F_T \tag{47}$$

The most common use of turning-effort diagrams is in connection with the determination of flywheel weight for electrical service, in which the load curve is a straight line. If it is desired to make such investigations for compressors of any type, whether arranged in tandem, or on a common shaft with different crank angles, these reversed diagrams must be plotted

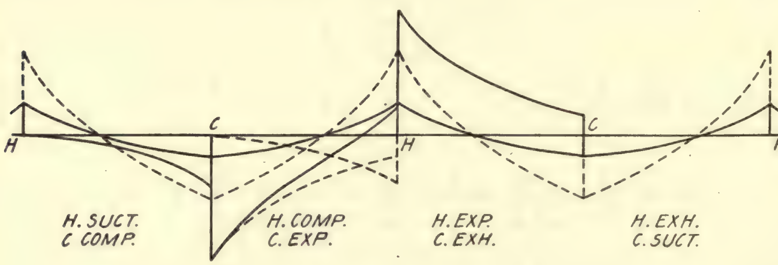


FIG. 193.

with their inertia diagrams and added algebraically to the values found by (47).

Where possible, the formulas are given in such a way that results may be obtained either by measuring the diagram or by calculation. In the latter case, P, W or F are multiplied by some factor depending upon the value of angle θ. To facilitate calculation a number of these values are given in Table 53 for a complete revolution, the crank circle being divided into 24 equal parts.

An approximation to Formula (30) may be written:

$$a_X = R\omega^2\left[\pm \cos\theta \pm \left(1 - \frac{l}{L}\right)\frac{\cos 2\theta}{n}\right] \tag{48}$$

The error is less than 2 per cent. if n is 4, being maximum when θ is 90 degrees. This formula is used for finding values in Table 53, l being zero at the center of the crosshead pin. The minus sign in the table indicates that the curve has crossed the plotting line, but has no other significance.

For a 4-cycle internal-combustion engine the values are all repeated, but P_T, on the second revolution is different, as it depends upon the pressure on the piston.

Steam Engine Diagram.—As an example of application, data will be assumed for a simple steam engine, the values given in Table 54, then plotted in Fig. 194. The indicator and inertia diagrams for the example are given in Figs. 185 and 186. Data for the problem will be for the 20- by 48-in. Corliss engine designed in Chap. XII, the parts of which are designed in later chapters. The data necessary for the problem are: Weight of reciprocating parts = 1550 lb. Weight of connecting rod = 1330 lb. $n = 6$, $N = 100$, $\omega^2 = 109$. Other data for the rod found from methods in Par. 99 are: $I = 78{,}654$, $r^2 = 59$, $L_G = 6.32$, $L_P = 8.07$.

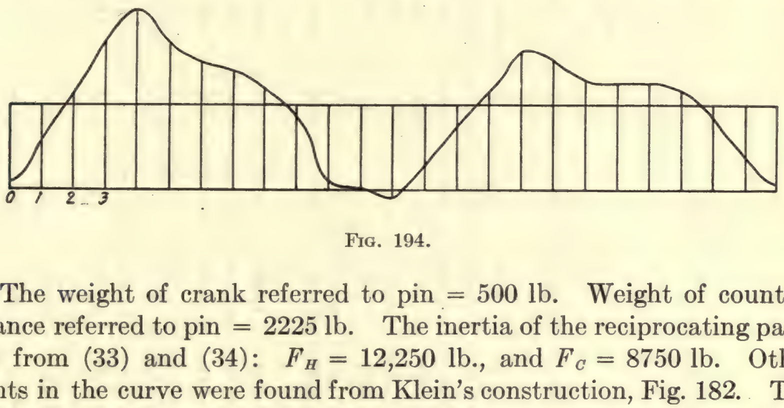


Fig. 194.

The weight of crank referred to pin = 500 lb. Weight of counterbalance referred to pin = 2225 lb. The inertia of the reciprocating parts are, from (33) and (34): $F_H = 12{,}250$ lb., and $F_C = 8750$ lb. Other points in the curve were found from Klein's construction, Fig. 182. The connecting rod is not included with the reciprocating parts but is treated separately. Total pressure P includes the effect of inertia of the reciprocating parts proper.

The counterbalance overbalances the crank by 1725 lb., referred to the crank-pin center. The signs for C_T are then opposite to those given in Table 50.

If the area of Fig. 194 is found by a planimeter and divided by the length the mean ordinate may be found, and a line representing the mean turning effort drawn as shown. The fluctuation of turning effort may thus be plainly seen, and the areas between the curve and this line represent energy fluctuations. The ratio of the maximum area thus enclosed to the area of the rectangle formed by the mean line and the plotting line (for one stroke, one revolution or one cycle, as desired) is called the coefficient of the fluctuation of energy. Referred to one stroke, this quantity for Fig. 194 is 0.333. The mean line must always be found from the diagram for the entire cycle.

TABLE 54

Force		Form.	Crank position											
			1	2	3	4	5	6	7	8	9	10	11	12
P			27,500	29,400	32,250	31,800	23,200	19,200	18,000	17,400	15,000	10,300	1,500	−6,000 8,500
P_T		(10)	8,260	16,800	25,400	29,900	23,400	19,200	16,620	13,800	9,350	440	325	0
G_T*		(16)	−676	−607	−495	−350	−182	0	182	350	495	607	676	700
C_T		(18)	1,662	1,492	1,220	862	446	0	−446	−862	−1,220	−1,492	−1,662	−1,725
F_X			−9,329	−8,170	−6,375	−4,150	−1,720	711	2,953	4,865	6,375	7,450	8,100	8,300
F_T	F_{XT}	(35)	−2,420	−4,085	−4,500	−3,600	−1,660	711	2,850	4,220	4,500	3,725	2,100	0
	F_{XNT}	(36)	−1,840	−280	−253	−143	−35	0	−35	−142	−253	−280	−184	0
	F_{YT}	(40)	945	1,635	1,870	1,635	945	0	−945	−1,635	−1,870	−1,635	−945	0
F_T		(41)	−1,659	−2,730	−2,883	−2,108	−750	711	1,870	2,443	2,377	1,810	971	0
T		(47)	7,587	14,955	23,242	28,340	22,914	19,911	18,226	15,731	11,002	1,365	310	−1,025

Force		Form.	Crank position 13	14	15	16	17	18	19	20	21	22	23	24
P			28,800	29,200	30,000	31,250	24,000	17,700	15,470	15,000	15,150	12,600	3,600	−3,750 27,000
P_T		(10)	6,250	12,870	18,700	24,750	22,100	17,700	15,630	14,100	11,950	7,220	1,082	0
G_T*		(16)	676	607	495	350	182	0	−182	−350	−495	−607	−676	−700
C_T		(18)	−1,662	−1,492	−1,220	−862	−446	0	446	862	1,220	1,492	1,662	1,725
F_X			−8,100	−7,450	−6,375	−4,865	−2,953	−711	1,720	4,150	6,375	8,170	9,329	9,725
F_T	F_{XT}	(35)	−2,100	−3,725	−4,500	−4,220	−2,850	−711	1,660	3,600	4,500	4,085	2,420	0
	F_{XNT}	(36)	184	280	253	142	35	0	35	143	253	280	184	0
	F_{YT}	(40)	945	1,635	1,870	1,635	945	0	−945	−1,635	−1,870	−1,635	−945	0
F_T		(41)	−976	−1,810	−2,377	−2,443	−1,870	−711	750	2,108	2,883	2,730	1,659	0
T		(47)	4,288	10,175	15,598	21,795	19,960	16,989	16,644	16,720	15,558	10,835	3,727	1,025

*For a vertical engine G_{XT} and G_{NT} would be determined.

Combined Turning-effort Diagram.—When a steam engine has more than one cylinder, the diagrams may be combined. This is especially interesting for twin or cross-compound engines with cranks not in the same plane; the most common angle between 2-cylinder engines is 90 degrees.

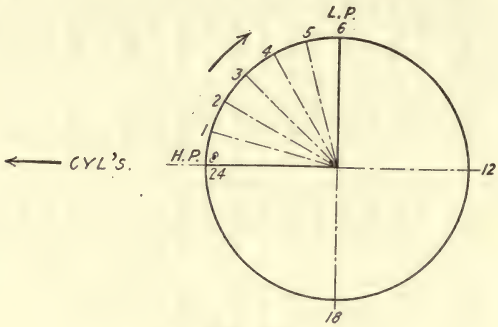


FIG. 195.

While the crank-effort diagrams for the high- and low-pressure cylinders of a compound steam engine would not be just alike, or the same as Fig. 194, they may be assumed the same and a combined diagram plotted. This will in reality be for twin simple engines, and the combined diagram will be the same whichever crank leads; with cross-compound engines, however, a more uniform turning effort may sometimes be obtained with a certain crank leading, and this may be determined by trial.

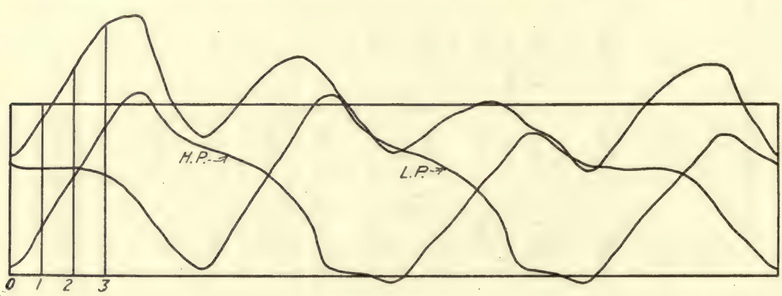


FIG. 196.

It is convenient to refer all forces to one crank circle, such as the high-pressure, the numbers on the rectified crank circle on which the diagram is plotted referring to this circle. Thus Fig. 195 is a crank circle for a cross-compound engine in which the low-pressure crank leads; therefore when the high-pressure crank is on position *O* (or 24) of its diagram (which

is also the zero of the combination diagram), the low-pressure crank is on position 6 of its diagram, and the force at this point must be plotted at position O of the combined diagram. When the high-pressure crank is at 18, the low-pressure crank is at O; then if the low-pressure diagram were drawn on tracing cloth and moved along so that its O position falls upon the 18 position of the combined diagram, the right relation would be established. This is shown in Fig. 196. If the ordinates of these diagrams are now added algebraically, the combined diagram formed by the highest curve is formed.

The mean line may be drawn as for Fig. 194. It will be noticed that the energy fluctuations are more frequent but less in intensity.

By changing the angle between the cranks it is sometimes possible to reduce the maximum fluctuation of energy; the writer's attention was first called to this in the paper by Mr. Astrom in vol. xxii, *Trans.* A.S.M.E. referred to at the end of Chap. XVIII. This is seldom done in practice, and unless the improvements were very marked, would probably not usually be considered worth while.

An examination of Figs. 185, 186 and 194 shows quite a reduction of the crank-end areas on account of the reduced piston area due to the piston rod; by making the cut-off for the crank end longer than that of the head end, the work may be equalized, probably resulting in a more uniform turning effort. This adjustment would be possible with Corliss gear without any change in design.

Internal-combustion Engine Diagram.—A combined indicator and inertia diagram for a 4-cycle, single-acting internal-combustion engine is shown in Fig. 187. This was drawn to scale for a 3¼- by 4-in. gasoline engine running 1500 r.p.m. and may be used as an illustration for plotting crank-effort diagrams. The indicator diagram used is conventional, and plotted from Par. 80, Chap. XIV, assuming an absolute compression pressure of 100 lb., and that the m.e.p. is 88 lb. Modifications were made as suggested; the suction and exhaust lines are taken as atmospheric. The connecting rod is not included with the reciprocating parts but is treated separately. The total pressure P, however, includes the inertia of the reciprocating parts proper. The piston is taken as cast iron. Other necessary data are as follows: Weight of piston and pin = 2.5 lb. Weight of connecting rod = 2 lb. $I = 0.636$, $\omega^2 = 24{,}500$, $L_G = 0.77$, $n = 4$. The inertia of the piston at the head end is 397 lb. and at the crank end 238 lb. Table 55 is prepared from these data. The effect of gravity being relatively small, is neglected.

The turning effort diagram for a single cylinder is plotted in Fig. 197. Due to the idle strokes during exhaust and suction, the effort is negative

TABLE 55

Force		Form.	Crank position 1	2	3	4	5	6	7	8	9	10	11	12
F_X		(32)	−256	−227	−179	−120	−53	14.6	78.2	128	179	212	232	232 −238
F_T	F_{XT}	(35)	−61.3	−113	−126	−104	−51.1	14.6	75.7	111	127	106	60	0
	F_{XNT}	(36)	−3.68	−5.65	−5.22	−3.39	−0.78	0	−1.16	−3.3	−5.22	−5.3	−3.33	0
	F_{YT}	(40)	45.5	78.8	91	78.8	45.5	0	−45.5	−78.8	−91	−78.8	−45.5	0
F_T		(41)	−21.5	−39.9	−40.2	−28.1	−6.4	14.6	29	28.9	30.3	21.9	11.2	0
P/A			290	213.5	179.4	122.2	100.2	89	83.2	77.2	74.8	71.2	62.6	53.8 −28.8
P_T		(10)	770	1038	1240	985	862	738	622	497	360	230	102	0
T		(47)	749	998	1200	957	856	723	593	468	330	209	91	0

Force		Form.	13	14	15	16	17	18	19	20	21	22	23	24
F_X		(32)	−232	−212	−179	−128	−78.2	−14.6	53	120	179	227	256	268 −268
F_T	F_{XT}	(35)	−60	−106	−127	−111	−75.7	−14.6	51.1	104	126	113	61.3	0
	F_{XNT}	(36)	3.33	5.3	5.22	3.3	1.16	0	0.78	3.39	5.22	5.65	3.68	0
	F_{YT}	(40)	45.5	78.8	91	78.8	45.5	0	−45.5	−78.8	−91	−78.8	−45.5	0
F_T		(41)	−11.2	−21.9	−30.3	−28.9	−29.0	−14.6	6.38	28.1	40.2	39.9	21.5	0
P/A			−28.6	−28.2	−26.8	−23.2	−17.2	−8.95	1.81	13.8	26.6	43.5	45	47.8 −47.8
P_T		(10)	−41.7	−91	−129	−145	−127	−74	15.5	112	177	220	120	0
T		(47)	−52.9	−113	−159	−174	−156	−89	22	140	217	260	141	0

Force	Crank position												
	Form.	25	26	27	28	29	30	31	32	33	34	35	36
P/A		−45	−43.5	−26.6	−13.8	−1.81	8.95	17.2	23.2	26.8	28.2	28.6	28.8 −28.8
P_T	(10)	−120	−220	−172	−112	−15.5	74	127	145	129	91	41.7	0
T	(47)	−141	−260	−217	−140	−22	89	156	174	159	113	52.9	0

Force	Form.	37	38	39	40	41	42	43	44	45	46	47	48
P/A		−29.6	−30.6	−29.6	−27.2	−23.2	−19	−14.2	−13.2	−15.4	−18.5	−49	−82.8 82.8
P_T	(10)	−42	−99	−142	−170	−173	−157	−121	−107	−107	−104	−130	0
T	(47)	−53	−121	−172	−199	−202	−172	−115	−79	−67	−64	−109	0

during the first part of the stroke, inertia only having any effect (if suction and exhaust are considered as atmospheric).

The mean line may be drawn as described under steam diagrams, and the fluctuation of energy determined; the maximum value of ΔE (Chap. XVIII) is obvious.

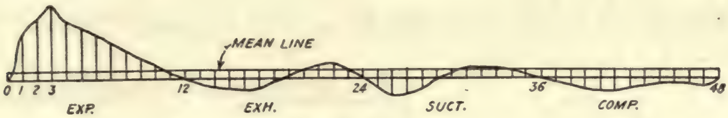


FIG. 197.

Combined Turning-effort Diagrams.—Fig. 197 may be considered a diagram for a single-cylinder engine, or cylinder No. 1 of a multi-cylinder engine. Some of the most common arrangements will now be given, the diagrams for the different cylinders being first shown separately to avoid confusion, and then combined.

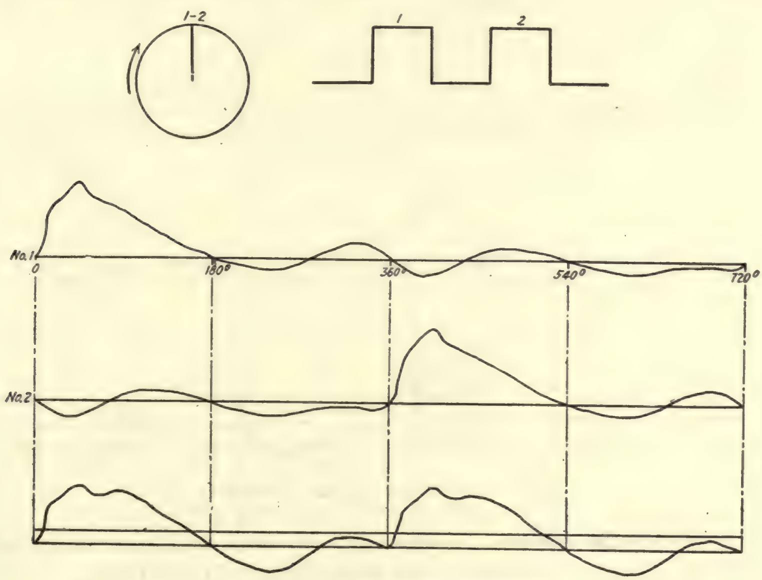


FIG. 198.—Two-cylinder diagrams.

Twin cylinder engine with cranks in unison. Order of firing, 1–2. In the crank diagram of Fig. 198, the head-end dead center is assumed to be at the top of the circle, as in a vertical engine. The turning-effort diagram is dimensioned in degrees, starting from the head-end dead center of cylinder No. 1. The combined diagram is given below that of cylinder

No. 2. The crank is assumed to turn clockwise, but to follow the order of firing, the crank circle should be followed in a counter-clockwise direction.

Three-cylinder engine with cranks at 120 degrees. Order of firing, 1-3-2. See Fig. 199.

Four-cylinder engine with cranks at 180 degrees. Order of firing, 1-3-4-2. See Fig. 200.

Six-cylinder engine with cranks at 120 degrees. Order of firing, 1-5-3-6-2-4. See Fig. 201.

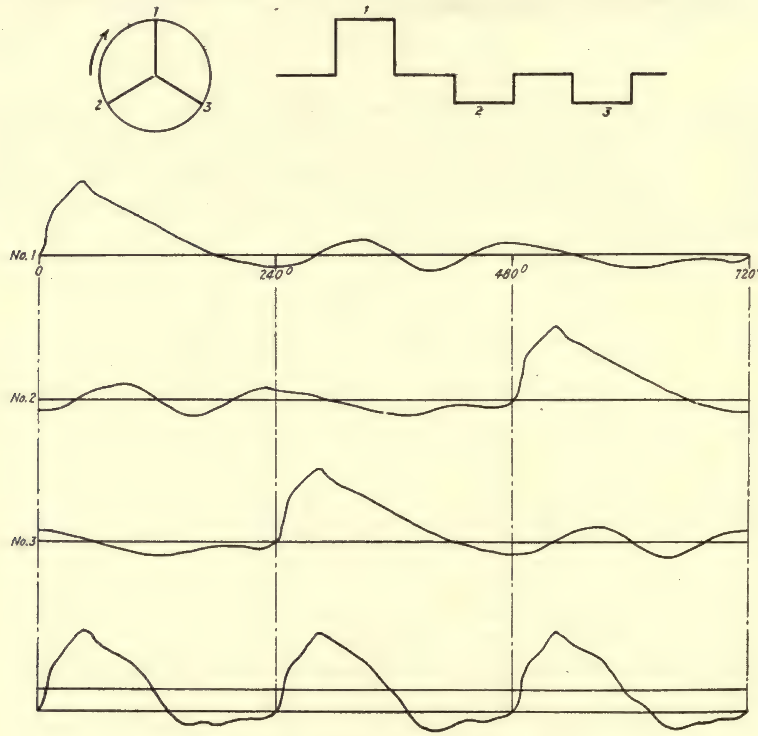


FIG. 199.—Three-cylinder diagrams.

The cranks are differently arranged on some shafts as shown in Fig. 413, Chap. XXVIII, for which the firing order is 1-2-4-6-3-5.

Crank diagrams for 8-cylinder and 12-cylinder engines are shown in Figs. 202 and 203 without the crank effort diagrams. These may be plotted in the same way when the order of firing is known; this will be considered in Chap. XX. The dotted lines of the crank diagrams show equivalent positions of the cranks to give the same turning effort if the cylinders were all vertical.

It will be seen that in all cases the order of firing is such as to divide the impulses around the circle evenly; aside from this there is not a fixed standard, but the more usual timing is given in this chapter.

107. Reactions on the Frame.—With the exception of the connecting rod in vertical and angle engines, gravity of the moving parts always exerts a downward effect only. The magnitude of the normal component

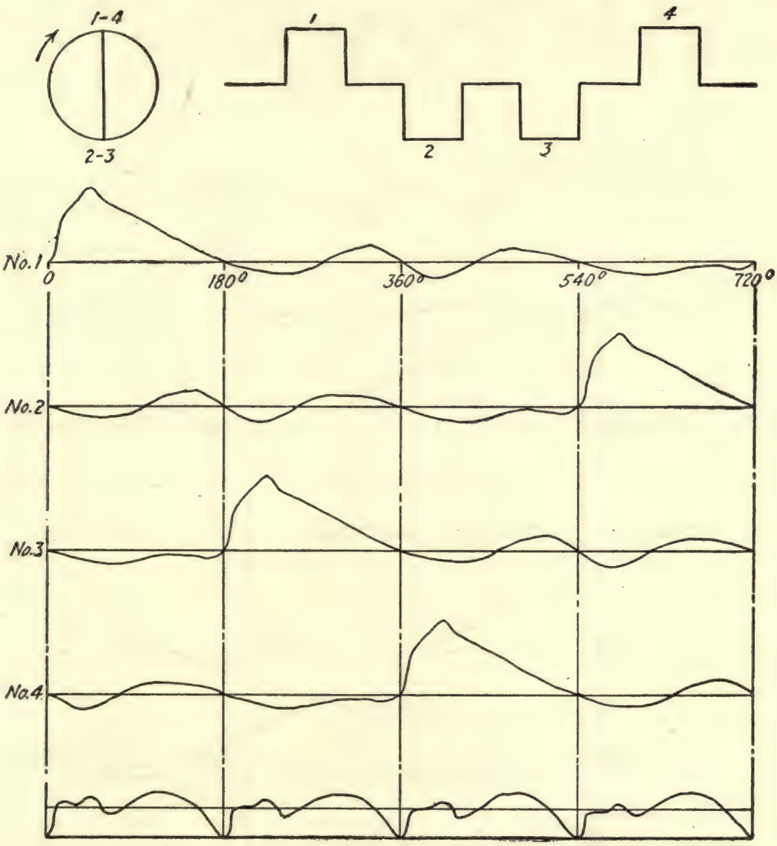


FIG. 200.—Four-cylinder diagrams.

of gravity is given by (19), and it always acts toward the center line of the engine at the crosshead end when the cylinder is above the crank, and away from the engine at the crank. The weight of the reciprocating parts of a vertical engine is included in P, the total piston pressure. In horizontal engines the gravity of these parts have no effect except to produce some pressure on the cylinder walls and guide. Under certain

unusual conditions their changing position may have a slight effect upon balance.

Forces due to gas or steam pressure and to acceleration are independent of the position of the engine, and their effect upon the frame will be

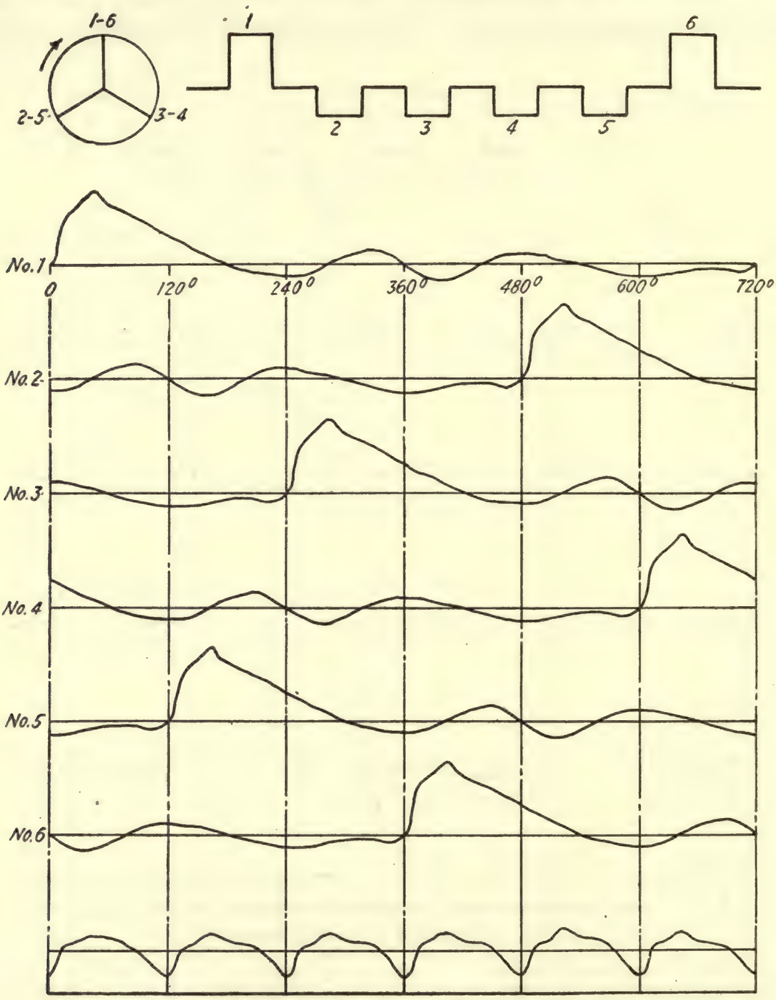


Fig. 201.—Six-cylinder diagrams.

given here. Force P in this paragraph had best be taken for steam or gas pressure only (including weight of reciprocating parts for vertical or angle engines), the inertia of the reciprocating parts being computed separately, as all forces except P (and consequently P_N) vary directly as

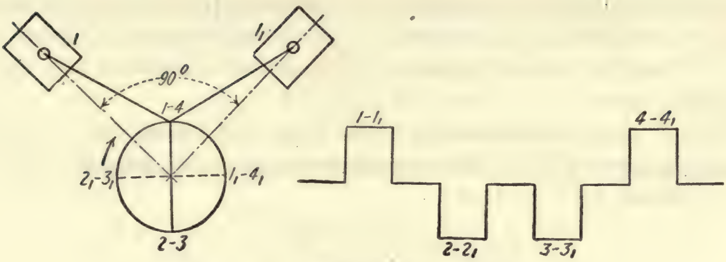


FIG. 202.—Eight-cylinder diagram.

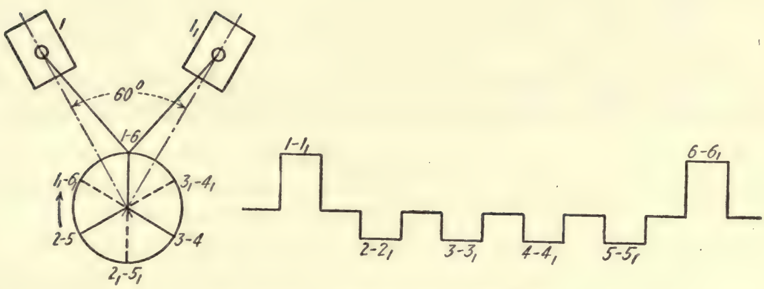


FIG. 203.—Twelve-cylinder diagram.

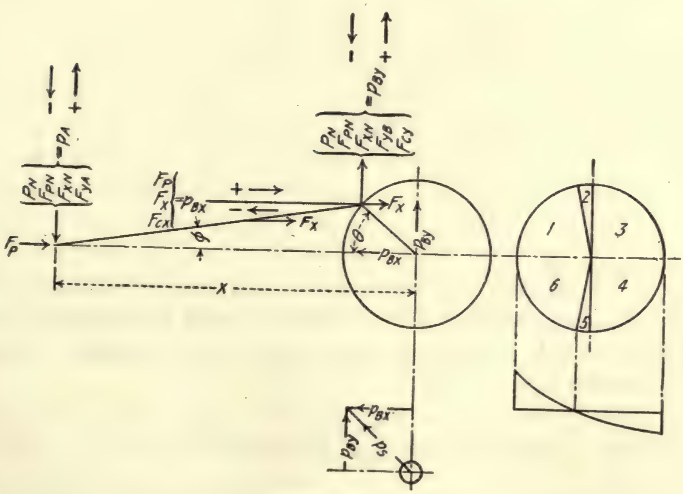


FIG. 204.

ω^2; these may then be combined and plotted, and the effect of different rotative speeds easily determined by changing the scale. The reactions due to P may be added to any of these inertia diagrams and the total effect determined.

Force P has an unbalanced effect upon the frame only through its normal component P_N. The normal component of F_P may be found from (43) by taking $L_G = O$; then:

$$F_{PN} = F_P \cdot \frac{\sin\theta}{n\sqrt{1-\left(\frac{\sin\theta}{n}\right)^2}} \tag{49}$$

The signs for P assume the force to be acting in the direction of piston motion; when the reverse is true the signs are reversed. This may be determined from diagrams similar to those given in Figs. 186 and 187.

Fig. 204 shows the numbering of the sectors, the meaning of the signs, and the action of the different forces, while Table 56 gives the signs.

TABLE 56

Force		Formula	Sector of circle					
			1	2	3	4	5	6
p_A	P_N	(12)	−	−	−	−	−	−
	F_{PN}	(49)	+	−	−	+	+	−
	F_{XN}	(43)	+	−	−	+	+	−
	F_{YA}	(42)	+	+	+	−	−	−
p_{BY}	P_N	(12)	+	+	+	+	+	+
	F_{PN}	(49)	−	+	+	−	−	+
	F_{XN}	(43)	−	+	+	−	−	+
	F_{YB}	(39)	+	+	+	−	−	−
	F_{CY}	(50)	+	+	+	−	−	−
p_{BX}	F_P	(32)	−	+	+	+	+	−
	F_X	(32)	−	+	+	+	+	−
	F_{CX}	(51)	−	−	+	+	−	−

If the revolving parts are not balanced, or are overbalanced due to a portion of the reciprocating parts being balanced at the crank, the centrifugal force of this weight will react at the main bearing. Normal to the line of stroke this force is:

$$F_{CY} = \Sigma\left(\frac{r_C}{R}\frac{C}{g}\right)R\omega^2\sin\theta \tag{50}$$

This forms a part of p_{BY}, the sign being as in Table 56 if the crank and pin

are underbalanced; if overbalanced the signs are reversed. The effect parallel to line of stroke is:

$$F_{CX} = \Sigma \left(\frac{r_C}{R}\frac{C}{g}\right) R\omega^2 \cos \theta \tag{51}$$

This is added to p_{BX}, the sign being as in Table 56 for the unbalanced crank and pin.

The forces of Fig. 204 and Table 56 may be plotted or tabulated separately, similar to Table 57, and combined if desired. They do not all act in the same plane, and may produce other forces and couples. Their effect may be taken up within the frame itself in some cases, so as to have little or no effect upon the foundation or other support, especially in multi-cylindered engines. Some of the forces are insignificant in some cases, but with different types, sizes, speeds, etc., it is well to prove this by calculation; the formulas are simple and easily applied with the aid of Table 53 and the diagrams, and their use may account for various things giving trouble to engine builders.

The forces F_X (due to the connecting rod) and F_P (due to the reciprocating parts) act along, or parallel to the line of stroke, but may act on the frame either at the cylinder end or the bearing end, or may be divided between them, depending upon the steam or gas pressure acting upon the piston. It must be borne in mind, however, that if the restraint of the foundation bolts is neglected, the forces acting upon the frame in the line of stroke are limited by the steam or gas pressure in the cylinder; this must be determined by the indicator diagram alone, inertia having no effect. If inertia is great, part of it may be balanced by compression, in which case it is transmitted to the foundation from the cylinder end of the engine; any excess inertia is transmitted to the frame and thus to the foundation at the shaft end of the engine. The inertia effects normal to the line of stroke cause bending and torsional stresses in the frame, but these are influenced by the attachment to the foundation.

Divisions 2 and 5 in Fig. 204 are not quite the same for F_P and F_X, as the inertia curves do not cross the line at the same point.

The action of the forces p_{BX} and p_{BY} along the crank in a line from the center of the shaft to the center of the crank pin is:

$$p_R = p_{BX} \cos \theta + p_{BY} \sin \theta \tag{52}$$

All or part of the forces of which these are composed may be taken in finding p_R. Formula (52) is convenient as practically all balancing is done along the crank center line.

The forces of Fig. 204, added algebraically are:

$$p_A = P_N + F_{PN} + F_{XN} + F_{YA} \qquad (53)$$
$$p_{BX} = F_P + F_X + F_{CX} \qquad (54)$$
$$p_{BY} = P_N + F_{PN} + F_{XN} + F_{YB} + F_{CY} \qquad (55)$$

Formula numbers for the various forces are given in Table 56. It may be convenient at this place to state the meaning of the notation contained in Formulas (53) to (55).

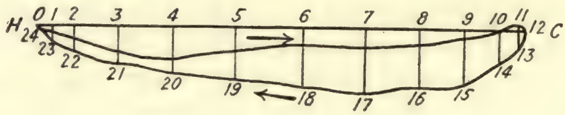


FIG. 205.

P_N = normal component of force P due to steam or gas pressure. If P includes the inertia of the reciprocating parts, P_N includes F_{PN}. Should the inertia of the connecting rod be included as in rough calculations. P_N includes F_{XN}.

F_P = the inertia of the reciprocating parts.

F_{PN} = the normal component of F_P.

F_X = the inertia of the connecting rod parallel to line of stroke, assumed as concentrated at mass center of rod.

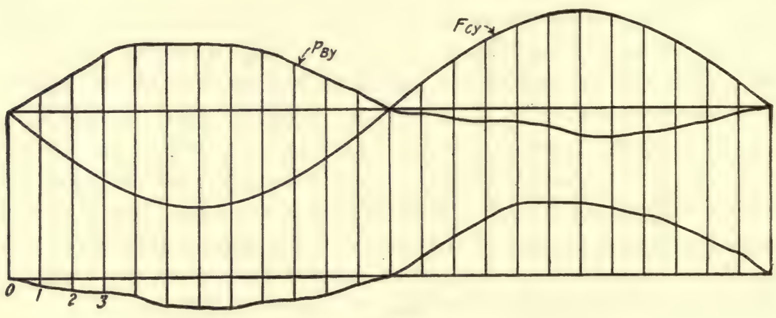


FIG. 206.

F_{XN} = normal component of F_X.

F_{YA} = effect at crosshead pin of inertia of connecting rod normal to line of stroke.

F_{YB} = same at crank pin.

F_{CX} = effect of centrifugal force of crank or counterbalance parallel to line of stroke.

F_{CY} = same normal to line of stroke.

In vertical engines G_N, the gravity effect of the connecting rod, should

be added to p_A and p_{BY}; its value is given by (19) and the signs are the same as for P_N in Table 56.

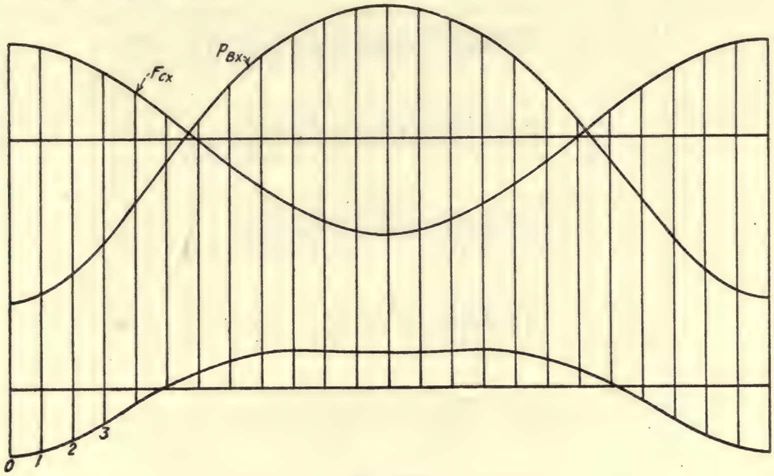


FIG. 207.

With the data for the Corliss engine already considered, and by the use of Fig. 204 and Table 56, Table 57 has been computed. The method of

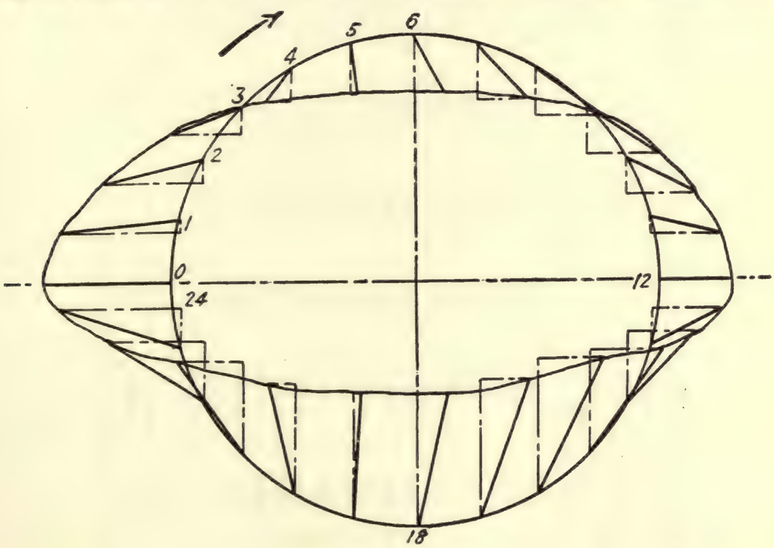


FIG. 208.

determining the counterbalance will be given in the next chapter. The values of p_A are plotted along the crosshead path in Fig. 205. Values

TABLE 57

	Force	Form.	Crank position											
			1	2	3	4	5	6	7	8	9	10	11	12
Crosshead	P_N	(12)	−1,693	−3,280	−4,650	−5,180	−3,920	−2,920	−2,210	−1,596	−877	−147	317	0
	F_{PN}	(49)	505	820	820	548	148	−330	−728	−940	−895	−720	−381	0
	F_{XN}	(43)	191	323	360	226	131	−56	−230	−336	−358	−295	−165	0
	F_{YA}	(42)	467	902	1,275	1,560	1,740	1,808	1,740	1,560	1,275	902	467	0
	p_A	(53)	−530	−1,235	−2,195	−2,846	−1,901	−1,498	−1,428	−1,306	−945	−260	238	0
Crank	P_N	(12)	1,693	3,280	4,650	5,180	3,920	2,920	2,210	1,596	877	147	317	0
	F_{PN}	(49)	−505	−820	−820	−548	−148	330	728	940	895	720	381	0
	F_{XN}	(43)	−191	−323	−360	−226	−131	56	230	336	358	295	165	0
	F_{YB}	(39)	953	1,840	2,600	3,180	3,550	3,680	3,550	3,180	2,600	1,840	953	0
	F_{CY}	(50)	877	1,690	2,400	2,930	3,270	3,380	3,270	2,930	2,400	1,690	877	0
	p_{BY}	(55)	2,824	5,667	8,470	10,516	10,461	10,366	9,988	9,182	7,130	4,692	2,693	0
	F_P	(32)	−11,700	−9,750	−6,900	−3,750	−900	1,950	4,450	6,450	7,580	8,550	8,850	8,900
	F_X	(32)	−9,329	−8,170	−6,375	−4,150	−1,720	717	2,953	4,865	6,375	7,450	8,100	8,300
	F_{CX}	(51)	−3,270	−2,930	−2,400	−1,690	−877	0	877	1,690	2,400	2,930	3,270	3,380
	p_{BX}	(54)	−24,299	−20,850	−15,675	−9,590	−3,497	2,667	8,280	13,005	16,355	18,930	20,220	20,580
	p_R	(52)	24,232	20,940	16,360	13,880	11,000	10,366	12,510	14,450	16,580	18,790	20,197	20,580
Balance	F_{CY}	(50)	−3,900	−7,550	−10,700	−13,100	−14,600	−15,114	−14,600	−13,100	−10,700	−7,500	−3,900	0
	F_{CX}	(51)	14,600	13,100	10,700	7,550	3,900	0	−3,900	−7,550	−10,700	−13,100	−14,600	−15,114

Force		Form.	Crank position											
			13	14	15	16	17	18	19	20	21	22	23	24
Crosshead	P_N	(12)	−1,620	−3,140	−4,450	−5,490	−4,650	−3,320	−2,377	−1,640	−965	−239	−432	0
	P_{PN}	(49)	381	720	895	940	728	330	−148	−548	−820	−820	−505	0
	F_{XN}	(43)	165	295	358	336	230	56	−131	−226	−360	−323	−191	0
	F_{YA}	(42)	−467	−902	−1,275	−1,560	−1,740	−1,808	−1,740	−1,560	−1,275	−902	−467	0
	p_A	(53)	−1,541	−3,027	−4,472	−4,774	−5,432	−4,742	−4,396	−3,974	−3,320	−2,284	−731	0
Crank	P_N	(12)	1,620	3,140	4,450	5,490	4,650	3,320	2,377	1,640	965	239	432	0
	F_{PN}	(49)	−381	−720	−895	−940	−728	−330	148	548	820	820	505	0
	F_{XN}	(43)	−165	−295	−358	−336	−230	−56	131	226	360	323	191	0
	F_{YB}	(39)	−935	−1,840	−2,600	−3,180	−3,550	−3,680	−3,550	−3,180	−2,600	−1,840	−953	0
	F_{CY}	(50)	−877	−1,690	−2,400	−2,930	−3,270	−3,380	−3,270	−2,930	−2,400	−1,690	−877	0
	p_{BY}	(55)	−756	−1,405	−1,903	−1,936	−3,128	−4,126	−4,164	−3,696	−2,855	−2,158	−702	0
	F_P	(32)	8,850	8,550	7,580	6,450	4,450	1,950	−900	−3,750	−6,900	−9,750	−11,700	−12,280
	F_X	(32)	8,100	7,450	6,375	4,865	2,953	717	−1,720	−4,150	−6,375	−8,170	−9,329	−9,725
	F_{CX}	(51)	3,270	2,930	2,400	1,690	877	0	−877	−1,690	−2,400	−2,930	−3,270	−3,380
	p_{BX}	(54)	20,220	18,930	16,355	13,005	8,280	2,667	−3,497	−9,590	−15,675	−20,850	−24,299	−25,385
	p_R	(52)	19,616	17,100	12,950	8,173	5,170	4,126	4,930	8,000	13,090	19,180	23,682	25,385
Balance	F_{CY}	(50)	3,900	7,500	10,700	13,100	14,600	15,114	14,600	13,100	10,700	7,550	3,900	0
	F_{CX}	(51)	−14,600	−13,100	−10,700	−7,550	−3,900	0	3,900	7,550	10,700	13,100	14,600	15,114

of p_{BY} and F_{CY} (for counterbalance) are plotted on a rectified crank circle in Fig. 206, and of p_{BX} and F_{CX} in Fig. 207. The resultant p_S of forces p_{BX} and p_{BY} may best be found graphically from Figs. 206 and 207. This has been done, taking the counterbalance into consideration, and plotted on a crank circle in Fig. 208, giving both direction and magnitude of the force acting during the revolution due to the moving parts of the

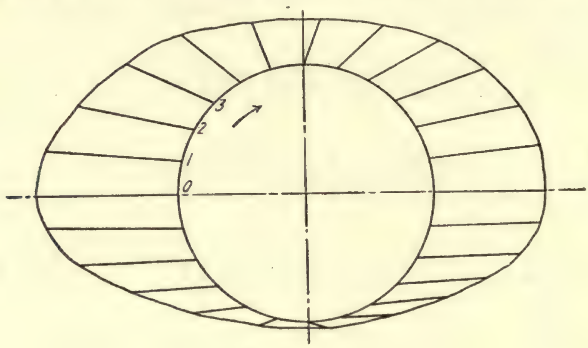


FIG. 209.

slider-crank mechanism. This force acts on the main bearing. To obtain the force acting at the crank pin, that due to the crank and counterbalance (F_{CY} and F_{CX}) must be omitted. This is shown in Fig. 209. The actual force on the bearing is somewhat different as it is not in the same plane; neither do the forces due to crank and counterbalance act in the same plane as the forces at the center of the crank pin, but for most calculations for strength and wear this may be neglected.

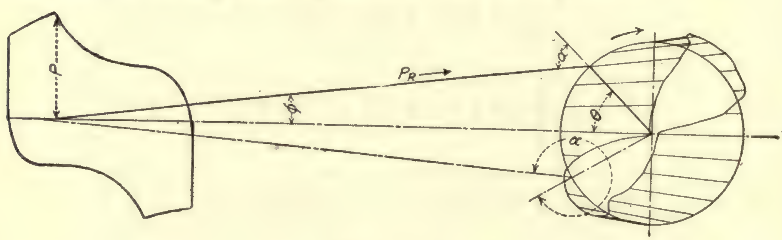


FIG. 210.

To obtain the total force acting on the crank pin or shaft at any point, the resultant of p_S and the steam or gas pressure must be obtained; when bearing pressure is desired the effect of gravity must be included (connecting rod, crank, shaft, wheel, etc.). The force due to steam or gas pressure may be found by the use of a stroke diagram such as Fig. 186 or 187, neglecting inertia. In Fig. 210 it may be seen that the force

P_R acts at an angle $\theta + \phi = \alpha$, from the point on the pin which is nearest the cylinder when the crank is on the head-end center, and that it moves around the pin in a counter-clockwise direction. This is also true of the shaft.

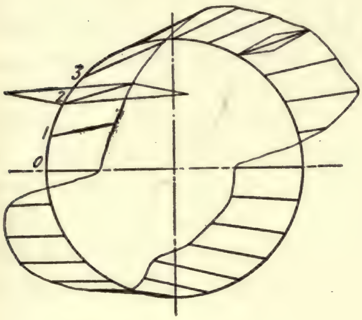


Fig. 211.

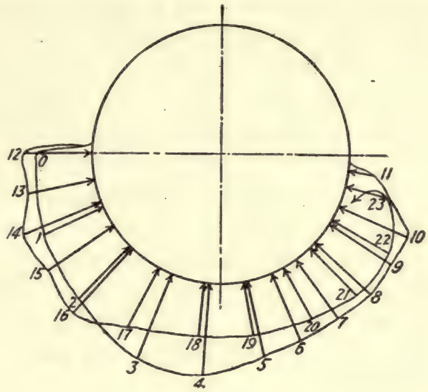


Fig. 212.

Fig. 208 or 209 may be combined with Fig. 210; then to find the forces acting on shaft or pin, these may be drawn in the position of head-end dead center; by revolving the resultant diagram on the same center in a counter-clockwise direction, the resultant forces may be transferred to

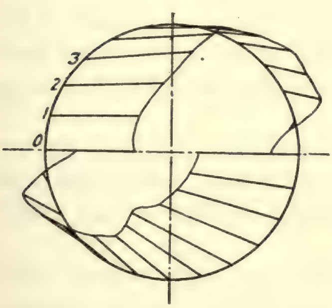


Fig. 213.

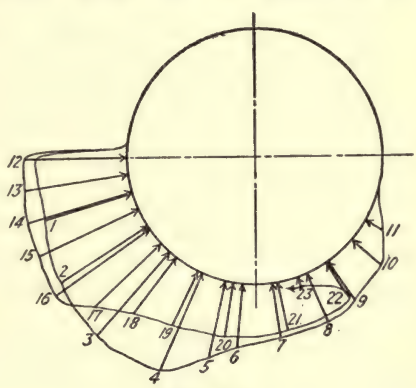


Fig. 214.

the pin or shaft in such a manner that they pass through the center if produced. Figs. 209 and 210 for the crank pin are combined in Fig. 211 and transferred to the pin in Fig. 212. The latter shows the normal force acting on all sides of the crank pin by means of which the maximum reversed or repeated loads may be determined.

Figs. 208 and 210 for the shaft are combined in Figs. 213 and 214 in the same way.

These diagrams are for the 20 by 48 in. Corliss engine previously mentioned, and it must be remembered that they are not strictly accurate due to the forces not being in the same plane. In all of these diagrams except Figs. 212 and 214, the force acts from the point on the circle from which the line starts, and the effect of gravity of the parts is neglected. In Figs. 212 and 214 the force curve is all drawn outside the circle which represents pin or shaft, and acts toward the circle; the radial force lines are numbered in order showing the direction of force change during the cycle. Reversed and repeated loads may be taken from these digarams (Figs. 212 and 214), the reversed loads being diametrically opposite. If two forces are more than 90 degrees apart they produce a certain amount of reversal; this usually produces bending. As the stress measured from any neutral axis is proportional to the distance from the axis, the intensity and kind of stress may be found at any point on the surface for forces inclined at any angle.

Various couples are produced by the constituents of p_A and p_{BY}, some of which tend to neutralize the others. Their effect is also influenced by the counterbalance if any is used.

The attachment of the frame to the foundation or other support influences the effect of these forces which might be determined by taking moments about certain points; but the origin of moments is so indefinite, and the behavior of the supports under active forces so uncertain that but little satisfaction is obtained by an attempt at anything like accurate analysis. For large engines, as careful a consideration as possible of the proportion of the reciprocating parts to be balanced, and for small light engines, a careful application of the best balancing methods, will bring the most satisfactory results.

In all that precedes, an absolutely uniform motion of the crank pin has been assumed. This is not strictly the case, but in most cases there is no great error in the assumption, and an attempt to account for this would be useless.

The treatment of the mechanics of the slider crank given here, while simple in principle, is rather complicated, and for most design problems only the direct forces due to gas or steam pressure and to heavy weights such as the flywheel, are used. However, no one is absolutely sure of the wisdom of neglecting certain forces until he has worked through a number of problems, and for turning effort diagrams used in flywheel design, the neglect of the effect of acceleration results in a method but little short of a rough rule of thumb. Also, when inertia is combined with the stroke

diagram of Fig. 187, the resulting force diagram differs considerably from the original diagram due to gas pressure only; were the connecting rod included, giving the forces acting on the crank pin, the difference would be still greater.

Perfect balance is claimed for certain engines of ordinary construction; there is no such thing, and the statement either shows lack of knowledge or an attempt at a "snappy" piece of advertising.

The heat engine designer cannot afford to be ignorant of the simple principles of this chapter even though he uses them but little. If the forces are carefully calculated the factor of judgment may be reduced in selecting the factor of safety; if, as is usually the case, it is considered unnecessary to make such elaborate calculations, the factor of judgment should be such as to cover discrepancies. This will be further discussed in Par. 166, Chap. XXI and in connection with the various details in later chapters.

Reference

Balancing of engines.................................... W. E. Dalby.

CHAPTER XVII

BALANCING

108. Introduction.—The subject of the balancing of reciprocating engines is well covered by Prof. W. E. Dalby in his excellent work by this title, and it is not possible to claim a complete treatment in a short chapter; however, an attempt is made to bring the essentials of balancing into simple practical form, so that they may be used to solve all balancing problems connected with the usual forms of reciprocating engines. It might be more correct to say that the lack of balance may be determined by these principles with a view to keeping it a minimum.

Notation.

W = weight in general in pounds.
W_B = weight to be balanced referred to the crank pin.
W_P = weight of reciprocating parts in pounds.
w = unit weight in pounds.
C = weight of crank and pin in pounds referred to pin center.
P = force in general in pounds.
C_F = radial inertia (centrifugal force) of a revolving weight in pounds.
Q = a couple.
M = moment in pound-feet or pound-inches of weights to be balanced.
A_B = area in square inches or square feet of face of counterweight.
t = thickness of counterweight in inches or feet.
l = dimensions in general in inches or feet.
r = radii in general.
L = length of connecting rod from center to center in feet.
R = crank radius in feet.
ω = angular velocity in radians per second.
N = r.p.m.
$n = L/R$.
$g = 32.16$.

109. Simple, or Primary Balancing.—Centrifugal force tends to displace a revolving weight in a radial direction from the axis about which

it revolves. The magnitude of this force is equal to the product of its mass and radial acceleration; or:

$$C_F = \frac{W}{g} \cdot R\omega^2 = \frac{W}{g} \cdot R\left(\frac{\pi N}{30}\right)^2 \tag{1}$$

Had W been taken as an indefinitely small division of the weight and R its distance from the axis, the sum of these products would equal the total weight multiplied by the distance of its center of gravity from the axis. It then follows that two weights diametrically opposite, revolving about an axis, will be in balance if the product WR is the same for each,

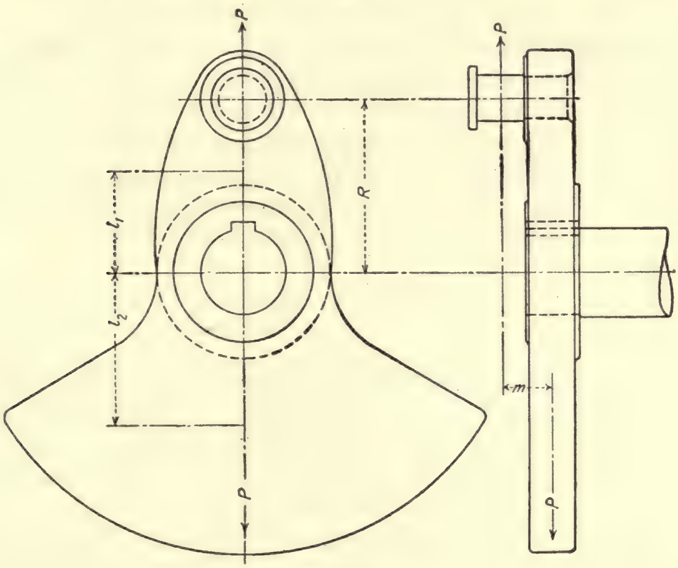


Fig. 215.

and this is the general principle of practical counterbalancing of machine parts.

The balancing of parts revolving in a circle is simple, and practically consists in suitably placing the balance weights so as to make a neat design. However, for over-hung cranks there is an unbalanced couple due to the impossibility of placing the counterbalance in the same plane as the portion of the weight to be balanced. This is shown in Fig. 215.

So far as forces are concerned, balance is obtained if the connecting rod is disconnected when:

$$W_2 l_2 = W_1 l_1 + WR \tag{2}$$

where W is the weight of the portion of the pin in the bearing, W_1 the weight of the crank and W_2 the weight of the counterbalance. The crank and counterbalance, as shown, are in the same plane, but the pin is not in the plane with the portion of the weight which balances it, and the couple:

$$\frac{W}{g} R\omega^2 m$$

is formed, which has a tendency to cause rotation in a plane normal to the plane of rotation. In practice this is usually small relatively and is ignored. With the center-crank engine the counterbalance is divided between the two crank arms and the couples balance.

Reciprocating Parts.—Let the effect of angularity of the connecting rod be temporarily neglected and its weight assumed added to the reciprocating parts. These parts must be brought to rest at the end of each stroke, the force required being the product of their mass and the radial acceleration of the crank. If C be the weight of crank and pin referred to the center of the crank pin, and W_p the weight of the reciprocating parts, the force acting at the crank pin at dead center is:

$$\frac{(C + W_P)}{g} \cdot R\omega^2.$$

If the crank and pin only are balanced, then at the dead center, W_p is unbalanced, and this is true in a lesser degree for every other position of the crank except when it is at right angles to the line of stroke. If W_p is also balanced by a revolving weight, the system is in balance only at the dead centers, the counterbalance being too heavy by the amount W_p when the crank is at right angles to the line of stroke. A compromise is usually made by balancing all of the revolving parts and a certain percentage of the reciprocating parts; at dead center the engine is underbalanced and at right angles, overbalanced.

The percentage of reciprocating parts to be balanced depends upon the type of engine and location. For locomotives, a special formula is used in which the total engine weight is a factor, but the limits are usually between 55 and 65 per cent.

In all engines with but one rod connecting with the crank pin this state of unbalance must exist, even though the system as a whole may be balanced in a multi-cylinder engine.

In many small engines with slow or moderate speed and heavy frames, the counterbalance is omitted, or if added by the use of a disc crank, no calculations are made in proportioning it; but with large engines or high

speeds, counterbalance is of advantage, not only for smoothness of operation but for relieving undue wear and strain.

110. Secondary Balance.—If, in addition to balancing the revolving parts, a weight be placed opposite the crank so that the product of weight and distance to mass center equals the product of crank radius and weight of reciprocating parts, the radial inertia of this weight at any point in its path is equal to that given by (1). From (33) and (34), Chap. XVI, the radial inertia of the reciprocating parts is greater than this at the head end of the stroke and less at the crank end. If the rod were infinitely long, or a Scotch yoke were used, there would be perfect balance at the dead centers, and this is known as *primary balance.* The balance along the line of stroke which would be necessary to compensate for rod angularity is known as *secondary balance.* It does not include the inertia of the rod itself, but only such masses as may be considered concentrated at the crosshead pin.

From (32) and (48) of Chap. XVI, taking l as zero in the latter, the inertia of the reciprocating parts along the line of stroke is:

$$F = \frac{W\omega^2}{g} \cdot R\left[\cos\theta + \frac{R}{L}\cos 2\theta\right] \tag{3}$$

This is the approximate formula, but is sufficiently accurate for the present purpose. The negative signs are omitted, but $\cos\theta$ and $\cos 2\theta$ have signs as explained in connection with Formula (30), Chap. XVI.

Removing the brackets and multiplying and dividing the term containing $\cos 2\theta$ by 4, (3) may be written:

$$F = \frac{W}{g} \cdot \omega^2 R\cos\theta + \frac{W}{g}(2\omega)^2 \cdot \frac{R}{4L} \cdot R\cos 2\theta$$
$$= \text{primary force} + \text{secondary force} \tag{4}$$

This shows the following:

1. *The primary force* along the line of stroke is equivalent to that which would be produced if the reciprocating parts were concentrated at the crank-pin center and revolved at the same speed as the crank.

2. *The secondary force* along the line of stroke is equivalent to that produced by a mass equal to the reciprocating parts concentrated at a radius equal to the fraction $R/4L$ of the crank radius, and whose rotative speed is twice that of the crank. When the crank is at either dead center, this mass is always at the head-end dead center.

The relation of the crank with this imaginary crank is shown in Fig. 216.

A graphical expression of (3) is given in Fig. 217, the radius of the imaginary crank in this case being R/n. The algebraic sum of the dis-

tances from the vertical center line to the projections of the two cranks on the horizontal center line gives the inertia to the same scale that R gives the primary force at dead center.

It is obvious that for engines with a single cylinder, the balancing of the secondary force is impracticable, therefore the reciprocating parts do not permit of perfect balance with revolving weights.

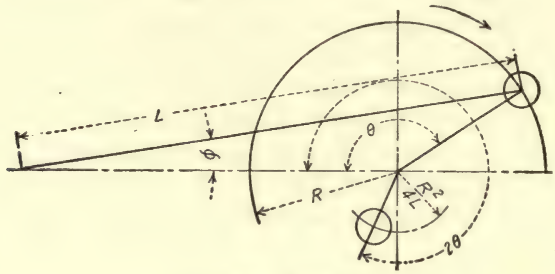


FIG. 216.

Complete balance of the reciprocating parts by a revolving weight is usually undesirable, so that the importance of secondary balance is only seen in certain combinations of multi-cylinder engines.

It should not be inferred from what has preceded that revolving weights at crank pin and imaginary crank-pin centers would produce the same forces in all directions as those produced by the reciprocating parts; the

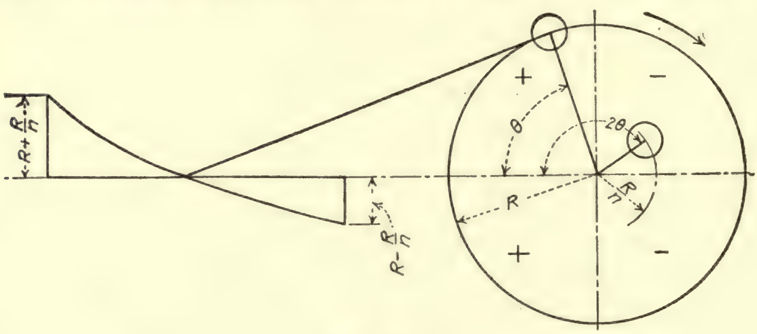


FIG. 217.

effect is identical only along the line of stroke, the reciprocating parts having no other influence upon the frame except through the normal component F_{PN}, given by (49), Chap. XVI.

111. Multi-cylinder Engines.—In this paragraph a number of common arrangements will be discussed relative to the balancing of revolving parts and purely reciprocating parts along the line of stroke, leaving the

connecting rod and the normal components of cylinder pressure and inertia to later paragraphs.

Complete balance includes the balance of *forces* and *couples*. *Unbalanced forces* may be determined by taking the algebraic sum of the components of all forces normal to a given diameter, assuming them to act in the same plane normal to the axis of revolution, as in Fig. 218. Perfect balance is obtained if $P_1 + P_2 + P_3 = 0$.

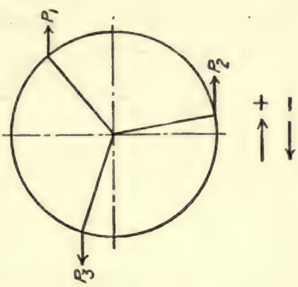


Fig. 218.

Unbalanced axial couples may be determined by taking moments about some axis cutting the shaft center at right angles, of the components of all forces in their respective planes of revolution, normal to a plane containing both this axis and the shaft center line. This is shown in Fig. 219.

The center of gravity of the upward forces is given by:

$$l_U = \frac{P_1 l_1 + P_4 l_4}{P_1 + P_4} \tag{5}$$

and for the downward forces:

$$l_D = \frac{P_2 l_2 + P_3 l_3}{P_2 + P_3} \tag{6}$$

The couple tending to cause rotation of the shaft in an axial plane is the product of the distance between the center of gravity and the lesser of the two forces, $P_1 + P_4$ or $P_2 + P_3$. Let this be denoted by P_Q; then:

$$Q = P_Q(l_D - l_U) \tag{7}$$

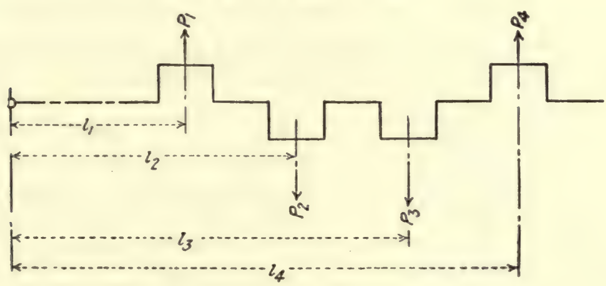


Fig. 219.

the subscripts D and U denoting down and up forces respectively.

It is often necessary to find a resultant couple due to the forces p_{BX} and p_{BY} of Fig. 204, Chap. XVI; this is given by:

$$Q = \sqrt{Q_X^2 + Q_Y^2} \tag{8}$$

If the forces are not in balance, there will be a force and a couple acting on the engine. The force may form a couple with some other force due to the reaction of the supports, which may act in the same or opposite direction of the first couple.

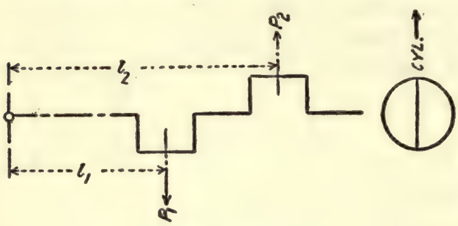


FIG. 220.

If all forces and couples are not balanced in the engine itself considered as a free body, it is practically impossible, due to the uncertainty in locating the origin of moments, to determine the couple acting on the system; but usually this is not necessary.

In some engines it is difficult to determine the maximum couple without plotting values for the entire cycle.

Two-crank engines with cranks in the same phase are treated the same as engines with a single cylinder so far as inertia is concerned. With cranks at 180 degrees, the primary force is balanced, but the small imaginary crank is always at the head-end dead center when the actual crank is at either center; consequently this gives an unbalanced effect equal to twice the secondary force. Referring to Fig. 220, if P_1 is at the crank-end dead center it is smaller than P_2, and:

$$Q = P_1(l_2 - l_1) \tag{9}$$

If the crank and pin are not balanced, their centrifugal force, referred to the crank-pin center, should be added to P_1 in (9).

There is no lateral component in the dead-center position, which gives a maximum couple. To obtain the best results the cylinders should be placed as close together as possible, and the cranks (and perhaps a portion of the reciprocating parts) should be balanced.

Four-crank engines such as Fig. 219, with cranks in the same plane, equal spacing of cylinders and weight of reciprocating parts, are in balance for the primary force but not the secondary force; both primary and secondary couples are balanced. The system as a whole is in no better balance if the revolving parts are balanced, but bending strains in frame and shaft, and pressure on bearings are relieved thereby.

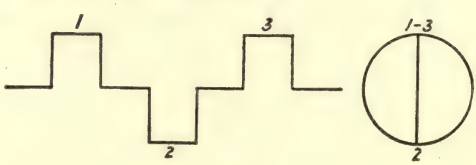


FIG. 221.

Three-cylinder engines with cranks opposed as in Fig. 221 have a primary force error unless the reciprocating parts of the center engine

are equal to the weight of the other two, which is not practical. There is a secondary force error, but if the cranks are evenly spaced there is no couple error.

Angle Engine.—This is shown in diagram in Fig. 222, with centers at an angle of 90 degrees. If the reciprocating parts are the same weight in both engines, and all the reciprocating and revolving parts are balanced by a revolving weight placed opposite the crank, the primary force is balanced. There still remains the unbalanced secondary force, which acts along a line inclined 45 degrees from the line of stroke as shown at 1–2, Fig. 222, and is a maximum as the crank passes each dead center, which is four times a revolution. As both connecting rods engage with the same crank pin the unbalanced axial couple is small. This type of

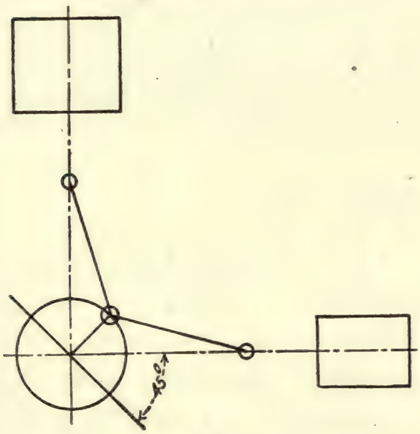


Fig. 222.

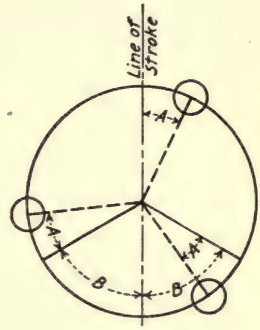


Fig. 223.

engine is the best balanced of any type of 2-cylinder engine of usual construction.

Three-crank engines with cranks at 120 degrees, cylinders in the same plane and on the same side of the shaft, have no force error due to purely reciprocating and revolving parts. It has been shown that the effect of the reciprocating parts along the line of stroke is the same as if their weight were attached to the crank to produce the primary force. Then assume three cranks at 120 degrees as shown in Fig. 223 with such weights attached Assume the cranks to move from a symmetrical position through the angle A. Then from trigonometry:

$$\cos (A + B) = \cos A \cos B - \sin A \sin B.$$

and:

$$\cos (A - B) = \cos A \cos B + \sin A \sin B.$$

Adding together gives:

$$\cos(A + B) + \cos(A - B) = 2 \cos A \cos B.$$

If $B = 60$, $2 \cos B = 1$; and:

$$\cos(A + B) + \cos(A - B) = \cos A.$$

This shows that the projection of centrifugal force upon the center line of engine, or any other diameter, of three equal weights revolving at equal radii with the same angular velocity, are in equilibrium in all positions.

As the secondary force along the line of stroke is equivalent to the projection of the centrifugal force due to the mass of the reciprocating parts concentrated at the center of an imaginary crank pin traveling on a circle $R/4L$ times that of the engine crank, and at a velocity twice as great, it follows that these imaginary cranks are always 120 degrees apart on the crank circle. According to Fig. 223, they are then in equilibrium among themselves. This is true of any revolving weights, such as the cranks, if they are of equal weight and their mass centers are the same distance from the shaft center.

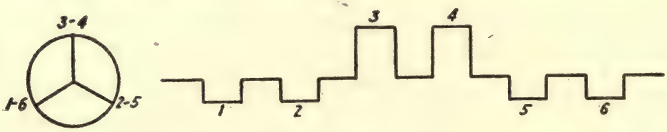


FIG. 224.

As previously stated, the force due to the reciprocating parts is not caused by a revolving weight, and is equivalent to a revolving system only along the line of stroke. The normal component F_{PN}, due to inertia of the reciprocating parts is not in balance in a 3-cylinder engine; this will be considered later.

There is a couple error, which is a maximum for the primary force when the center crank is at right angles to the line of stroke. For the reciprocating parts this is equal to the product of the inertia, the cosine of 30 degrees and the distance between the center of the first and third cylinders. If the cranks are not balanced, the radial inertia of crank and pin, referred to the pin center, must be added to the inertia. The actual couple due to reciprocating parts is some less due to the secondary effect.

Six-crank engines with cylinders in the same plane and on the same side of the shaft are balanced along the line of stroke for both primary and secondary forces, the same as the 3-crank engine. It is practically the same as two 3-cylinder engines placed end to end as shown in Fig. 224.

Applying Formulas (5), (6) and (7) shows that if the arrangement of

cylinders about the line midway between the two center cylinders is symmetrical and the reciprocating parts and cranks weigh the same for each cylinder, there is no couple error, either primary or secondary. The only advantage of balance weights in this engine is to relieve frame and shaft strains and bearing pressures, and to reduce this to a minimum, part of the reciprocating parts should be balanced. The effect of the forces acting on shaft and bearing, with and without counterbalance may be seen in Figs. 208 and 209, Chap. XVI.

112. The Connecting Rod.—Forces due to the mass of the rod were discussed in Chap. XVI, and their effect upon the frame may be separately considered if desired.

In computing counterbalance it has commonly been assumed that if the rod is divided between the reciprocating and revolving parts inversely as the mass center divides the rod, it is correctly disposed of. As the proportion of reciprocating parts to be balanced is either fixed experimentally as in locomotive practice, or arbitrarily assumed, such a rule is probably accurate enough. It is, in fact, correct along the line of stroke, as may be seen by adding

$$\frac{L_G}{L} \cdot \frac{W}{g} a_1 \text{ to } \left(1 - \frac{L_G}{L}\right)\frac{W}{g} a_2$$

where W is the weight of the rod, $a_1 = R\omega^2 \cos \theta$, the acceleration of the crank pin, and a_2 is the acceleration of the crosshead pin as given by (30), Chap. XVI, when l is zero. The result is identical with (30) and (32), Chap. XVI which gives the inertia of the rod along the line of stroke when $l = L_G$.

Normal to the line of stroke, the force due to the rod is the algebraic sum of F_{YB} and F_{XN} given by (39) and (43), Chap. XVI, the signs being taken from Table 55 of the same chapter. For a rod of uniform section, five cranks long, this gives:

$$0.353 \frac{WR\omega^2}{g}$$

while the divided rod method gives:

$$0.5 \frac{WR\omega^2}{g}$$

an error of 42 per cent.

Neglecting the balance of the crank, let it be assumed that 60 per cent. of the reciprocating parts is to be balanced; then the balance weight required to balance this according to the usual method is:

$$0.6 \left(W_P + \frac{W}{2}\right) + \frac{W}{2}$$

where W_P is the weight of the reciprocating parts. If $W = kW_P$, the constant radial inertia (centrifugal force) of the weight, assumed to have its center of gravity on the crank circle is:

$$(0.6 + 0.8k)\frac{W_P R\omega^2}{g}.$$

The inertia of the reciprocating parts at the head end of the stroke, from (33), Chap. XVI is:

$$F_H = 1.2\frac{W_P R\omega^2}{g}.$$

At the head end, from (34):

$$F_C = 0.8\frac{W_P R\omega^2}{g}$$

The inertia of the rod along the line of stroke at the head-end dead center is, from (30) and (32), Chap. XVI:

$$F_{XH} = 1.1\frac{WR\omega^2}{g} = 1.1k\frac{W_P R\omega^2}{g}.$$

At the crank end:

$$F_{XC} = 0.9\frac{WR\omega^2}{g} = 0.9\frac{W_P R\omega^2}{g}.$$

The total head-end inertia is:

$$(1.2 + 1.1k)\frac{W_P R\omega^2}{g}$$

and at the crank end:

$$(0.8 + 0.9k)\frac{W_P R\omega^2}{g}$$

Comparing with the counterbalance, the amount unbalanced at ends of stroke is:

For head end, $(0.6 + 0.3k)\dfrac{W_P R\omega^2}{g}$

For crank end, $(0.2 + 0.1k)\dfrac{W_P R\omega^2}{g}$

Average for two ends, $(0.4 + 0.2k)\dfrac{W_P R\omega^2}{g}$

Normal to the line of stroke, when the crank is at right angles to the line of stroke, the inertia as just given, is:

$$I_Y = 0.353\frac{WR\omega^2}{g} = 0.353k\frac{W_P R\omega^2}{g}.$$

The overbalance in this position is:

$$(0.6 + 0.447k)\frac{W_P R\omega^2}{g}.$$

With the divided rod method it was assumed that

$$I_Y = 0.5\,\frac{WR\omega^2}{g} = 0.5k\,\frac{W_PR\omega^2}{g}.$$

This would give an overbalance of:

$$(0.6 + 0.3k)\,\frac{W_PR\omega^2}{g}$$

which is less than the actual. The ratio of the actual to the assumed is:

$$\frac{\text{actual overbalance}}{\text{assumed overbalance}} = \frac{0.6 + 0.447k}{0.6 + 0.3k}$$

If $k = 1$, the ratio is 1.162. For the Corliss engine which is being carried through the book, $k = 0.86$, and the ratio is 1.15, an error of 15 per cent.

It may be observed that the method is more correct the nearer the mass center of the rod is to the crank pin. This also permits the rod to be more nearly balanced by a revolving weight and reduces the force and couple errors due to the rod mass.

The rod cannot be perfectly balanced in engines of usual form, even in 6-crank engines.

113. Turning Effort.—The turning force is transmitted to the engine frame at the guide (or cylinder, when a trunk piston is used), giving the force p_A. This is plotted for the 20- by 48- in. Corliss engine in Fig. 206, Chap. XVI. It forms a couple equal to the turning couple, but may be offset, at least for part of the cycle, by the effect of counterbalance and other forces normal to the line of stroke acting at the shaft center. This may be seen in Fig. 206, Chap. XVI, which is drawn to the same force scale as Fig. 205. These forces are periodic and tend to cause vibration normal to a plane containing the engine and shaft center lines; they may not well be balanced.

In many engines with massive frames and heavy foundations the effect of turning effort is unimportant and is seldom given any thought. In ships and automobiles, in which the engines are made as light as practicable, vibration may be considerable, due to this cause, but is offset in part by arranging cylinders and cranks so as to obtain as uniform a turning effort as possible; that is, to provide more impulses per cycle. The vibrating forces are then more frequent but vary less from the mean force applied.

With uniform turning effort as given by steam turbines and electric motors, the couple is constant, and this condition is approximated in multi-cylinder engines.

As previously stated, the nature of supports or foundation determines to a great extent the effect of unbalanced forces. If the periodic motion is in synchronism with the natural period of vibration of the supports the

consequences may be serious; but if not, a small amount of unbalanced force will usually have no serious effect.

114. Balance weights are of various forms, some of which are shown in Chaps. XXVII and XXVIII. The required balance referred to the crank pin may be determined, then when the general form of the counterweight is selected, it may be drawn to scale with assumed dimensions. With certain simple forms the center of gravity may be calculated; then the area may be calculated or found by a planimeter and the moment determined.

Some designers prefer to draw the weight to scale, then cut it out of card board or thin wood and either balance it on a point, or hang it from two different points so that it is free, allowing a plumb line to hang from the point of suspension; the point where the lines cross in the two positions is the center of gravity.

Sometimes the first trial will suffice, any deviation from the required weight being provided for by varying the thickness.

When there is not room to accommodate the required weight, the casting is sometimes cored and filled with lead.

The *segment* of a circular ring is a form rather commonly used, and this lends itself to simple means of finding dimensions. Fig. 225 shows a crank with a counterweight of this kind. The moment of the counterweight about the shaft center must equal the sum of the moments of all weights to be balanced; or:

$$wA_Btl_B = \Sigma(Wl) = M \tag{10}$$

where w is unit weight, A_B the area of the segment forming the counterbalance, t its thickness and l_B the distance of its center of gravity from the shaft center. All dimensions must be either in feet or inches; if the latter, w is the weight per cu. in.

From (10):

$$A_Bl_B = \frac{M}{wt} \tag{11}$$

Without taking space for the derivation of the formula:

$$l_B = \frac{240}{\pi\delta}\cdot\frac{r_0^2 + r_0r + r^2}{r_0 + r}\cdot \sin\frac{\delta}{2}.$$

It is plain that:

$$A_B = \frac{\pi\delta}{360}(r_0^2 - r^2) = \frac{\pi\delta}{360}(r_0 + r)(r_0 - r).$$

Then letting $r_0/r = q$:

$$A_Bl_B = \frac{2}{3}(r_0^3 - r^3)\sin\frac{\delta}{2} = \frac{2}{3}r^3(q^3 - 1)\sin\frac{\delta}{2} \tag{12}$$

For a plain balance with no limits for r_0,

$$q = \sqrt[3]{1 + \frac{3A_Bl_B}{2r^3 \sin \frac{\delta}{2}}}$$

$$= \sqrt[3]{1 + \frac{3M}{2wtr^3 \sin \frac{\delta}{2}}} \tag{13}$$

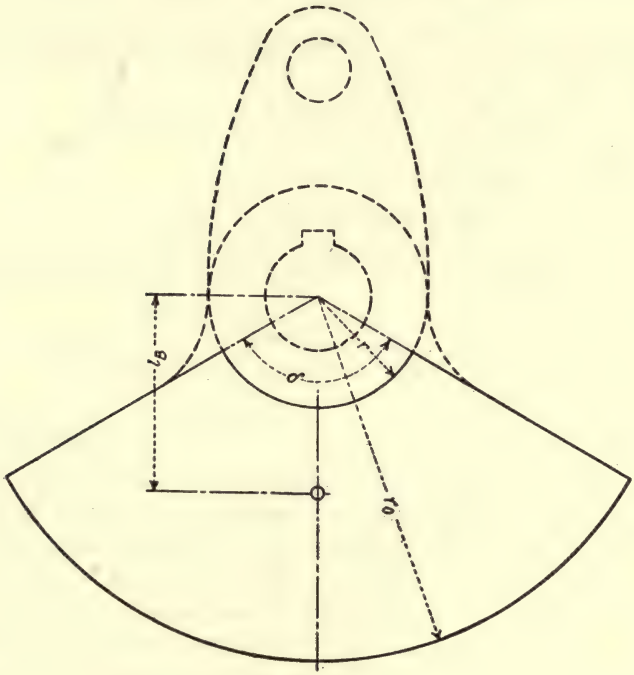


FIG. 225.

Then:

$$r_0 = qr \tag{14}$$

If a disc crank is used which fixes both r and r_0, and if t is assumed:

$$\sin \frac{\delta}{2} = \frac{3M}{2wtr^3 (q^3 - 1)} \tag{15}$$

If δ is assumed:

$$t = \frac{3M}{2wr^3 (q^3 - 1) \sin \frac{\delta}{2}} \tag{16}$$

With a disc crank, the thickness of the disc should not be counted as part of either arm or balance thickness.

In the Corliss engine for which data is tabulated in Chap. XVI, the values of p_R from Table 57 were taken at crank positions 6, 12, 18 and 24, and averaged. This method is arbitrary, and in this case gave slightly smaller weight than by taking 0.6 of the reciprocating parts added to the revolving parts, the rod being divided inversely as the mass center. The average gives the radial inertia required of the counterweight and is:

$$C_F = \frac{10{,}366 + 20{,}580 + 4126 + 25{,}385}{4} = 15{,}114.$$

From (1), solving for W (W_B in this case) gives the weight referred to the crank pin; or:

$$W_B = \frac{gC_F}{R} = \frac{32.16 \times 15{,}114}{2 \times 109} = 2225 \text{ lb.}$$

Then from (10), in pound-inches.

$$M = 2225 \times 24 = 53{,}400$$

From (13), more conveniently taking dimensions in inches, and assuming a cast-iron crank:

$$q = \sqrt[3]{1 + \frac{3 \times 53{,}400}{2 \times 0.26 \times 6.375 \times 12^3 \times 0.866}} = 3.22.$$

Then:

$$r_0 = 3.22 \times 12 = 38.75 \text{ in.}$$

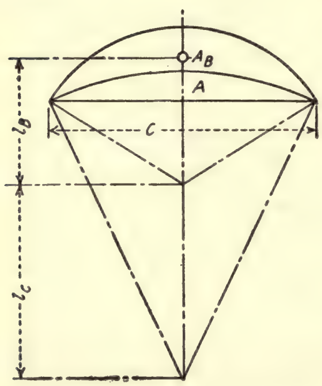


FIG. 226.

This assumes the thickness of balance as 6⅜ in., which is the same as the arm thickness for the crank designed for the 20-in. Corliss engine in Chap. XXVII. The angle δ was taken as 120 degrees.

The *crescent* and the *segment* of a circle are forms used in locomotive balancing. Fig. 226 shows a crescent, the distance between the centers of the two arcs being denoted by l_C. A_B is the area of the crescent and A the area of the segment between this and the chord. A simple exact formula for l_B was published by the author in the American Machinist some years ago and is:

$$l_B = \frac{Al_C}{A_B} \tag{17}$$

or:

$$Al_C = A_Bl_B = \frac{M}{wt} \tag{18}$$

If the counterbalance is a segment, $A = 0$ and $1_C = \infty$; then the formula fails; then:

$$l_B = \frac{c^3}{12A_B} \tag{19}$$

or:

$$A_B l_B = \frac{c^3}{12} \tag{20}$$

CHAPTER XVIII

REGULATION DURING THE CYCLE. FLYWHEELS

115. Introduction.—It is evident from the turning-effort diagrams of Par. 106, Chap. XVI that the force applied to the crank pin in the direction of motion varies considerably during the engine cycle, even when a number of cylinders are employed. Unless the resistance varies in identically the same way, which it never does, a change of velocity must occur during the cycle, the number of fluctuations depending upon the number of times the turning effort varies from the mean; also the displacement of the crank pin from the place it would occupy at any instant with perfectly uniform motion, occurs.

The function of the flywheel is to limit speed fluctuation and displacement to an amount found by practice to give satisfactory operating results.

The method of controlling speed fluctuation is simpler and is much more commonly used for designing wheels for all purposes. The displacement method is more difficult and is used only when engines are designed to drive alternating-current generators which are to operate in parallel. The relation between the two methods will be shown in Par. 118.

In applying either of these methods to practical work certain assumptions are made and refinements neglected, as the varying condition of operation of any type of engine does not permit of great accuracy. Attention is called to these approximations in the proper places.

Notation.

D = diameter of wheel in feet.
R = radius of crank circle in feet.
r = radius of gyration in general, in feet.
w = weight in pounds in general.
W_R = weight of wheel rim in pounds.
W_W = total weight of wheel in pounds.
W = weight in pounds concentrated at crank-pin center, which would give the same effect as the weight of the entire wheel concentrated at its center of gyration.

M = mass assumed concentrated at center of crank pin, corresponding to W ($= W/g$).
F = turning effort in pounds, above or below the mean effort.
E = mean energy per piston stroke in foot pounds.
ΔE = fluctuation of energy above or below the mean; usually the maximum $= \epsilon E$.
H = indicated horsepower.
V = mean velocity of rim in feet per second.
V_1 = maximum velocity of rim.
V_2 = minimum velocity of rim.
v = velocity of crank pin in feet per second.
N = r.p.m.
a = linear acceleration of crank pin in feet per second per second.
t = time of one engine cycle in seconds.
s = displacement in feet of the crank pin from its mean position.
ϵ = coefficient of fluctuation of energy.
δ = cœfficient of fluctuation of velocity.
Δ = change of.
α = displacement of crank pin from mean position in electrical degrees.
c = number of cycles of alternator per second.
p = number of poles.
m = time scale = number of seconds per inch.
n = force scale = number of pounds per inch.
q = scale of Mv = number of units of Mv per inch.
k = scale of Ms = number of units on Ms per inch.
x, y and l = measurements on diagrams in inches.
C and K = constants in Formula (8) and Table 60.
$g = 32.16$.

116. The Control of Speed Fluctuation.—This method may be applied where the resistance varies, so long as the cycle is constant at a given load. In most cases where such calculations are made the resistance is assumed to be uniform for a given load, so that the turning effort diagram shown in Fig. 194, Chap. XVI will be used to illustrate the methods of determining flywheel weight in this chapter. This is reproduced in Fig. 227.

As it is the variation from mean which causes change of speed, the areas above and below the mean effort line are the areas considered, and these are shaded. Each one of these lobes bounded by the curve and the mean line represents the *fluctuation of energy* as the crank pin passes between two intersections of these lines, and is denoted by ΔE. The

crank pin has a certain velocity at A (and the flywheel rim a proportionate velocity). In passing from A to B energy is taken from the wheel equivalent to the area of the lobe between A and B, and its velocity is reduced. At B the velocity begins to increase and continues to do so until C is reached, the kinetic energy stored in the wheel during this time, as the crank pin travels from B to C being represented by the lobe between B and C.

Kinetic energy varies as the square of the velocity; then it is obvious that the maximum fluctuation of velocity is caused by the maximum fluctuation of energy, and the largest lobe ΔE must be used in the following calculations.

Let E be the energy of the cycle in ft. lb. divided by the number of strokes, or the mean energy per stroke. Then the *coefficient of the fluctuation of energy* is:

$$\epsilon = \frac{\Delta E}{E} \tag{1}$$

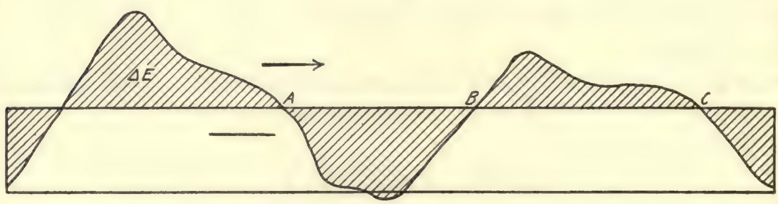


FIG. 227.

Basing ϵ upon the energy per stroke has advantages over energy per cycle in simplifying the final equations.

If N = r.p.m., there are $2N$ strokes per minute; the i.h.p. then is:

$$H = \frac{2NE}{33,000} = \frac{NE}{16,500}.$$

Then for all engines the mean energy per stroke is:

$$E = \frac{16,500H}{N} \tag{2}$$

As just stated, the change of kinetic energy of the rim due to the maximum fluctuation of velocity from V_2 to V_1 ft. per sec., or *vice versa*, is equivalent to the maximum fluctuation of energy along the crank-pin path; then from (1):

$$\Delta E = \epsilon E = \frac{W_R(V_1^2 - V_2^2)}{2g} = \frac{W_R(V_1 - V_2)(V_1 + V_2)}{2g} \tag{3}$$

If V is the mean velocity and δ the *coefficient of the fluctuation of speed:*

$$V_1 - V_2 = \delta V = \frac{\pi \delta DN}{60} \tag{4}$$

Also, nearly enough for practical purposes:

$$\frac{V_1 + V_2}{2} = V = \frac{\pi DN}{60}$$

or:

$$V_1 + V_2 = \frac{\pi DN}{30} \tag{5}$$

where D is the wheel diameter in ft.

Substituting (2), (4) and (5) in (3) gives:

$$\frac{16{,}500\epsilon H}{N} = \frac{\pi^2 \delta W_R DN}{60 \times 30 \times 2g}.$$

Solving for W_R gives:

$$W_R = 194{,}000{,}000 \frac{\epsilon H}{\delta D^2 N^3} \tag{6}$$

Stresses in flywheels will be treated in Chap. XXX, but to limit these to safe values the rim speed is not allowed to exceed a certain limit; a common speed allowed for cast-iron wheels is one mile (5280 ft.) per minute. Then:

$$\pi DN = 5280 \text{ and } DN = 1680.$$

Substituting this in (6) gives:

$$W_R = 70 \frac{\epsilon H}{\delta N} \tag{7}$$

Formula (7) is convenient in estimating when it is likely that the wheel will run up to the limiting speed; but it gives a lighter rim than does (6) should the rim velocity be reduced, and this must be kept in mind.

If values of ϵ and δ are known, (6) and (7) may be written:

$$W_R = C \frac{H}{D^2 N^3} = K \frac{H}{N} \tag{8}$$

The value of ϵ in Fig. 227 is 0.33. This was plotted for the 20- by 48-in. Corliss engine designed in Chap. XII with a ¼ cut-off. Goodman gives the values of ϵ for double-acting steam engines in Table 58.

Table 58

Cut-off	Single cylinder	Two-cylinder cranks at 90	Three-cylinder cranks at 120
0.1	0.35	0.088	0.040
0.2	0.33	0.082	0.037
0.4	0.31	0.078	0.034
0.6	0.29	0.072	0.032
0.8	0.28	0.070	0.031
Full	0.27	0.068	0.030

It will be seen that ϵ decreases when the cut-off, and therefore the load increases; but not in the same ratio. It would seem from this that a value of H near the maximum should be used. It is ΔE (which varies as ϵH) that causes the fluctuation of speed, and not ϵ $\left(= \frac{\Delta E}{E}\right)$.

For 4-cycle internal-combustion engines working on the Otto cycle, Goodman gives the values in Table 59.

TABLE 59

Exploding at	Values of ϵ for internal-combustion engines				
	Single-acting			Double-acting	
	1-cyl.	2-cyl.	4-cyl.	1-cyl.	2-cyl.
Every cycle	3.7 to 4.5	1.5 to 1.8	0.3 to 0.4	2.3 to 2.8	0.3 to 0.4
Alternate cycles	8.5 to 9.8	2.5 to 3.0			

TABLE 60

Service	ϵ	δ	C	K
Pumping	0.4	1/30	2,320,000,000	824
Machine shop	0.4	1/35	2,710,000,000	960
Textile, paper and flour mills	0.4	1/40	3,100,000,000	1,100
Spinning machinery	0.4	1/100	7,740,000,000	2,750
Gas compression	0.4	1/100	7,740,000,000	2,750
Rolling mill	2.0	1/30	11,620,000,000	4,110
Cable railway	0.4	1/120	9,300,000,000	3,300
Electrical machinery (belted)	0.4	1/150	11,620,000,000	4,110
Direct-connected alternator	0.4	$1/5p$	$387,500,000p$	$140p$
Sugar mill—grinder	0.4	1/42	3,200,000,000	1,160
Sugar mill—crusher	0.4	1/73	5,600,000,000	2,010

In Chap. XIV, Par. 78, reference was made to hit-and-miss governing at light loads. From (6) and (7) it may be seen that the speed fluctuation of a wheel of given weight varies as the product ϵH. If every alternate impulse is missed the value of H is one-half full load. Then taking values from Table 59, to give the same regulation at half load a single-cylinder gas engine would need a wheel 10 per cent. heavier, and a two-cylinder engine 83 per cent. as heavy as for full load. This may not hold true for lighter loads, but by making a wheel on a hit-and-miss engine some 20 per cent. heavier than the formula indicates, fair regulation should be

obtained for quite a wide range of load. If combination governing were used, the hit-and-miss principle would not be applied except at the lower range; if at one-half load, one-quarter load could be carried by missing every alternate impulse.

Many tables of δ have been published and there is considerable variation. Table 60, for double-acting, single-cylinder steam engines is given as an example in tabulating C and K of Formula (8), also to give some values of δ which have been taken from various sources. The value for the direct-connected alternator will be further explained in Par. 3; p denotes the number of poles. The table has been used by the author for several years and part of the data are from his practice. It must be used with judgment; in some cases the values of C and K may give too small values to enable the proper construction of a wheel for strength, especially if the rim speed is near the maximum limit.

The value of ϵ is taken a little high to provide for discrepancies. From Table 58 it will be seen that the value of ϵ varies inversely as the square of the number of alternations of turning effort. This would mean that a cross-compound steam engine requires a wheel but one-quarter as heavy as a single-cylinder engine of the same power. But with compound engines, load distribution between high- and low-pressure sides is considerably affected by different valve settings, and this in turn affects the form of the turning effort diagram, often increasing the value of ΔE. Then too, a compound engine does not respond to the governor quite as quickly as a simple engine, especially if the high-pressure cylinder only is directly governed. It therefore seems that some allowance should be made in using values of ϵ found from carefully designed compound diagrams. With large units which are to have frequent applications of the indicator this need not be very much, possibly an increase of 25 per cent. In a good many cases the author has determined wheel weights by the use of Table 60 for a simple engine, and taken from 75 to 100 per cent. of this weight for cross-compound engines of the same power, and while this may seem excessive, it has been satisfactory.

For internal-combustion engines with more than one cylinder it is probably not necessary to make much allowance, although there may be a small difference in the indicator diagrams of the different cylinders; increasing the value of ϵ about 10 per cent. over the value found with equal indicator diagrams will probably give ample allowance.

For strict accuracy the diameter D used in the formulas should be equal to twice the radius of gyration of the entire wheel, or, more strictly, of all parts revolving on the shaft, and W_R should include the entire weight of wheel and other revolving parts. If w is the weight of a part

of the wheel or other revolving part and r its radius of gyration about the shaft center, then:

$$W_R \left(\frac{D}{2}\right)^2 = \Sigma(wr^2) \tag{9}$$

The calculation of wr^2 requires much time and is usually an unnecessary refinement. If D is taken as the outside diameter of the wheel, the weight of the wheel rim found by (6), (7) or (8), and the weight of arms, etc. neglected in the calculation, the energy of the wheel will be equal to or greater than that required if it is a belt wheel; for a heavy flywheel with a thick rim (radially) the weight may fall a little short if calculated in this way, but increasing the weight so found by 10 per cent. will make ample provision. The value of ϵ in Table 60 for simple steam engines will cover this.

In estimating it is convenient to know the weight of the entire wheel. After finding rim weight W_R from (6), (7) or (8), sufficient metal will be allowed for a well-built wheel of the more common designs if the total weight W is as follows:

For belt—or rope wheels. . . . . $W_W = 1.7$ to $1.8W_R$ (10)

For heavy-rimmed flywheels . . $W_W = 1.5$ to $1.6W_R$ (11)

117. The Control of Displacement.—Alternators operating in parallel must be in synchronism to give good results. They must not only have the same r.p.m. but the angular distance between corresponding poles (in electrical degrees) must not exceed a certain limit or cross currents will be set up, lowering the average voltage and causing loss of effectiveness and efficiency.

The general effect of irregularity of turning effort upon speed fluctuation during the cycle has been discussed in the preceding paragraph; it now remains to study the effect upon the displacement of the wheel from some mean position it would occupy if revolving with a perfectly uniform velocity.

To try and make this clear before proceeding with a mathematical determination of the displacement curve, a rather imaginary piece of apparatus shown in Fig. 228 will be examined. Assume two wheels running side by side with the same r.p.m. Wheel No. 1 runs with absolute uniformity of speed and has on it two paper drums operated by gears with a uniform motion. The arrow shows the direction the paper moves. Wheel No. 2 is an engine flywheel subject to the variation of turning effort. Arm A, containing a pencil is fastened to a spoke of this wheel. During the revolution the relative angular position of the two wheels changes, causing the pencil to move across the spoke of wheel

1 and draw a curve as the paper moves at right angles to the pencil movement. This is a *displacement curve* and with a constant load on the engine it should be exactly alike for each cycle of the engine.

A line drawn in the direction of paper motion in such a way that the algebraic sum of the areas enclosed between it and the curve for a complete cycle is zero, is the mean line; this is shown dotted, and if the pencil of wheel 2 is brought over to this line it will show the mean position of wheel 2 relative to wheel 1 as the two wheels revolve.

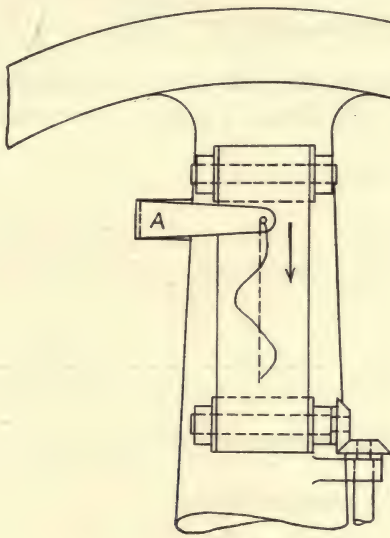


Fig. 228.

A similar curve may be drawn from the crank-effort diagram of any engine. When the turning effort is above or below the mean it causes positive or negative acceleration of the wheel. It is convenient to refer all force and motion to the crank-pin center. Then if M is the mass ($= W/g$, where W is the weight in lb.) giving the same effect as the wheel concentrated on the crank circle, a the linear acceleration of the crank pin and F the turning effort in lb. above or below the mean:

$$F = Ma$$

or:

$$a = \frac{F}{M} \tag{12}$$

Then a is directly proportional to F, and Fig. 227 may be converted into an acceleration diagram by changing the scale.

In practice the displacement is very slight, so that with the usual spacing of the crank circle, if it be assumed that equal increments of time are accompanied by equal increments of space as the crank moves around the circle, the degree of accuracy will be in keeping with that of the whole method. Then by changing the scale, space may be changed to time; then the time of one cycle in seconds which is represented by the length of the diagram is:

$$\left.\begin{array}{l} \text{For a 2-stroke cycle, } t = \dfrac{60}{N} \\ \text{For a 4-stroke cycle, } t = \dfrac{120}{N} \end{array}\right\} \qquad (13)$$

Fig. 227 is reproduced in Fig. 229 as an *acceleration-time* diagram. The dimensions x, y and l are always in inches, and areas in sq. in.

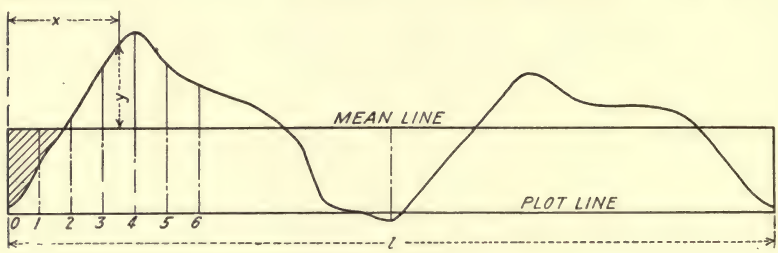


FIG. 229.—Acceleration-time diagram.

A general expression for acceleration is:

$$a = \frac{dv}{dt}$$

or:

$$dv = a \cdot dt \qquad (14)$$

It is then obvious that by integrating the acceleration-time curve above and below the mean line, a curve may be plotted with change of velocity as ordinates and time as abscissas—a *velocity-time* diagram.

A general expression for velocity is:

$$v = \frac{ds}{dt}$$

or:

$$ds = v \cdot dt \qquad (15)$$

After finding the mean line for the velocity-time curve, this curve may be integrated and a *displacement-time* curve plotted. From the mean line found for this curve the maximum displacement from mean may be found.

All integrations will be made with a planimeter, the mathematical expressions being used to make the operations clear.

Fig. 230 is a velocity-time curve and Fig. 231 a displacement-time curve.

To make practical use of these diagrams scales must be used for the different quantities, and it avoids much confusion if these are determined as the work proceeds.

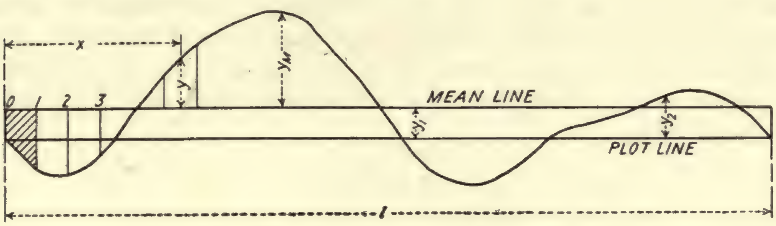


Fig. 230.—Velocity-time diagram.

For all curves: 1 in. $= m$ seconds $= \frac{t}{l}$, where t is found from (13). Then:

$$dt = mdx \tag{16}$$

where dx is in inches.

Let: 1 in. $= n$ lb. $= n$ units of F $(= Ma)$.

Or: 1 in. $= \frac{n}{M}$ units of a $= \frac{n}{M}$ ft. per sec.^{2}

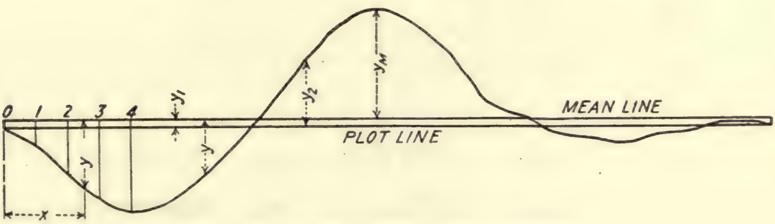


Fig. 231.—Displacement-time diagram.

n has already been determined for plotting Fig. 227. Then:

$$a = \frac{n}{M} y \tag{17}$$

To integrate the acceleration-time curve to obtain change of velocity, the mathematical expression may be reduced by substituting (16) and (17) in (14) as follows:

$$\Delta v = \int_{t_1}^{t_2} a \cdot dt = \int_{x_1}^{x_2} \frac{n}{M} \cdot ym \cdot dx = \frac{nm}{M} \int_{x_1}^{x_2} y \cdot dx \tag{18}$$

The quantities n and m are known. If it is desired to find the displacement of a wheel already designed or built, M is also known. The quantity in the integration sign is the area in sq. in. between any two ordinates found by the planimeter. If this is the first space of Fig. 229, shown shaded, it is below the mean line and must be laid off at point 1 of Fig. 230 from and below the plotting line, the mean line not having been determined. The mean line is found by taking the difference of the areas above and below the mean line and dividing by the length in inches. If the area above the line is greater, the mean line will be drawn above the plotting line, or vice versa. If y_1 is the distance from mean to plotting line and y_2 from plotting line to curve, this may be expressed algebraically thus:

$$y_1 = \int_0^l y_2 \cdot dx \div l \tag{19}$$

To fix the scale for the velocity-time curve, determine the total height that it is desired to make it, then add together the maximum and minimum values of Δv (not algebraically) and divide by the desired height in inches and round up to some convenient figure.

In most design problems M is unknown and may be carried through the calculation as an unknown quantity. We may then write (18) as follows:

$$M \cdot \Delta v = nm \int_{x_1}^{x_2} y\, dx \tag{20}$$

Then let: 1 in. = q units of Mv;

or: $$1 \text{ in.} = \frac{q}{M} \text{units of } v = \frac{q}{M} \text{ ft. per sec.}$$

Then:

$$v = \frac{q}{M} \cdot y \tag{21}$$

To integrate the velocity-time curve to obtain displacement the expression is found by substituting (16) and (21) in (15), thus:

$$\Delta s = \int_{t_1}^{t_2} v \cdot dt = \int_{x_1}^{x_2} \frac{q}{M} \cdot yn \cdot dx = \frac{qm}{M} \int_{x_1}^{x_2} y \cdot dx \tag{22}$$

If M is unknown:

$$M \cdot \Delta s = qm \int_{x_1}^{x_2} y \cdot dx \tag{23}$$

The scale is found as for the velocity-time curve; then let:

1 in. = k units of Ms;

or:

$$1 \text{ in.} = \frac{k}{M} \text{ units of } s.$$

The mean line is found as before from (19).

The maximum ordinate y_M from the mean line must now be found and this corresponds to the greatest displacement s_M in feet. Then:

$$s_M = \frac{k}{M} \cdot y_M \tag{24}$$

If M is unknown and to be determined:

$$M = \frac{k y_M}{s_M} \tag{25}$$

The value of s_M is determined from electrical conditions. There are 360 electrical degrees between two poles of the same sign. If the alternator has p poles, the number of electrical degrees in the entire circle is:

$$360 \cdot \frac{p}{2} = 180p.$$

If the allowable displacement either side of mean is α electrical degrees (degrees of phase), the maximum allowable displacement of the crank pin in feet is:

$$s_M = \frac{\alpha}{180p} \cdot 2\pi R = \frac{\alpha R}{28.65p} \tag{26}$$

There seems to be a scarcity of data regarding α in electrical engineering hand books, but 2.5 is given in papers from the Trans. A.S.M.E. referred to at the end of this chapter.

If the number of electrical cycles per second is denoted by c, the relation between the cycle, number of poles, and speed is given by:

$$p = \frac{120c}{N} \tag{27}$$

It is convenient to tabulate data as the work proceeds as shown in Table 61. The minus sign indicates that the larger part of the area up to that number is below the mean line; when it is above the mean line the sign is plus. Obviously the final value should be zero, bringing the curve back again to the plotting line to the point from which it started. In determining the areas for the table, the planimeter may be started at the beginning of the curve each time if the diagram is not too large; or, each division may be measured and added algebraically.

From (6), the weight varies inversely as the square of the diameter

TABLE 61

Ordinate	1	2	3	4	5	6	7	8	9	10	11	12
Area (*vt*)	− 0.50	−0.58	0.28	−0.36	1.00	1.43	1.77	1.92	1.96	1.56	1.00	0.32
Mv	− 250	− 290	− 140	180	500	715	885	960	980	780	500	160
Area (*st*)	− 0.34	−0.86	−1.35	−1.56	−1.46	−1.03	−0.54	0.19	0.92	1.63	2.08	2.15
Ms	− 8.5	−21.5	−33.7	−39.0	−36.5	−25.3	−13.5	4.75	23.0	40.7	52.0	53.8

Ordinate	13	14	15	16	17	18	19	20	21	22	23	24
Area (*vt*)	−0.34	−0.65	− 0.68	−0.43	0	0.23	0.37	0.55	0.73	0.75	0.46	0
Mv	− 170	− 325	− 340	−215	0	115	175	275	365	375	230	0
Area (*st*)	1.94	1.39	0.69	0.28	−0.06	−0.19	−0.26	−0.24	−0.13	0.03	0.06	0
Ms	48.5	34.8	17.2	7.00	−1.5	−4.75	−6.5	−6.0	−3.25	0.71	1.50	0

at which it is applied; then if the weight ($= gM$) assumed concentrated at the radius R of the crank circle is referred to diameter D, we have:

$$W_R \left(\frac{D}{2}\right)^2 = WR^2$$

or:

$$W_R = W \left(\frac{2R}{D}\right)^2 \tag{28}$$

The general notes in reference to weight of arms, etc. of the preceding paragraph apply to wheels designed by the displacement method also. It would be a farce, however, to take the trouble to calculate a wheel by this method for a simple engine, to use on a cross-compound engine.

118. Comparison of Methods.—For a given engine with certain conditions of speed, steam pressure, valve setting, etc., Formula (6) may be written:

$$W_R = \frac{\text{constant}}{\delta} \tag{29}$$

Also from (25), (26) and (28):

$$W_R = \frac{p}{\text{constant}} \tag{30}$$

Equating (29) and (30) we may write:

$$p\delta = \text{constant}$$

or:

$$\delta = \frac{1}{p \times \text{constant}} \tag{31}$$

This is the form for the value of δ for alternators in Table 60, and if the displacement in electrical degrees is to be the same for all alternators running at a given speed, δ must vary inversely as the number of poles, or as the number of cycles per second. This would indicate that the arbitrary use of the usual values of δ for all sorts of electrical machinery is in error.

If the constant in (31) is carefully chosen (and it depends upon the crank-effort diagram and is as reliable as ϵ), the simple method of wheel weight determination in Par. 116 is reliable and saves much time; but in new work it is usually desirable to investigate by more elaborate means, and errors may sometimes be caught in this way; it is for this reason that the displacement method is given in this book.

119. Application to Practice.—A wheel will be designed to regulate an alternating-current generator, neglecting any flywheel effect of the generator. The engine is the 20 by 48 in. Corliss engine of Chap. XII. The turning-effort diagram calculated in Chap. XVI has been reproduced in

Fig. 227 of this chapter and is the basis of calculation for the methods of both Pars. 116 and 117.

For a rim speed of 1 mile per minute the wheel diameter could be 16.8 ft. Assume it to be 16 ft. in diameter. While the maximum power at which the alternator is expected to give the best service should probably be taken, the diagrams have all been worked out for the rated horsepower of 450, and this will serve as an application of the methods. The diagram factor was omitted in plotting the indicator diagrams, but this is probably on the safe side.

Weights will first be determined from the method of Par. 116, using Formula (8) with the constant C; as the speed is less than 5280 ft. per min., K can not be used. The value C in line 8 of Table 60 has been used for electric generators for direct or alternating currents, whether belt-driven or direct connected. This gives:

$$W_R = \frac{11{,}620{,}000{,}000 \times 450}{16^2 \times 100^3} = 20{,}500 \text{ lb.}$$

Line 9 for a 25-cycle (30-pole) generator gives:

$$W_R = \frac{30 \times 387{,}500{,}000 \times 450}{16^2 \times 100^3} = 20{,}500 \text{ lb.}$$

For a 60-cycle (72-pole) generator, line 9 gives:

$$W_R = \frac{72 \times 387{,}500{,}000 \times 450}{16^2 \times 100^3} = 49{,}000 \text{ lb.}$$

From this it is apparent that if line 8 had been used for a 60-cycle machine, there might have been trouble.

The displacement method is given in Par. 117. The areas and values of Ms and Mv are given in Table 61. The length of the diagrams as originally plotted was 12 in. and it was intended to have the height about 3 in.

As the steam engine is a 2-cycle engine, the time of one cycle is, from (13):

$$t = \frac{60}{100} = 0.6 \text{ sec.}$$

then:

$$m = \frac{t}{l} = \frac{0.6}{12} = 0.05.$$

As already determined for plotting the turning-effort diagram:

$$n = 10{,}000 \text{ lb.}$$

From Table 61, the maximum plus value of Mv is 980, the maximum minus value 340, and their sum (not algebraic) 1320. For a height of 3 in. the scale would be:

$$q = \frac{1320}{3} = 440; \text{ or, take it as } 500.$$

In the same manner the scale of *Ms* is:

$$k = 30.$$

The maximum ordinate measured from the mean line of Fig. 231 is:

$$y_M = 1.72 \text{ in.}$$

From (27), for a 25-cycle alternator:

$$p = \frac{120 \times 25}{100} = 30.$$

Taking $\alpha = 2.5$, and as $R = 2$, (26) gives:

$$s_M = \frac{2.5 \times 2}{28.65 \times 30} = 0.00582 \text{ ft.}$$

From (25), at the crank pin:

$$M = \frac{30 \times 1.72}{0.005820} = 8850 \quad \text{and} \quad W = 32.16 \times 8850 = 285{,}000 \text{ lb.}$$

From (28), the weight referred to the rim is:

$$W_R = 285{,}000 \times \left(\frac{2 \times 2}{16}\right)^2 = 17{,}800 \text{ lb.}$$

In the same manner for a 60-cycle machine we have:

$$p = \frac{120 \times 60}{100} = 72 \qquad s_M = \frac{2.5 \times 2}{28.65 \times 72} = 0.002425 \text{ ft.}$$

$$M = \frac{30 \times 1.72}{0.002425} = 21{,}250 \text{ and } W = 32.16 \times 21{,}250 = 683{,}000 \text{ lb.}$$

$$W_R = 683{,}000 \times \left(\frac{2 \times 2}{16}\right)^2 = 42{,}750 \text{ lb.}$$

Tabulating the results for comparison, taking $W_W =$ about 1.6 W_R, gives Table 62.

TABLE 62

Method	W_R	W_W
Formula (8), line 8 of Table 60	20,500	33,000
Line 9 of Table 60—30 poles	20,500	33,000
Line 9 of Table 60—72 poles	49,000	80,000
Displacement method—30 poles	17,800	29,000
Displacement method—72 poles	42,750	70,000

No allowance having been made in the displacement method for possible discrepancies, the values are some smaller than by the simpler method, which in the present problem may be said to give equally good results. Allowance is made for assuming the radius of gyration as the radius of the outside of the rim.

In Fig. 230 the height of the diagram is 2.7 in. and the scale is q/M. Then the speed fluctuation of the crank pin in ft. per sec. is:

For the 25-cycle machine,

$$v_2 - v_1 = \frac{2.7 \times 500}{8850} = 0.153.$$

For the 60-cycle machine,

$$v_2 - v_1 = \frac{2.7 \times 500}{21{,}250} = 0.0637.$$

The mean velocity of the crank pin is:

$$v = \frac{2\pi RN}{60} = \frac{\pi \times 2 \times 100}{30} = 21.$$

Then the coefficient of speed fluctuation is:

$$\text{For the 25-cycle,} \quad \delta = \frac{0.153}{21} = \frac{1}{137}$$

$$\text{For the 60-cycle,} \quad \delta = \frac{0.0637}{21} = \frac{1}{330}$$

Equating these with (31) gives:

$$\delta = \frac{1}{4.58p}$$

It is very clear that δ must depend upon the number of poles when the engine is to drive alternators in parallel.

In Par. 106, Chap. XVI, attention was called to the inequality of crank effort for the two ends of the cylinder due to the reduction of effective piston area by the piston rod, although these diagrams were drawn for equal cut-off. In some small steam engines with simple gear, the head-end cut-off is longer than the crank-end, increasing the inequality. When no provision is made for equalizing cut-off, this should be taken into account in plotting diagrams. With gears which will permit it, making the crank-end cut-off longer will probably give a more uniform turning effort; but most erecting and operating engineers would probably try to obtain equal cut-offs, so it is safer to ignore such a possible improvement in drawing the diagrams.

A flywheel for this problem will be designed in Chap. XXX.

References

Determination of flywheels to keep the angular variation of an engine within a fixed limit. *Trans.* A.S.M.E., vol. 22, p. 955.

Flywheel capacity for engine-driven alternators. *Trans.* A.S.M.E., vol. 24, p. 98.

CHAPTER XIX

REGULATION DURING CHANGE OF LOAD. GOVERNORS

120. Introduction.—The *governor*, or *regulator*, is a device for controlling the speed of heat engines and other motors. It is therefore closely allied with the valve-gear mechanism and it is sometimes a little difficult to locate the dividing line between the two. Commercially, some small throttling governors are all there is of valve gear as well as the valve, and needs only to be connected with the steam line and belted to the engine shaft.

In general the parts directly associated with the forces which are in equilibrium at a certain speed (called the tachometer by Zeuner), and the stand or wheel to which they are attached, comprise the governor; the same governor may be applied to various governor problems and in different ways.

There are two general methods of applying the governor to the control of speed;

1. The *independent* method, in which the only connection with the engine to be governed is that required to turn the governor shaft. This method may be *direct*, when there is a positive connection—sometimes links and levers—between the governor and the valve controlling the working fluid or fuel; or *indirect*, when the governor directly controls the supply of fluid to a steam or hydraulic cylinder or the current to solenoids, which in turn operate the main controlling valve or valves.

2. The *coördinate* method, in which the governor coördinates with *valve gear* operated by eccentrics or cams which receive their motion from the engine shaft and bear a positively timed relation to it.

The governor and all parts connected with the independent method will be treated in this chapter; the valve gear and its relation to the governor will be considered in Chap. XX.

Due to more or less contradiction of terms in governor nomenclature, and the fact that different names are given to the same thing by different builders and writers, the schedule of notation will take the form of definition to some extent, making reference easy.

Notation.

C = centrifugal force in pounds of governor weights when governor is in perfect equilibrium at a certain speed, neglecting friction or any

externally applied force. This is balanced by the centripetal force caused by springs or dead weights belonging to the governor proper.

F = external resistance in pounds due to valve-gear adjusting mechanism, opposing change of position of governor parts in either direction, thereby increasing or decreasing the effective centripetal force as the speed is increased or decreased respectively. F is sometimes called the *speed regulating force.* It is measured at the point of attachment to the valve-gear controlling mechanism, as the sliding sleeve of the fly-ball governor. F may also be assumed to include the friction of the governor bearings.

m = the total movement in inches of the part on which the force F acts. It is sometimes called the *stroke* or the *sleeve lift.*

Fm = the measure in inch-pounds of the ability of the governor to overcome external resistance, or to do work on the valve-gear adjusting mechanism with a given speed variation. This assumes the mean value of F and is called the *work capacity with a given speed variation.* It is also known as the *power* of the governor but this is a misnomer as power involves the time element.

ΔC = change of centrifugal force from any value C, required to overcome resistance F and allow the governor to assume another position corresponding to a new load. In any governor with inertia effect this aids in overcoming F.

ϵ = the ratio of ΔC to C $(= \Delta C/C)$ The ratio F/ϵ is sometimes known as the energy of the governor (this is a misnomer as F/ϵ is a force), and Fm/ϵ as the *total work capacity* in inch-pounds. F/ϵ is the effect on the point where F is measured, of the total centrifugal force of the revolving weights.

N = r.p.m. of the governor weights in any position of equilibrium corresponding to C.

ΔN = change of N corresponding to ΔC.

δ = the ratio of ΔN to N $(= \Delta N/N)$. This is called the *speed variation* by some makers of governors. It is also sometimes known as the *coefficient of sensitiveness.* F and δ are interdependent, but as F in a working governor is the actual resistance to be overcome, ϵ and δ must depend upon it. As δ measures either increase or decrease in speed, the total range of variation—or fluctuation due to F is the sum of δ in both directions, or practically 2δ, as demonstrated in Par. 130.

N_1 = minimum value of N, for maximum load on engine. A load causing a lower speed is out of control of the governor.

N_2 = maximum value of N, for minimum load on engine. A limit is sometimes assumed at zero load.

N_M = mean of N_1 and N_2 (= $(N_2 + N_1)/2$).

v = coefficient of fluctuation of speed ($=(N_2 - N_1)/N$). This is sometimes called the *coefficient of speed regulation*, and by some writers is considered as the measure of sensitiveness. If v is too small, "it will react with the variation of turning effort within the cycle. This leads to a restless governor play known as 'hunting.'" This is also true of δ. The value of v depends upon governor proportions only.

α = the coefficient of speed fluctuation for the engine cycle dependent upon flywheel regulation, for any given load. The value of this coefficient depends upon the variation of turning effort and the kinetic energy of the flywheel.

W = weight of centrifugal weight in pounds. This may be a single weight or may be divided into two or more parts.

r = the distance in inches from the axis of rotation to the center of gravity of the weight W for any position. This should include, referred to the center of W, all parts having a centrifugal action. If subscripts 1 and 2 are used, they correspond to the same numbers applied to N.

M_C = centrifugal moment without friction or external resistance.

M_T = centripetal moment without friction or external resistance.

M_F = moment of the force F.

P = force on spring in pounds.

For other notation see diagrams.

121. Governor Types.—The names most commonly applied to governors used on modern heat engines are:

Centrifugal governors.

Inertia, or centrifugal-inertia governors.

Relay governors.

Safety, or over-speed governors.

These will be defined in a general way in Pars. 122 to 127. A mathematical treatment will be given centrifugal governors in Pars. 128 to 133, these being of the only type capable of satisfactory analysis.

Governors of the different types selected from practice will be illustrated in Pars. 135 to 139.

122. Centrifugal Governors.—In these governors the centrifugal force of revolving weights is balanced at a certain speed by centripetal force due to dead weights or springs, or both. Two simple forms are shown in

Fig. 232. In Fig. 232–*A*, if W is the weight and C the centrifugal force of one ball, the *centrifugal moment* about pivot x is Ch; the *centripetal moment* about the same point is Wr. These moments must be equal to be in equilibrium, so for some definite speed.

$$Ch = Wr.$$

Likewise for Fig. 233–*B*.

$$Cb = Pa$$

where P is the spring load balanced by one weight. A more extended treatment will presently be given.

Any change in the load on the engine tends to change the speed; this changes the centrifugal force of the revolving weights, causing it to balance the centripetal force in a new position, at a slightly different speed. This new position adjusts the valve gear (including the mechanism of the independent method) to the new load.

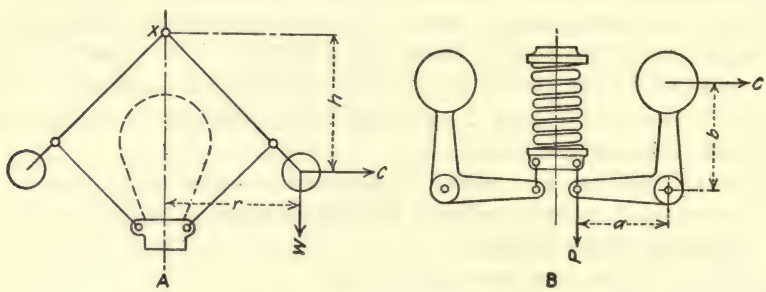


Fig. 232.

Stability.—A governor is *stable* if a change of position necessitates a change of speed in order to maintain equilibrium, an increase of speed always being accompanied by an increase of centrifugal moment, which, in turn, involves an increase in the radius of rotation of the revolving weights.

Isochronism.—In an *isochronous* governor the centrifugal and centripetal forces are in equilibrium for all positions of the governor weights at the same speed. If the load changes, the slightest hint at change of speed causes the revolving weights to fly to a new position (if the external resistance to be overcome is relatively small), adjusting the valve gear to suit the new load, at the original speed. On account of friction, strict isochronism is not possible, and a governor theoretically designed for such is usually unsatisfactory and unreliable. A certain amount of stability is desirable even for service requiring the closest regulation.

Isochronism is the limit of stability. If this limit is passed an engine would run faster with heavy than with light load. One engine builder states that their governor has regulated satisfactorily when so adjusted; but it is of the centrifugal-inertia type and supplied with a dashpot, without which such adjustment would be impossible.

A *conical pendulum* governor is shown in diagram in Fig. 232–*A*. This is sometimes called the Watt governor. If a central weight is added as shown dotted, it is called a *loaded* governor, or *Porter* governor. The pendulum governor must always be a vertical-spindle governor. Sometimes a spring is used instead of the central weight, but the gravity effect of the revolving weights is still an important factor, so a vertical spindle must be used.

A *spring governor* is shown in diagram in Fig. 232–*B*. This particular form is sometimes known as the Hartnell, or Wilson Hartnell governor. The gravity effect of the weights is small compared with the spring load and this type may be placed in any position. In fact, when in a horizontal position there is absolutely no gravity effect, while for the vertical position there is a slight gravity moment except when the weights are exactly in line vertically with the pivots. There are different forms of spring governors, some of which will be shown in later paragraphs.

Shaft, or *flywheel* governors are located on the engine shaft, and must always be spring governors. They may have either one or two weights.

123. Inertia, or Centrifugal-inertia Governors.—About the earliest example of this type to be practically applied was the Rites governor, which has been adopted by various builders and modified in some cases to suit their requirements. They are usually, though not always, shaft governors.

Revolving masses tend to maintain a uniform speed unless acted upon by some external force. If a weight is caused to revolve around a governor shaft, being connected with the shaft by a system of linkage in such a way that a change of angular relation between shaft and weight operates upon the valve gear, it is obvious that any change in speed of the shaft will, due to the inertia of the weight, effect changes of adjustment. As force is the product of mass and acceleration, the ability of the revolving weight to overcome the resistance due to the valve gear and change the position of the gear depends upon its weight and the time in which the change of angular relation between shaft and weight occurs.

As the inertia weight fixes no speed limits, but acts only upon sudden change of speed, it is always used in conjunction with a centrifugal governor.

The principle of the Rites inertia governor may be illustrated by Fig. 233. A bar containing a weight at each end is pivoted on the governor wheel at A. The eccentric rod is attached at B and swings through the arc of a circle as the governor makes adjustments. The right end of the weight is heavier than the other, being practically in gravity balance about the pivot A. The center of gravity is therefore at F As the wheel revolves, the centrifugal force C of the entire weight produces a centrifugal moment Cb about A. A spring connects the fixed point E on the wheel with point D on the weight, producing a centripetal moment Pa in an opposite direction (clockwise) to Cb, exactly as in the centrifugal governor described in Par. 122, and at a certain speed:

$$Cb = Pa.$$

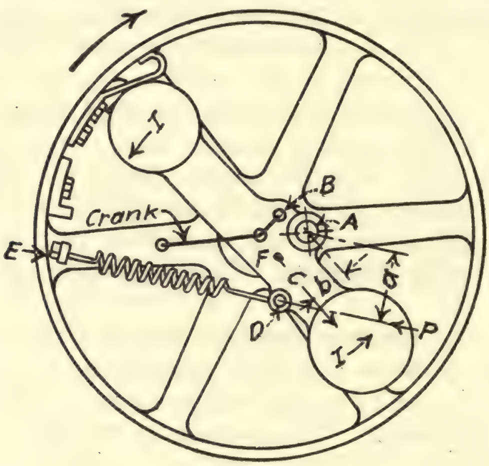


Fig. 233.

These forces are not great enough in some governors of this type to overcome the resistance of the valve gear without too great speed variation, but the necessary force is furnished by inertia I of the weight. If the wheel is turning clockwise as shown by the arrow, and should suddenly increase in speed as load is thrown off the engine, the wheel would plunge ahead. The weight, due to its inertia, would tend to revolve uniformly, the inertia I acting counterclockwise as indicated in Fig. 233. The increase in centrifugal force due to increase in speed would tend to move the weight in the same way about pivot A. Should the wheel decrease in speed, both inertia and centrifugal force would act to move the weight in a clockwise direction relative to the wheel.

There is some inertia effect at some positions in many centrifugal gov-

ernors, especially of the shaft-governor type. When present the *work capacity* of the governor is determined by the sum of the centrifugal and inertia effects if they are in the same direction, and by their difference if opposed. This may be illustrated by Fig. 234. Assume a wheel to be rotating in the direction R (clockwise) shown by the full-line arrow. Let a weight be pivoted at x, and assume that the centrifugal force C of the weight is just balanced by the spring, holding the weight in the position shown, at a certain speed. Now assume the wheel to suddenly increase in speed. The centrifugal force of the weight will be increased; the weight will fly out until the tension of the spring balances it in a new position. During the sudden change of speed the weight "hung back" due to its inertia, exerting the force I. While the centrifugal force exerted a positive or clockwise moment about pivot x, the inertia exerted a

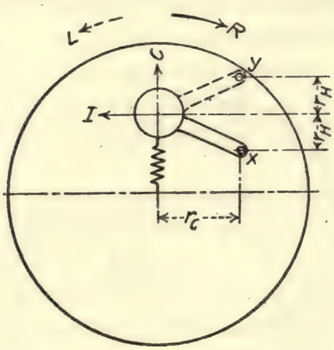


FIG. 234.

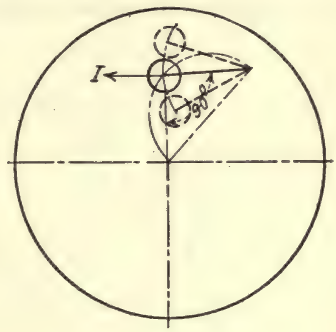


FIG. 235.

negative or counterclockwise moment. The total moment is then:

$$M = Cr_C - Ir_H$$

If the centrifugal moment is greater the weight would go outward, making the proper adjustment of the valve gear; but if the inertia moment is greater the reverse would be true.

Suppose the governor weight to be pivoted at y as shown dotted. This causes the moment of I to act in the same direction as that of C; then:

$$M = Cr_C + Ir_H$$

If the wheel turns counterclockwise the moments are in the same direction for the full-line position of the weight, and opposed for the dotted position. It is clear that the effect of I will be zero when the lines connecting weight center with wheel center and with pivot form an angle of 90 degrees as shown in Fig. 235. It is known from geometry that in

this position the weight center will fall on a circle whose diameter is a line joining the wheel center and pivot. Then from what has preceded, if the effect of inertia and centrifugal force are to work together in gear adjustments, the weight should always remain inside the circle for clockwise motion of the wheel, and outside the circle for counterclockwise motion. This is shown in Fig. 236.

In some governors the inertia effect is nearly negligible and such are known as centrifugal governors. If the inertia effect is great they are called inertia, of centrifugal-inertia governors.

Inertia governors act quickly and have been found to give exceptionally good regulation under widely varying and suddenly changing loads, although there are some prominent engine builders who do not favor much inertia effect. It is obvious that when inertia is present in any governor, its moment must be of the same sign as the centrifugal moment to obtain the best results. However, all governors are not so built.

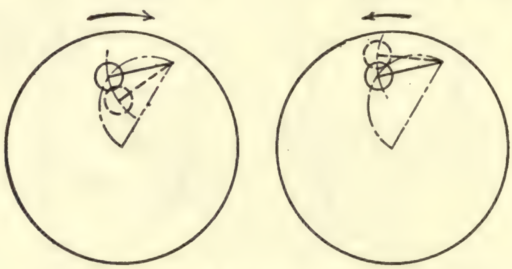


Fig. 236.

Due to the impossibility of predicting the relative angular acceleration of shaft and inertia weight during change of load, the design of inertia governors is dependent upon the results of practice, and no attempt will be made to treat it mathematically.

124. Relay Governors.—When considerable force is required to make adjustments of the valve gear a hydraulic cylinder, usually operated under oil pressure, is employed. The controlling valve of the oil cylinder is a small balanced piston valve requiring a very small force to move it; this in turn is controlled by a governor of ordinary capacity.

125. Safety, or Over-speed Governors.—These are auxiliary governors used in addition to the regular governor, more usually on the steam turbine. They consist of weights and spring so proportioned that at a certain excess of speed the weight will overcome the spring tension and operate a trigger, release a heavy weight or strong spring which shuts the throttle valve and stops the turbine. In some cases the relay principle

is used, the trip mechanism admitting or releasing steam from an auxiliary cylinder whose piston is connected with the valve. The safety governor is a simple centrifugal governor and is adjustable.

126. Dashpots.—Under certain load conditions, especially with extremely sensitive governors, there is a tendency toward a restless movement of the governor weights, sometimes periodic and sometimes accompanying load changes. This action is called *hunting*, and is sometimes prevented by the use of *dashpots*. A dashpot consists of a cylinder containing a fluid such as oil, and supplied with a piston which allows the fluid to pass between it and the cylinder from one side of the piston to the other as displacement occurs. The piston rod is attached to some moving part such as the governor-sleeve crosshead, and yields readily under gradual changes, but offers considerable resistance to a sudden change. Sometimes less clearance is allowed between the piston and cylinder walls, the connection from one side of the piston to the other being through a small pipe; a valve is placed in the pipe so that the resistance may be adjusted.

Dashpots may be seen on some of the governors illustrated in later paragraphs.

127. Speed Adjustments.—The speed of a governor, and consequently the speed of the engine, may be changed by changing the weight (except in the simple Watt governor) or spring. With some governors this may be done while the engine is running. Where several engines are direct-connected to alternators running in parallel, small motors operated from the switch-board—sometimes automatically—are in some cases attached to the governors in such a way that the tension of the spring may be varied and the alternators brought into synchronism.

128. Speed Fluctuation.—By mean speed, the speed at rated load is usually meant, and this is usually assumed to be:

$$N_M = \frac{N_2 + N_1}{2} \tag{1}$$

in which N_2 and N_1 are maximum and minimum r.p.m. respectively, the engine is designed to run under the control of the governor—possibly from no load to maximum load.

The coefficient of the fluctuation of speed is:

$$v = \frac{N_2 - N_1}{N_M} = \frac{2(N_2 - N_1)}{N_2 + N_1} \tag{2}$$

If v is assumed, we have from (1) and (2):

$$N_1 = \left(1 - \frac{v}{2}\right)N_M \tag{3}$$

and

$$N_2 = \left(1 + \frac{v}{2}\right)N_M \tag{4}$$

If a governor is designed with this range of speed between maximum and zero load, it is not at all certain, or perhaps even likely, that at the position the governor will assume in running the speed N_M the gear will be adjusted to carry the rated load; but this is not essential, and Equations (3) and (4) are convenient in the design of spring governors.

Speed fluctuation and the coefficient of speed fluctuation depend upon the construction of the governor and the relation of the forces and not upon the magnitude; they are independent of friction. It is true that the actual fluctuation may be greater than that given by (2), due to friction, but this is considered in Pars. 129 and 133.

129. Speed Variation.—The centrifugal force of a revolving weight in lb. is:

$$C = \frac{WrN^2}{35,230} \tag{5}$$

where W is the weight in lb., N the r.p.m. and r the radius of revolution in inches, of the center of gravity of the weight.

At a change of load the speed changes, increasing for decrease of load and vice versa. However, before the governor can change, a certain amount of external resistance F, due to friction and weight of valve-gear parts, has to be overcome, so the speed changes an amount ΔN before C changes enough from normal to overcome the resistance and move the governor. The change of centrifugal force is ΔC, and as this change is made before the governor makes its adjustment, it occurs at a constant value of r. Then, for this change, it may be seen that in (5), C varies directly as N^2. Let the speed variation be denoted by δ; then:

$$\delta = \frac{\Delta N}{N} \tag{6}$$

The ratio of corresponding values of centrifugal force is, for an *increase* in speed:

$$\epsilon = \frac{\Delta C}{C} = \frac{(N + \Delta N^2)^2 - N^2}{N^2} = \frac{2N \cdot \Delta N + \Delta n^2}{N^2}$$

$$= \frac{2\Delta N}{N} + \left(\frac{\Delta N}{N}\right)^2 = 2\delta + \delta^2 \tag{7}$$

Completing the quadratic and solving for δ gives:

$$\delta = \sqrt{\epsilon + 1} - 1 \tag{8}$$

Or, approximately:

$$\delta = \frac{\epsilon}{2} + 1 - 1 = \frac{\epsilon}{2} \tag{9}$$

This may also be obtained from (7) by neglecting the minute quantity.

For a decrease in speed:

$$\epsilon = \frac{\Delta C}{C} = \frac{N^2 - (N - \Delta N)^2}{N^2} = \frac{2\Delta N}{N} - \left(\frac{\Delta N}{N}\right)^2 \tag{10}$$
$$= 2\delta - \delta^2$$

From which:

$$\delta = 1 - \sqrt{1 - \epsilon} \tag{11}$$

Or, approximately as before:

$$\delta = \frac{\epsilon}{2}$$

Then approximately:

$$\epsilon = 2\delta \tag{12}$$

In practice ϵ ranges from 0.02 to 0.05. Taking the largest value:

From (7), $\epsilon = 0.10 + 0.0025 = 0.1025$
From (10), $\epsilon = 0.10 - 0.0025 = 0.0975$
From (12), $\epsilon = 0.10$.

It is obvious that (12) is as accurate as practice will warrant.

The total range of variation of speed without adjustment is obviously 2δ.

130. General Equations for Centrifugal Governors.—For perfect equilibrium at any speed the centrifugal moment must equal the centripetal moment; or, neglecting friction:

$$M_C = M_T \tag{13}$$

At the instant of change of relative position of the governor parts, the moment of the external resistance F must equal the fraction ϵ of the moment of the centrifugal force C before the change of load begins: or:

$$\epsilon M_C = M_F \tag{14}$$

From (13) and (14):

$$\epsilon M_T = M_F \tag{15}$$

Equations (13) to (15) are general and apply to all centrifugal governors. From (15) it is obvious that if ϵ is to be constant, M_F/M_T must be constant. But M_T must increase for light engine loads. If the external resistance F is constant, the moment arm of F must increase relative to the moment arm of the centripetal force (or centrifugal force). Usually ϵ, and consequently δ, varies with change of position when F is uniform,

but sometimes a compensating device is employed as in the Jahns governor described by means of Fig. 251.

131. Equations for Conical Governors.—As all general principles were deduced from some specific case, so it is necessary to explain them in the same way. For the purpose the equation of centrifugal and centripetal moments was illustrated by the simple sketches of Fig. 232. The equation is then given a general form by (13).

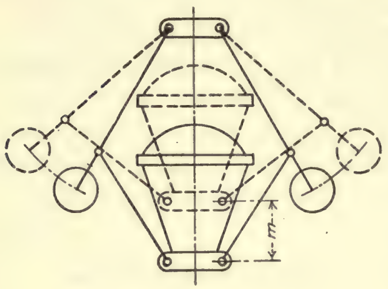


FIG. 237.

It is quite common in elementary works dealing with governors to substitute the value of C in the equation for the simple Watt governor with arms pivoted at the center line of spindle (Fig. 232-A), and show thereby that the r.p.m. is proportional inversely to the square root of the *height* h. As this often clings to the mind as a general rule, such a treatment has been purposely avoided and a more general method employed. This method covers the simple type, but includes factors which modify results considerably when this type is not strictly adhered to, as it seldom is.

Fig. 237 shows a conical governor with central weight in two extreme positions; the full lines for maximum load and lowest speed, the dotted lines for highest speed and lightest load (if this is taken as zero load it is an imaginary position, as the steam or fuel supply would be entirely cut off). This is known as the Porter, or loaded governor. Fig. 237 is made to include everything for the most general treatment of this type. Should the center weight not be used, the symbol for this may be taken as zero in the final equations. When this weight is used it adds to the centripetal force only. To offset this the centrifugal force must be increased to keep the governor parts in the same relative positions. As increasing the weight of the revolving weights affects centripetal and centrifugal forces equally, it may be seen from (5) that the speed must then be increased. The revolving weights are usually made smaller when the center weight is used.

In deriving the equations one revolving weight only need be considered. The component W_O of the center weight W_C acting on one side may be determined, also the component W_F of external resistance F.

Fig. 238 is a diagram of one side of a pendulum governor containing notation used in the equations. The center weight is omitted from the

sketch but included in the calculations. At the right are shown force diagrams for finding components W_O and W_F of the center weight and resistance respectively. The arrows indicate the direction of the forces C, W and $W_C \pm F$. The last shows that the effective centripetal force at the sleeve is increased as the governor rises, requiring a centrifugal force greater than C (by the amount $\Delta C = \epsilon C$) to overcome it before the governor begins to rise; and on descending, the effective centripetal force is reduced by the amount F, requiring a centrifugal force less than C (by the amount ϵC) before the weights can begin to descend (see Par. 129).

Determination of Weights.—In practical design it is necessary to first determine the weights required to keep the speed variation δ $(= \epsilon/2)$

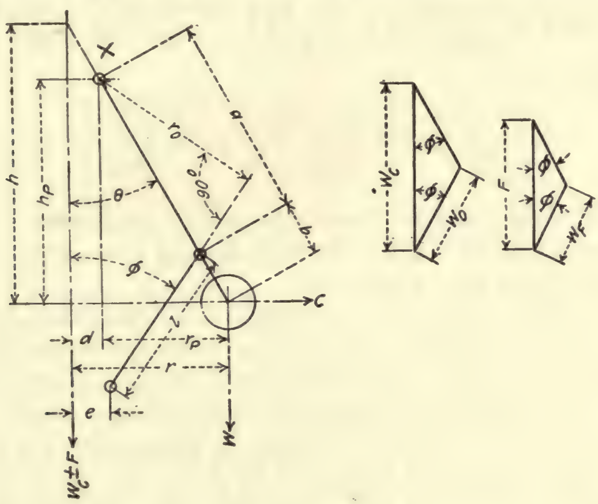


FIG. 238.

within the desired limit, then the linkage may be arranged to give room for these weights.

The centripetal moment is due to both central and revolving weights; then from Fig. 238, for one side of the governor, taking moments about x, the point of suspension of the arm:

$$M_T = Wr_P + W_O r_O \tag{16}$$

Also:

$$M_F = W_F r_O \tag{17}$$

Substituting (16) and (17) in (15) gives:

$$\epsilon(Wr_P + W_O r_O) = W_F r_O \tag{18}$$

Equation (18) may be used for solving governor problems by trial and error; but while the design of a governor is largely a drawing board

problem, the work may be shortened by making a number of substitutions in (18). Referring to Fig. 238, let:

$$\frac{a+b}{a} = q \quad \text{and} \quad \frac{W_C}{W} = k.$$

From construction:

$$r_P = h_P \tan\theta \text{ and } r_O = h_P\left(1 + \frac{\tan\theta}{\tan\phi}\right)\frac{\sin\phi}{q}.$$

From the force diagrams:

$$W_O = \frac{W_C}{2\cos\phi} = \frac{kW}{2\cos\phi} \quad \text{and} \quad W_F = \frac{F}{2\cos\phi}.$$

Substituting these values in (18) gives:

$$\epsilon W\left[\tan\theta + \frac{k\tan\phi}{2q}\left(1 + \frac{\tan\theta}{\tan\phi}\right)\right] = \frac{F\tan\phi}{2q}\left(1 + \frac{\tan\theta}{\tan\phi}\right)$$

or simplifying:

$$\frac{F}{\epsilon} = W\left[\frac{2q}{\frac{\tan\phi}{\tan\theta} + 1} + k\right] \tag{19}$$

from which W, F or ϵ may be found if the others are known or assumed.

In practice k may be taken from 10 to 15, and q ranges from 1 to 2, a common value being from 1.2 to 1.35.

If $\phi = \theta$ for all positions the equation is much simplified, but advantage is taken of a difference in the values to reduce the speed fluctuation (see Par. 128). The value of F depends upon the type of gear and size of engine. It may sometimes be determined on an engine already built by hooking on a spring balance and moving the parts. For a small or medium-sized Corliss engine it may be 3 or 4 lb. As previously stated, ϵ $(=2\delta)$ ranges from 0.02 to 0.05, depending upon the desired sensitiveness; the smaller the value of ϵ the more sensitive is the governor.

From given values of F and ϵ, it may be seen from (19) that W will have the greatest value when $\tan\phi/\tan\theta$ (or ϕ/θ) is a maximum. Then if ϵ is taken as the maximum desired value, W should be computed for the position when ϕ/θ is maximum.

Determination of Speed.—Solving for N in (5) gives:

$$N = 187.7\sqrt{\frac{C}{Wr}} \tag{20}$$

From (13), (16) and Fig. 238:

$$Ch_P = Wr_P + W_O r_O$$

from which:

$$C = \frac{Wr_P + W_O r_O}{h_P} \tag{21}$$

Observing that $r = h \tan \theta$, and substituting the values previously found in (21), and (21) in (20), gives:

$$N = 187.7 \sqrt{\frac{1 + \frac{k}{2q}\left(\frac{\tan \phi}{\tan \theta} + 1\right)}{h}} \tag{22}$$

It is obvious that N increases as h decreases; if $\tan \phi/\tan \theta$ decreases when h decreases and vice versa, it is obvious that the speed will change less for a given change of the governor. This means that to reduce speed fluctuation, ϕ/θ should have a maximum value when h is maximum, which is when the governor is in its lowest position (for all practical governors). It is in this position, then, that W should be determined from (19). To obtain this result, link l must be longer than a.

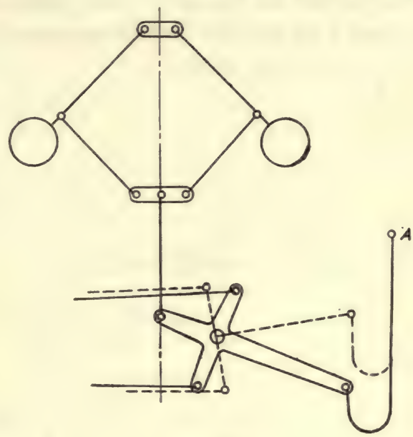


FIG. 239.

If a spring is added to the weight, or entirely takes the place of it, k will be variable, but equations (19) and (22) may still be used. If the spring is attached in any other way than to act on the sliding sleeve as does the center weight, the equation of Par. 130 should be used.

It is clear from (22) that if k is increased as the governor weights descend and decreased as they rise, N will vary less. In governors already built, a little closer regulation may be obtained by the addition of some compensating device. In Fig. 239, if an extension be added to the bell crank which operates the cam rods on a Corliss Engine, and a chain attached to this and to a fixed point A as shown, the chain will exert an upward pressure on the weight when the governor is in the lower position (shown dotted). The chain may lie on a flat support instead of being suspended from A, and in this way is more effective. The effect at the center weight support is inversely proportional to the length of lever arm,

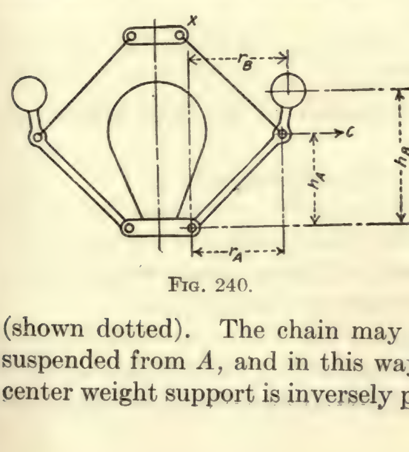


FIG. 240.

and this may be subtracted from W_C and the values of k found for the extreme positions and N found from (22).

Proll's governor is a modification of the pendulum governor having the weight on the lower arm, and so arranged that its effect is similar to the chain of Fig. 239. A diagram is shown in Fig. 240. The effect of the revolving weight may be referred to the juncture of the links and to the connection of the lower link with the sleeve, and their moments taken about x as for the Porter governor. The effect of the centrifugal force of the ball so referred is:

$$C = \frac{h_B}{h_A}\frac{Wr_BN^2}{35{,}230}.$$

Then:

$$M_C = \frac{h_Bh_P}{h_A}\cdot\frac{Wr_BN^2}{35{,}230} \tag{23}$$

The effective central weight is:

$$W_{CE} = W_C - 2W\frac{r_B - r_A}{r_A}.$$

Then:

$$M_T = W_{OE}r_O + \frac{r_Br_P}{r_A}W \tag{24}$$

And:

$$M_F = W_Fr_O$$

as before.

Speed variation is satisfactory if:

$$\frac{M_F}{M_T} = \epsilon \tag{25}$$

N may be determined by equating (23) and (24) and solving; or:

$$N = 187.7\sqrt{\frac{h_A}{Wh_Bh_Pr_B}(W_{OE}r_O + W\frac{r_Br_P}{r_A})} \tag{26}$$

W_{CE} and W_{OE} replace W_C and W_O respectively on the force diagram of Fig. 238.

By assuming $q = h_B/h_A$, W may be solved tentatively by (19). The value of k may be taken as for the Porter governor; the larger this is, the greater the effect of offsetting W_C in the high position of the governor, and the more nearly constant the speed. This governor is sometimes called an isochronous governor in manufacturers' catalogues.

132. Equations for Spring Governors.—Fig. 241 shows a spring governor which may be used to derive the equations. The weight of W is for both, or all of the revolving weights if a single spring is employed, as

is usually the case with governors of the type shown in Fig. 241. If separate springs are used as in some shaft governors, W may be for one weight, and F the part of the resistance to be handled by one weight. If two opposed weights are connected to a common spring, so that the deflection is double what it would be with a separate spring for each weight, the calculation for deflection and length must be for a spring one-half the length of the actual spring; the strength calculations are based upon one weight.

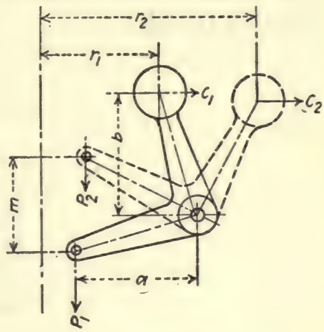


FIG. 241.

From (13):

$$Cb = Pa.$$

It is more general to assume F to act at a different lever arm from P; call this a_0 Then from (15):

$$\epsilon Pa = Fa_0.$$

From which:

$$P = \frac{F}{\epsilon} \cdot \frac{a}{a_0} \tag{27}$$

If the ratio a_0/a is constant, as it usually is, P should be calculated for the least spring tension in order that ϵ may not be less than the assumed value; then:

$$P_1 = \frac{F}{\epsilon_1} \frac{a_{01}}{a_1}.$$

Then for this position:

$$C_1 b_1 = P_1 a_1$$

or:

$$C_1 = \frac{P_1 a_1}{b_1} = \frac{W r_1 N_1{}^2}{35{,}230}.$$

We may find N_1 from (3) and N_2 from (4). Then solving for W gives:

$$W = \frac{35{,}230 P_1}{r_1 N_1{}^2} \cdot \frac{a_1}{b_1} = \frac{35{,}230}{r_1 N_1{}^2} \cdot \frac{F a_{01} a_1}{\epsilon a_1 b_1} \tag{28}$$

Also:

$$P_2 = C_2 \frac{b_2}{a_2} = \frac{W r_2 N_2{}^2}{35{,}230} \cdot \frac{b_2}{a_2} = P_1 \frac{a_1 b_2 r_2}{b_1 a_2 r_1} \left(\frac{N_2}{N_1}\right)^2 \tag{29}$$

By assuming F/ϵ constant in (27), adding subscripts 1 and 2 for extreme positions, then solving for F/ϵ and equating, we have:

$$\frac{a_{02}}{a_{01}} = \frac{P_2 a_2}{P_1 a_1} \tag{30}$$

If a_{01} was assumed in (27) when solving for P_1, a_{02} may be found. This relation provides constant speed variation δ for a constant value of F,

or a constant ratio F/ϵ. A special case will be treated under the Jahns governor illustrated in Fig. 251, Par. 135.

Springs.—Forces P_1 and P_2 give the minimum and maximum spring loads required for a desired speed fluctuation. It now remains to select a spring which will give these values with a change of deflection equal to m. For this purpose the diagram for helical springs in Fig. 242 is useful. The notation on the sketch is clear, all dimensions being in inches. In addition to this:

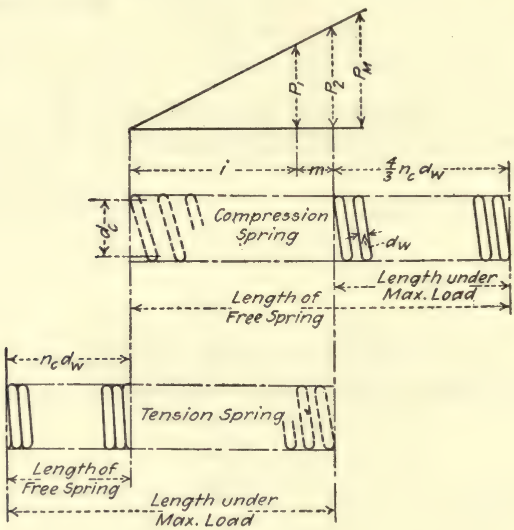


Fig. 242.

E_S = torsional modulus of elasticity of spring wire.
S = maximum allowable fiber stress in spring wire.
n = number of coils in the spring.
P_M = the maximum safe load in pounds.

The initial deflection i is required to produce load P_1, and the additional deflection m requires the load P_2. From Fig. 242, by similar triangles:

$$\frac{m + i}{i} = \frac{P_2}{P_1}.$$

From which:

$$i = \frac{m}{\frac{P_2}{P_1} - 1} \tag{31}$$

From a spring table, given in engineering handbooks, select a spring

with a safe load $P_S \overline{>} P_2$, and which will deflect the amount i with the load P_1, or $i + m$ with load P_2.

In the absence of spring tables the following formulas may be used:

$$P_M = \frac{0.392 S d_W^3}{d_C} \tag{32}$$

and:

$$i = \frac{8 d_C^3 n P_1}{E_S d_W^4} \tag{33}$$

An empirical formula for safe stress is:

$$\left.\begin{array}{l} S \overline{\gtrless} \dfrac{10{,}000}{d_W} + 30{,}000 \\ \text{Not to exceed } 100{,}000 \end{array}\right\} \tag{34}$$

Laminated springs are used in some governors, and while they are satisfactory and the theory is simple, it is not quite so easy to predict results as when helical springs are used. The formulas are based upon the bending of a triangular plate which has been cut up into leaves as shown in Fig. 243, in which the dimensions are in inches, and n the number of plates. The theoretical formula for maximum safe load is:

$$P_M = \frac{Snbt^2}{6l} \tag{35}$$

and for deflection:

$$i = \frac{6P_1 l^3}{Enbt} \tag{36}$$

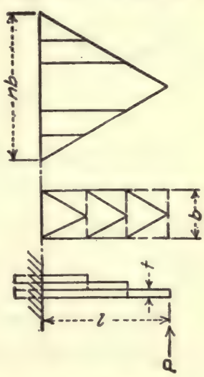


FIG. 243.

where E is the direct modulus of elasticity. The value of E for steel is 29 to 30 million, but Goodman says that 26 million should be used in calculation; the difference is probably from deflection due to shear, and the fact that the small central plate is left out. In practical springs the ends of the leaves are squared as shown dotted. There is often more than one full-length leaf, and when the number of such leaves is one-fourth the entire number, 5.5 is sometimes used instead of 6 in (36). Friction also alters the deflection of laminated springs; if well oiled the discrepancy is not great.

Sometimes a plate spring is used which curves nearly around the wheel as in Fig. 244. The application of force P as shown by the arrow produces a bending moment at any point in the spring equal to Px. If the free end of the spring were guided in the direction of application as indicated, the bending moment at a would be zero. It does not deflect in this way however, and the elastic curve is very complicated. The spring may be

checked for strength by (35), using values of x instead of l; rough deflection calculation may be made, but the design of such springs must be largely empirical.

Condition of Stability.—For any position of the governor:

$$P = \frac{b}{a} \cdot \frac{WrN^2}{35{,}230} = rN^2 \times \text{constant}$$

or:

$$N = \sqrt{\frac{P}{r}} \times \text{constant.}$$

If P/r is constant the governor is isochronous.

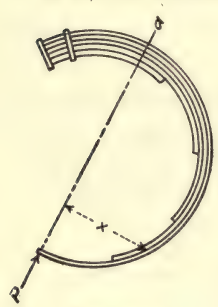


Fig. 244.

If P varies in the same direction as r but at a greater rate, N varies in the same direction as P and the governor is stable.

If P varies in the same direction as r but at a lesser rate, N varies in the opposite direction from P and the governor is unstable.

Some governors are so constructed that the moment arms for centrifugal and centripetal force vary in different ways throughout the range of governor action, and it is often wise to plot a curve showing these relations.

133. Practical Considerations.—To obtain the best results in practice, the speed fluctuation of the governor should be greater than the speed fluctuation of the cycle allowed by the flywheel; otherwise there will be governor action or *hunting* during every revolution of the wheel. This is also true of the fluctuation of speed due to change of load, or 2δ.

The speed fluctuation should never be less than twice the speed variation and should usually be greater. These relations may be expressed thus:

$$\left.\begin{aligned} v &> \alpha \\ 2\delta &> \alpha \\ v &\gtreqless 2\delta \end{aligned}\right\} \qquad (37)$$

Weight of Links.—When heavy weights or strong springs are employed, the weight of arms and links may usually be neglected. It is more accurate to include them and this may be easily done. In getting their centripetal moment, their weight may be assumed concentrated at their center of gravity but for their centrifugal moment this would not be correct. For the upper link, dividing the link into a number of parts, the centrifugal moment is:

$$M_C = \Sigma \frac{wrN^2}{35{,}230} \cdot a \qquad (38)$$

where w is the weight of one part (see Fig. 245). For the lower link, the effect must be transferred to the upper link at the juncture of the two; then:

$$M_C = \Sigma \frac{wrN^2}{35{,}230} \cdot \frac{x}{l} \cdot b \tag{39}$$

Gravity Balance.—As far as possible, all parts of a governor should be in gravity balance for all parts of the revolution. This applies especially to governors with horizontal shafts.

Selection of Constants of Regulation.—Goodman says: "If a governor be required to work over a very wide range of power, such as all the load suddenly thrown off, a sensitive, almost isochronous governor with dash-pots gives the best results; but if very fine governing be required over small variations of load, a slightly less sensitive governor without dash-pots will be the best." He further says: "Nearly all governor failures are due to their lack of power." The word power as used here is a misnomer, meaning the product Fm, the work capacity with a given speed variation. It is therefore safe to assume F amply large; on the other hand, if the actual value falls too far short of the assumed value, δ is reduced, and the relations given by (37) may not obtain.

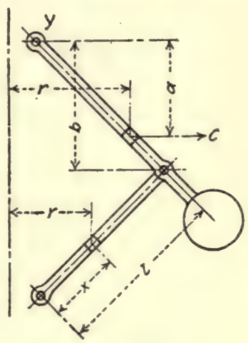


FIG. 245.

Machine Design.—Failure of a governor due to broken parts or loosened bolts is exceedingly serious, and great care should be exercised in design and construction. Friction of joints and bearings should be eliminated as much as possible and ample lubrication provided for. All governors should be thoroughly tested and adjusted before leaving the factory.

ILLUSTRATIONS FROM PRACTICE

134. Weight-loaded Conical Governors.—A good example of the loaded governor of the Porter type is given in Fig. 246, which is used on the Bass Corliss engine built by the Bass Foundry and Machine Co., Fort Wayne, Ind. In this governor, part of the center weight is hollow to receive shot for adjusting the weight. A sliding weight is also placed on the bell crank to allow for further adjustment. A dashpot is furnished to prevent wide fluctuation when the load is changed.

A safety pawl is provided which prevents the governor falling to its lowest position when starting the engine. It is so constructed that when the engine gains sufficient speed to lift the sleeve, the pawl falls by grav-

ity; then should the governor belt break or slip, the governor upon coming to rest may sink to its lowest position, in which it engages the safety cams and prevents the engine from taking steam. The so-called safety collar which must be turned to the safety position by hand after the engine has started, is more convenient when starting the engine, but is only a partial safety device; neglect to set the collar may result in disaster.

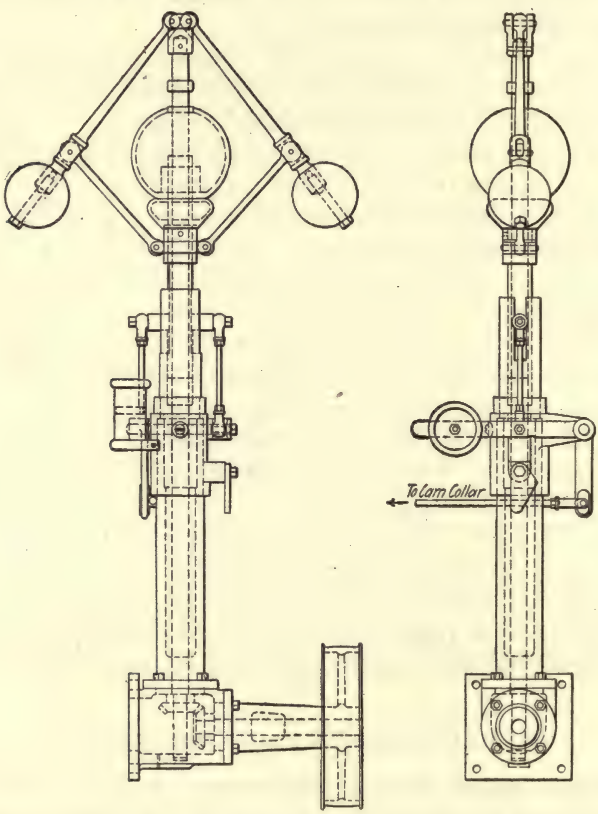
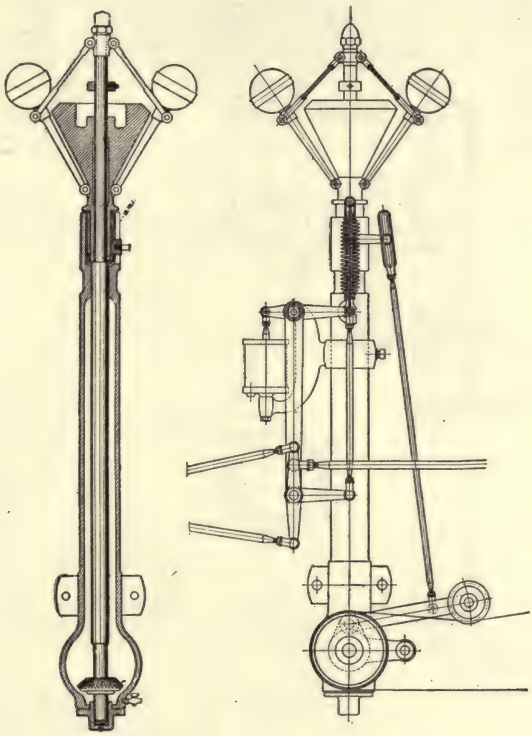


Fig. 246.—Bass-Corliss governor.

The Bass engine is sometimes fitted with an idler pulley running on the governor belt. This connects with the pawl, and in case of belt accident, falls, pulling the pawl out of position. This has the advantage of easy starting and stopping. This device is shown on a governor about to be described.

A governor of the Proll type, built by the Nordberg Manufacturing Co. for use on their Corliss engines is shown in section in Fig. 247-*A*, and

a side elevation in Fig. 247-*B*. Fig. 247-*B* is arranged for a simple engine of moderate size having the Nordberg long-range cut-off gear. It is provided with a dashpot and has an idler pulley which operates the safety device. In a portion of the connection between the governor crosshead and the double bell crank, a spring may be seen holding two parts together. Between these two parts is a separating strut with a projecting lever containing a pin which slides in a slot of the rod which is fastened



A *B*

FIG. 247.—Nordberg governor.

to the idler arm. Should the belt fail or run off, the idler would drop; the end of the slot would strike the pin, remove the strut and allow the spring to raise the bell-crank lever, which for this type of gear prevents steam admission, and the engine would stop.

A Nordberg governor of the Porter type is shown in Fig. 248. This is employed when the engine is used to drive an alternator in parallel. It is also designed for the long-range gear, but the details of design are different. A small motor, shown at the right, is provided; this is operated

from the switch board, and by driving a lead screw, shifts a counterweight, varying the speed and bringing the engine into synchronism. A spring is interposed between the gear-operating lever and the dashpot, and the safety latch releases a weight if the belt breaks, which forces the levers in the safety position.

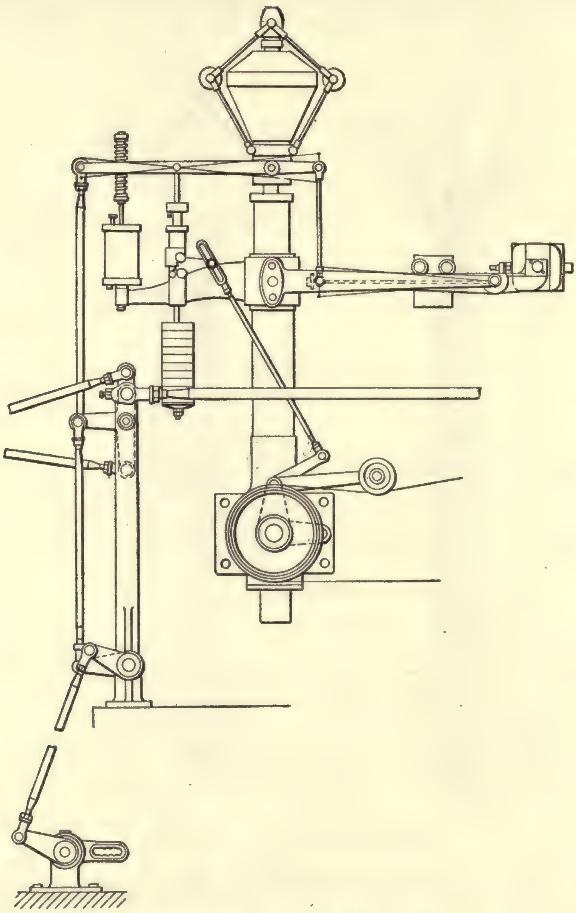


FIG. 248.—Nordberg governor.

135. Spring-loaded Flyball Governors.—Fig. 249 shows the simple spring governor used on the smaller sizes of "Economy" turbines built by the Kerr Turbine Co., Wellsville, N. Y. It is located on the end of the turbine shaft, its connection with the governor throttle valve being shown in Fig. 250. The governor weights 408 are of semi-annular cross-

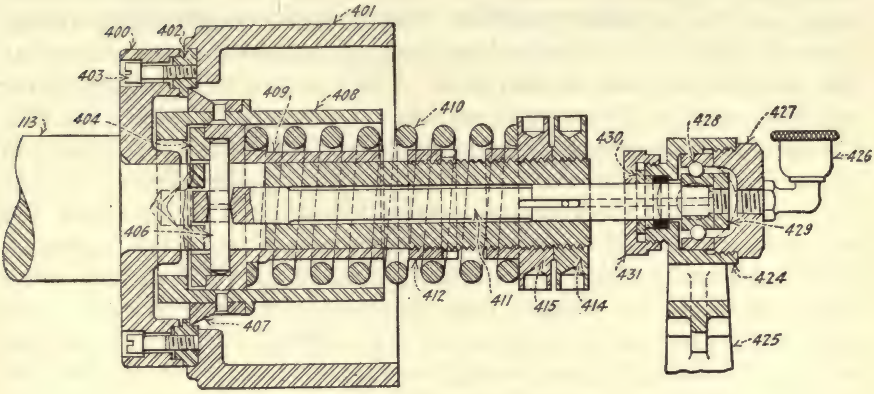


FIG. 249.—Kerr turbine governor.

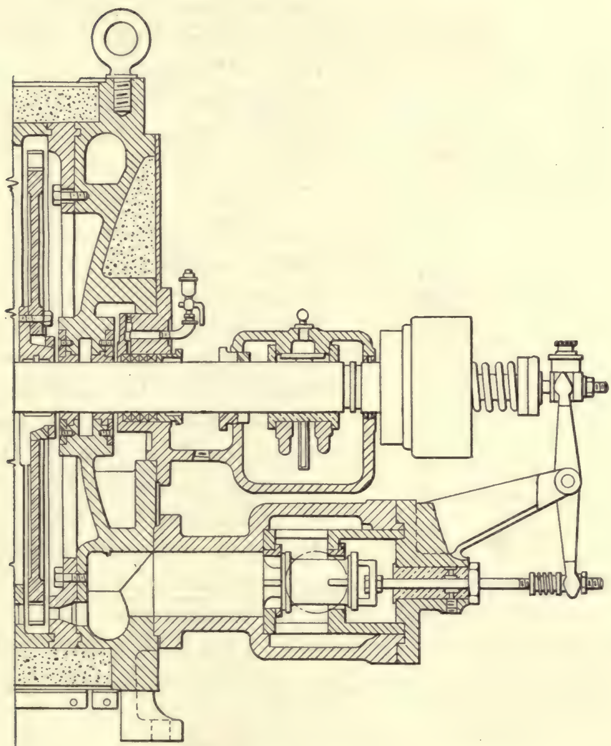


FIG. 250.—Kerr governor on turbine.

section, fitted with hardened tool-steel knife edges 402 which are securely fitted into the governor cup 400. The governor weights have rolling contacts shown by the dotted lines, and act against the governor collar 404, transmitting their motion to it. The governor collar in turn transmits its motion to the governor spindle 411 through the spindle pin 406. The spindle has a sliding clearance in the reamed hole provided for it in the end of the turbine shaft. A governor spring 410 adjusted by nut 415 works against the governor weights through the spring sleeve 409 and the governor collar. A stop nut 412 screwed up to such a position that the governor spring sleeve comes out against it at its extreme outer travel, prevents the weights from throwing out of their seats. The guard 401 is fastened to the governor cup, serving to protect the weights and supplementing the stop nut in preventing over-travel of the governor weights. Any movement of the governor is transmitted to the governor valve through the pivot block 424 and the governor lever 425. The thrust comes on a race of ball bearings 428. Lock nut 427 holds the ball race in place. Lubrication is supplied by an oil cup 426. Small cord packing keeps the oil from running out along the spindle. This packing is kept in place by gland 430 and nut 431.

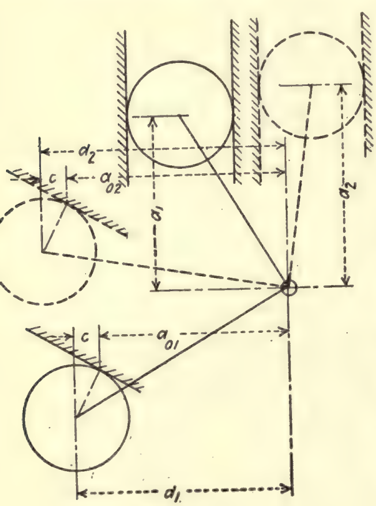


Fig. 251.

In some governors of the Jahns type, the movement of the sliding sleeve is effected by bell cranks engaging with the sleeves through roller-bearing connections, with races set at such angles that the movement of the weights due to centrifugal force is transmitted in such a manner that the force F exerted by the sleeve is practically constant for each position of its travel. This was mentioned in Par. 130, and may be illustrated by Fig. 251 which is somewhat exaggerated for the purpose. Using the same notation as in Par. 132, remembering that lever arm a is coincident with b, and taking d as the apparent arm for F, it is clear that:

$$\frac{d_2}{a_2} = \frac{d_1}{a_1}.$$

But due to the slanting roller race, arm a_0 is shorter for all positions

by the constant amount c. It is then clear that the ratio of arm a_0 to arm a is greater in position 2 than in position 1; or:

$$\frac{d_2 - c}{a_2} > \frac{d_1 - c}{a_1}$$

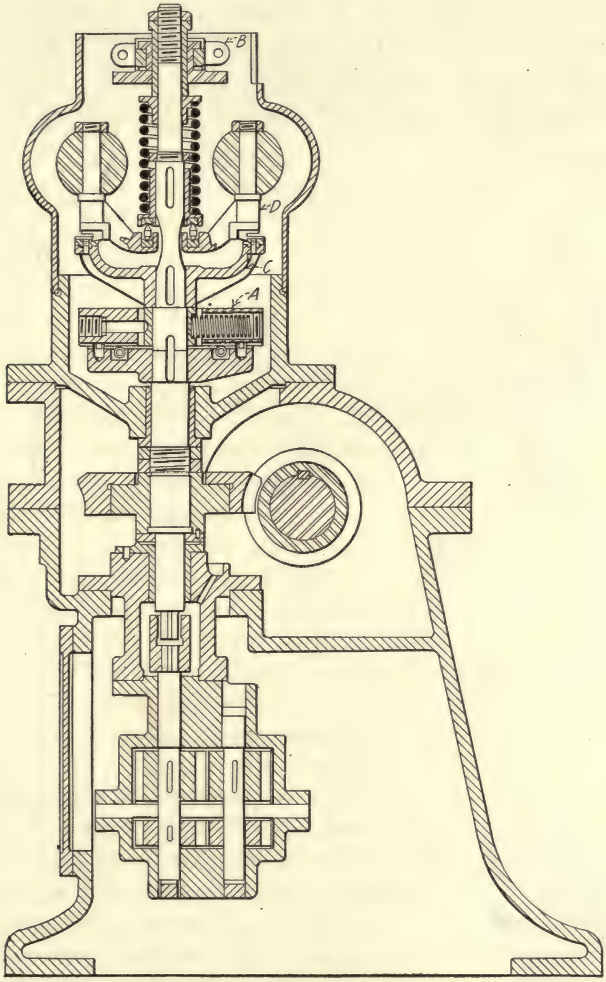


FIG. 252.—Kerr turbine governor.

To have ϵ and F constant, or ϵ vary as F, the following relation should hold:

$$\frac{a_1(d_2 - c)}{a_2(d_1 - c)} = \frac{P_2}{P_1}$$

or:
$$\frac{a_{02}}{a_{01}} = \frac{P_2 a_2}{P_1 a_1} \tag{40}$$
which is the same as (30).

This governor is used to quite an extent on steam turbines and internal-combustion engines.

Fig. 252 shows the governor used on the larger turbines built by the Kerr Turbine Co., and is used in connection with an oil relay system of governing. It is driven from the turbine shaft by spiral gears. Below the speed governor the safety governor A is located; this will be de-

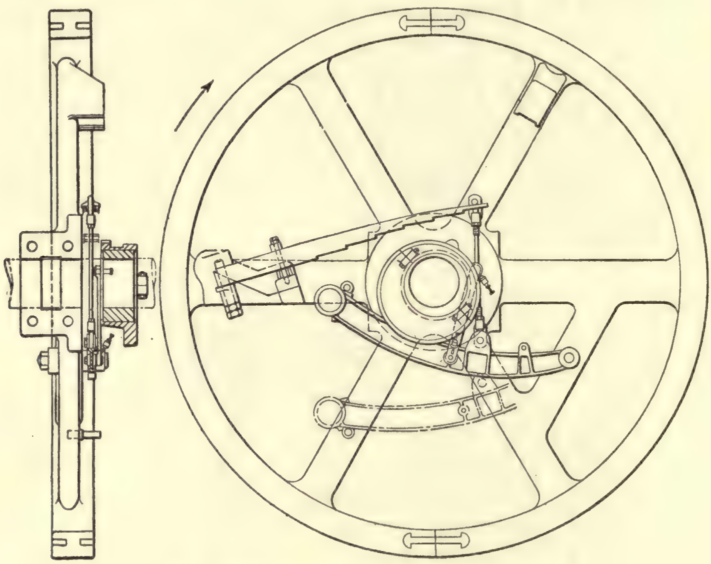
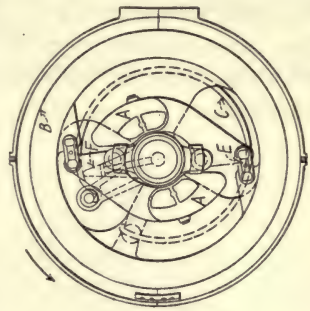


FIG. 253.—McIntosh & Seymour governor.

scribed later. At the lower end of the governor is a gear pump which supplies the oil relay cylinder and the lubricating system. The supply line is connected to the pump discharge line ahead of a 30-lb. relief valve. The oil supplied to the bearings is taken from between the discharge from the 30-lb. relief valve and a 3-lb. relief valve. This provides 30 lb. pressure for the relay cylinder and 3 lb. for the lubricating system.

Except in detail of design this governor is practically the same as the Kerr governor described at length earlier in this paragraph. The pivot block B which operates the governor lever is at the top of the spindle. The governor yoke C fitted with hardened tool-steel knife-edge blocks carries on it two governor weight-arms D. These compress the spring by means of two pivot struts.

136. Centrifugal Shaft Governor.—Fig. 253 shows the shaft governor built by the McIntosh and Seymour Corporation, Auburn, N. Y. and used on their steam engines. It is of the centrifugal type. To quote from Bulletin No. 52 of their Type *F* engines: "The governor is so designed that the combined inertia effect of the different parts due to change of speed, while it acts in the proper direction to assist the governor, is very slight, as a considerable inertia effect has proved to result in instability and unsatisfactory governing." It will be noticed that this is contrary to the views of other builders who use inertia governors.



The governing eccentric is placed upon a fixed eccentric in such a way that the radius of the eccentric path and the angular position of the eccentric relative to the crank are both changed as the governor weight changes position. A leaf spring is used to balance the centrifugal force of the weight at the desired speed.

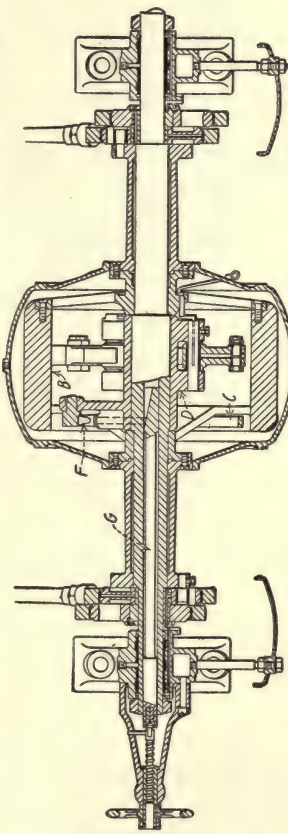


FIG. 254.—Lentz governor.

137. Inertia, or Centrifugal-inertia Governors.—Fig. 254 shows the centrifugal-inertia governor of the Lentz engine built by the Erie City Iron Works, Erie, Pa. The hub *D* is keyed to the lay shaft which is geared to the engine shaft. The centrifugal weights *A* are pivoted to this hub. They are also connected by links *E* to the inertia weight *B*. The spring *C* connects the inertia weight with an arm on hub *D*. The governor as shown in Fig. 254 turns counterclockwise. Should the load decrease and the engine speed up, the hub *D* tends to accelerate in a counterclockwise direction. Centrifugal force tends to throw the weights out; the inertia weight lags and has the same action on the weight levers.

A hand wheel on the end of the shaft moves the rod G, on the end of which is a taper. This raises or lowers the rod F which presses upon the spring near its support and changes the tension. Thus the speed of the engine may be changed while it is running. This may be operated by a small motor from the switchboard when the engine is driving an alternator in parallel.

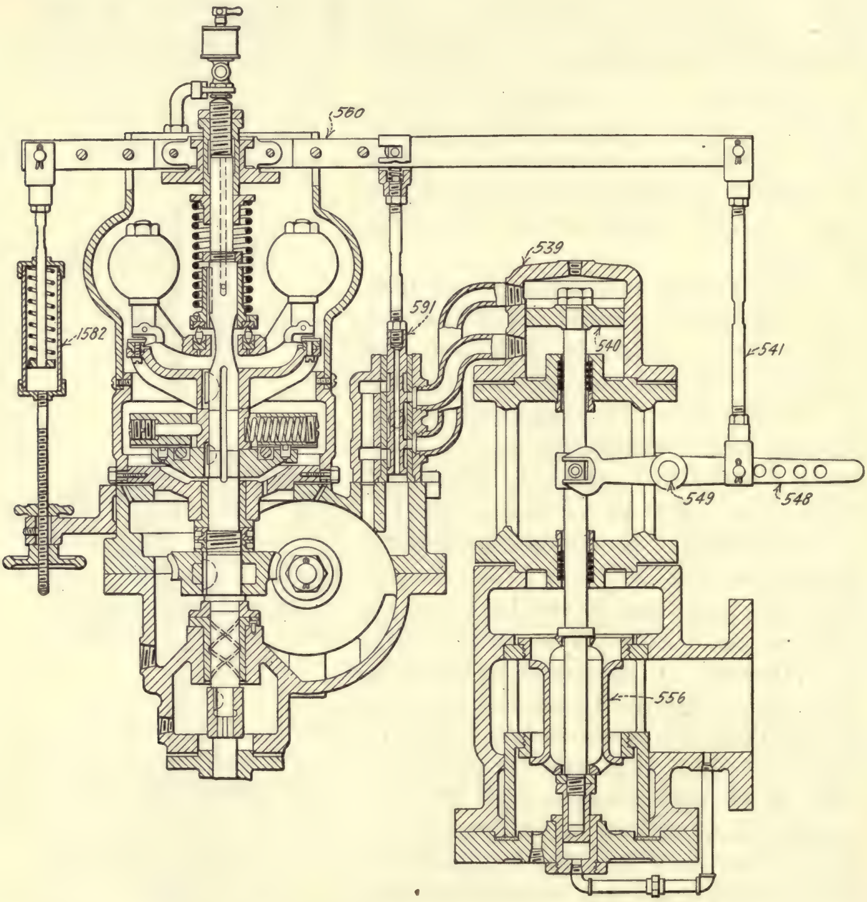


Fig. 255.—Kerr turbine relay governor.

138. Relay Governors.—The Kerr Turbine Co. relay governor is shown in Fig. 255. The pilot valve is shown at 591. Oil under pressure enters at the center of the valve chamber and is admitted to one end or the other of oil cylinder 539. The exhaust from the oil cylinder passes out near the ends of the valve chamber into the space surrounding the driv-

ing gears. The governor is shown in its extreme highest position with the governor valve 556 closed. This valve is a double-seated balanced valve and opens downward. Upon starting it is in its extreme lowest position.

The following description is taken from Bulletin 26 of the Kerr Turbine Co.:

"As the turbine is started up the weights are held in position by the governor spring until centrifugal force begins to compress the spring. As the speed increases the weights swing outward in a larger and larger circle transmitting their motion to the governor lever and giving it an upward motion. This causes an upward motion of the relay pilot valve 591 and uncovers the ports so as to release the pressure on the upper side of the oil relay piston 540 and admit pressure on the bottom side. This causes an upward travel of the piston and decreases the steam valve opening. At the same time the motion of the valve is communicated to the governor lever 560 through the starting lever 548, which is pivoted at 549 and the starting lever link rod 541. This downward travel of the lever at its outer end brings the oil relay pilot valve back toward its original position, gradually decreasing the flow of oil, and finally a position is reached where the steam valve opening admits the proper amount of steam for the load to be handled. The pilot valve will then be in a central position and the pressure on the top and bottom of the relay piston will be equal. When the speed decreases, the governor weights travel in toward the governor shaft, causing a downward motion of the governor lever and pilot valve which opens up the chamber below the piston to exhaust and the chamber above the piston to admission. This causing downward travel of the steam valve, admitting more steam, at the same time causing an upward movement of the governor lever, which brings the pilot valve back toward its original position until the speed and load become settled again. The pilot valve will now be in a central position and the pressure on the top and bottom of the relay piston will be equal.

"To receive close regulation the starting lever link rod 541 is moved toward the fulcrum 549 of the starting lever; and for wide speed regulation and great stability it is pivoted toward the outer end of the starting lever. The relay pilot valve will always run with the ports all closed as soon as the load is settled; in other words, the relay pilot valve is the fulcrum of the governor mechanism. A speed variation of from 2 to 5 per cent. can be secured while the turbine is running by shortening or lengthening the starting lever link rod 541, which has right- and left-hand threads on the ends. This speed variation can also be obtained on all turbines driving alternators by means of the synchronizer. This consists of a synchronizer spring case 1582 to which is attached the upper and lower covers. The upper one has a clearance hole through which the stem, which has a collar attached, can move. The upper end is connected to a continuation of the governor lever. By means of the adjusting wheel the spring may be

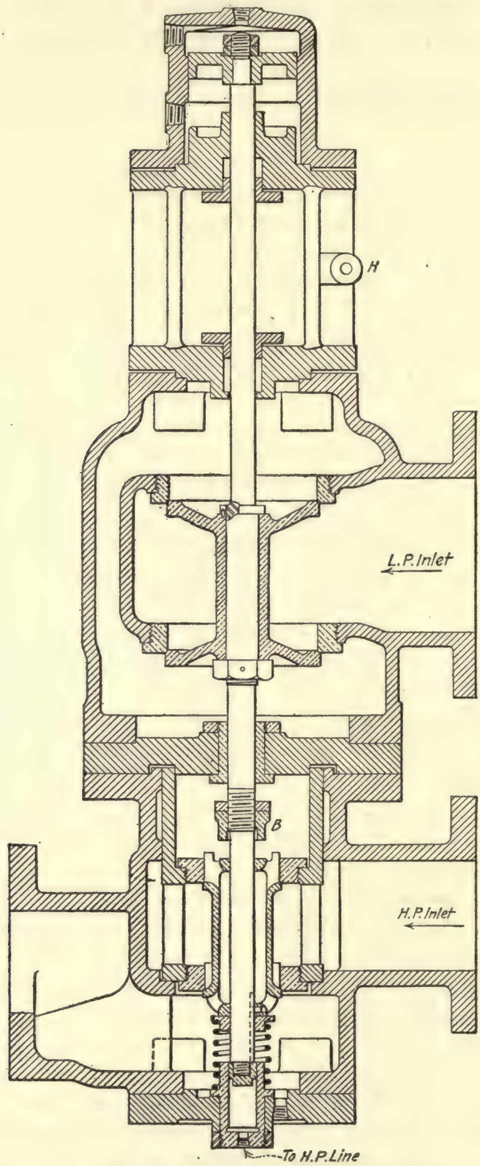


FIG. 256.—Kerr turbine mixed-pressure valve.

given any degree of compression desired, and locked there by means of the locking wheel. The greater the compression of the spring, the greater the speed of the turbine. A speed of about 5 per cent. above that given by the regular governor can be obtained."

The steam valve shown in Fig. 255 is the regular high-pressure valve. A mixed-pressure valve is shown in Fig. 256. The starting lever fulcrum is shown at *H*. The top valve is the low-pressure valve admitting steam to the low-pressure element on the turbine shaft. The bottom valve admits steam to the high-pressure element. When there is sufficient

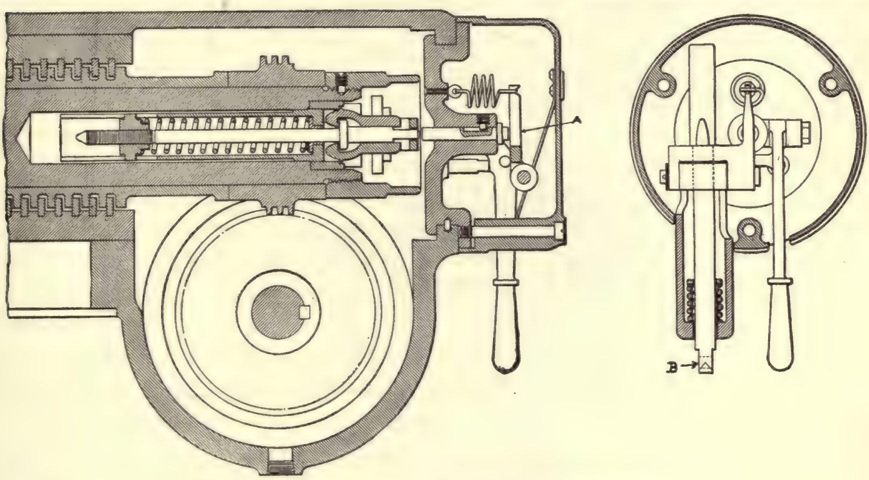


FIG. 257.—Allis-Chalmers safety governor.

low-pressure steam to carry the load, the operation is exactly as described for the regular high-pressure turbine; the valve stem then slides through the high-pressure valve without engaging with it. Should the low-pressure steam not be ample for the load, the slight decrease in speed causes the governor weights to take a new position toward the center, lowering the pilot valve and admitting pressure to the top side of the oil piston. The nut *B* then engages with the high-pressure valve, forcing it open against the spring, which forces the valve again to its seat when the stem ascends. The turbine now operates as a mixed-pressure turbine, or if there is no low-pressure steam, as a high-pressure turbine. A check valve in the low-pressure steam line prevents an outflow of steam from the turbine, should the pressure exceed that in the low-pressure main. A small piston on the lower end of the valve stem is connected with the high-pressure steam line for the purpose of balancing the valves, oil piston and stem.

139. Safety, or Over-speed Governors.—Fig. 257 shows the safety governor of the Allis-Chalmers Co. It is located in the end of the turbine shaft and does not depend upon gears. It is a simple spring governor, which, when some predetermined speed is reached, compresses the spring, forces out the spindle and releases the trigger *A*. This releases the rod *B*, to the end of which a heavy weight is hung. This weight in turn is attached to a lever which, when the weight drops, closes the main throttle valve and stops the turbine.

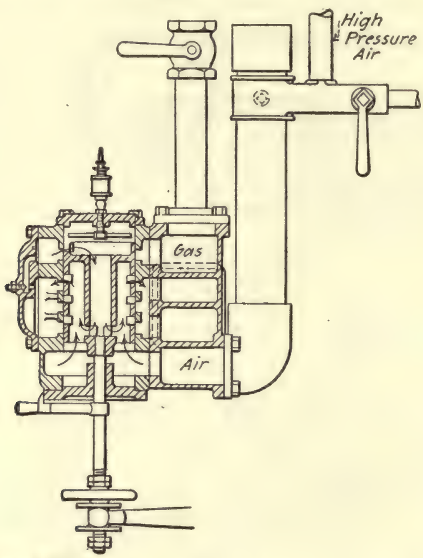


FIG. 258.—Bruce-Macbeth mixing valve.

140. Gas engine governors are usually of the spring-loaded type. In some cases they coördinate with the valve gear, and such arrangements will be treated in Chap. XX. More commonly they operate the throttle valve, which is usually a small butterfly valve in constant-volume oil engines. In gas engines the throttle valve is known as a governor valve or mixing valve. Such a valve, used on the Bruce-Macbeth engine for natural and illuminating gas is shown in Fig. 258.

CHAPTER XX

VALVES AND VALVE GEARS

141. Introduction.—The design of valve gears is essentially a drawing board proposition. The division of mechanics most used is kinematics, It is true that gear parts must have sufficient strength and wearing surface, and the analysis to determine forces acting is not usually very difficult; but the force required to move the valve is uncertain in many cases and assumptions must be made which makes the method largely a rule of thumb.

Corliss valves in large engines, though unbalanced, are easily moved by hand so long as they are kept in motion, showing that the film of steam between valve and seat partly balances them. By assuming poor lubrication, and taking the force required to move the valve after it has stood a while under pressure, forces acting on all parts of the gear are easily found.

There is no force theoretically required to move balanced valves if friction is neglected; they may be computed the same as unbalanced valves, but as one of the advantages of balanced valves is the reduction in weight of the gear, a certain proportion—such as one-half—of the force may be used.

The details illustrated later in this chapter will be given as a guide, but the computations will be omitted in most cases. The dimensions are usually the result of experience, and the stresses and bearing pressures may be found when the gear layout is complete.

The problem of treating the subject of valve gears in a single chapter is difficult and may only be done with any degree of satisfaction by discussing with reasonable thoroughness a few of the gears in common use which involve the principles common to other gears, illustrations of the latter being given with little or no discussion.

A number of gears are in use today on engines built some years ago, but these gears are not manufactured today; such gears are then practically obsolete from the standpoint of the designer, and little or no attention will be given to them. It may be thought by some that the Corliss gear is entering this class, but as this was practically the first to give a shortened valve travel with quick opening and is still used in both the

older and modified forms, with and without the releasing mechanism, it is treated in a fairly thorough manner.

No special valve diagram is championed; those selected will enable nearly every gear problem to be handled. The Zeuner diagram, while elegant, may be used for no problem which may not be better solved by one of the other diagrams given; neither does it clarify the principle of valve action better; therefore it is omitted.

For a larger collection of gear designs and a more thorough treatment of some phases of the subject the reader is referred to the excellent books mentioned at the end of this chapter.

Notation.

a = port area.
S = piston speed. Also stress, and on valve diagrams, steam lap.
A = piston area.
V = steam or gas velocity.
D = cylinder diameter.
w = width of port.
l = length of port = kD. Also lead on valve diagrams.
d = poppet valve diameter.
h = poppet valve lift.
$r = V_C/V_F$.
$q = d/D$.
$m = h/d$.

S, P, E, T, B, W and l are dimensions on diagrams.
A, Q, D, L, C, R and M are angles on diagrams.

142. Port Area.—About the first requirement in valve gear design is the port area. This is determined by assuming the port wide open throughout the stroke, and that the steam or gas is of constant specific volume, although this is really far from true, especially during the exhaust stroke. But with this assumption the areas of port and cylinder bore are inversely proportional to mean velocities of steam or gas and piston; or:

$$aV = AS \tag{1}$$

in which a and A are areas in sq. in. of port and cylinder bore respectively, S is the mean piston speed in ft. per min. and V the nominal mean gas or steam velocity in ft. per min. Then:

$$a = \frac{AS}{V} \tag{2}$$

The length of the port in steam engines (which will be taken as its greater dimension) varies from about 0.8 the cylinder diameter in small,

slow-speed steam engines to somewhat greater than the cylinder diameter for locomotives with flat valves. Corliss engines usually have ports equal in length to the cylinder diameter. For piston valves the length is much greater, while in poppet valves the area is equal to the product of lift and effective circumference.

As a general guide, it is considered desirable to have the port opening at any part of the piston stroke proportional to the piston velocity at that point, the maximum port opening corresponding to the maximum piston speed; it is obvious, however, that this could be possible only with engines in which the valve is open during the entire stroke unless opening and closing is instantaneous, but it gives a basis of comparison which will be used in the following paragraphs.

Allowable Fluid Velocity V.—For steam, the following formulas may be used as a guide. These are:

$$\text{Steam port,} \quad V = 7000 \sqrt[5]{\frac{D}{10}} \tag{3}$$

$$\text{Exhaust port,} \quad V = 5000 \sqrt[5]{\frac{D}{10}} \tag{4}$$

By varying the numerical coefficient these formulas may be used for the ports of internal-combustion engines. Values for these engines will presently be given in connection with poppet valves.

If double ports are used each opening may be taken one-half of the value found by (2).

By the term *port opening,* the amount that the valve uncovers the port is meant. In single-valve engines where the same ports are used for inlet and exhaust, the port is made large enough to accommodate the exhaust steam; then the inlet edge of the valve does not usually uncover the entire port. Also, due to certain characteristics of the valve motion the exhaust edge of the valve usually has considerable *over-travel.* The terms *steam* and *exhaust* are used in common parlance for the incoming and exhaust steam respectively.

Slide valves may be used to include all those having sliding surfaces. The relative proportions of such valves and their seats are determined by practice with the various kinds. As an example of design, double-ported Corliss valves will be taken. Fig. 259 shows sections of steam and exhaust valves at one end of the cylinder. By experience with the layout of Corliss valve diagrams the following proportions were found to give good results:

$$w = \frac{d}{6} \quad (5) \qquad b = 0.5d \quad (6) \qquad w_1 = 0.233d \quad (7) \qquad b_1 = 0.6d \quad (8)$$

$$c = 0.27d \quad (9)$$

Certain other dimensions are given which may be used in cylinder design. The values of b and b_1 were made ample to provide sufficient seal when the valves are closed; this may often be made less if desired and may be determined or checked by means of the valve diagram given later.

As separate valves are used, the ports may be entirely uncovered, and there is no necessity of excessive over-travel of the exhaust valve. Some

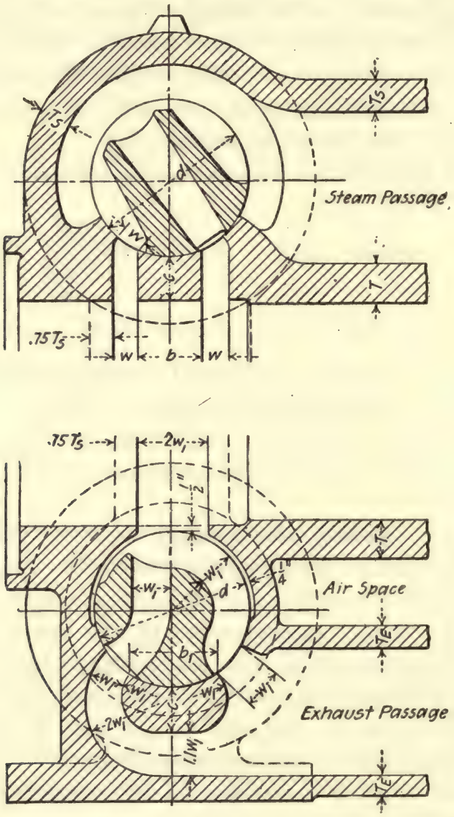


FIG. 259.—Corliss double-ported valves.

over-travel is advantageous as it gives a quicker opening and allows for adjustment. The valves in Fig. 259 are shown wide open (the steam valve moves in a counterclockwise direction in opening, and the exhaust valve clockwise), with some over-travel, also some clearance at the heel of the valve to insure against partly closing the second port in case of adjustment; this does not give an ideal steam passage, but is apt to give

better practical results than too little allowance. The operation of this valve will be better understood from a study of the gear, the layout of which is also determined experimentally and in connection with valve and port design; there is no distinctly logical order in proceeding.

The length of the port is usually equal to the cylinder diameter in Corliss engines, but a fraction of this length is taken up by ribs which connect the bridge with the two parts of the cylinder for strength and stiffness and to insure against distortion due to heat. The length may then be taken:

$$l = kD$$

where k may be from 0.85 to 0.9; or it may be any desired fraction for special designs. Then the area of the steam port in Fig. 259 is:

$$a = 2wl = 2kwD$$

The piston area is:

$$A = \frac{\pi D^2}{4}$$

Substituting these values in (2) gives:

$$w = \frac{\pi DS}{8kV} \qquad (10)$$

Then from (5):

$$d = 6w = \frac{3\pi DS}{4kV} \qquad (11)$$

If in (11), V is taken as the inlet velocity, the correct velocity will be obtained through the exhaust port, as w_1 from (7) is proportioned so that:

$$Vw = V_1w_1$$

Having determined d, other dimensions may be found by Formulas (6) to (9). As V is greater for larger cylinders, the valve diameters are relatively smaller. In some large low-pressure cylinders the diameter of the steam valves is less than of the exhaust valves, the relative proportions being the same.

Other types of valves may be treated in the same general way.

Single-ported poppet valves are used almost entirely for internal-combustion engines. Some have flat seats as in Fig. 260-*A*, but usually the seat is conical with an angle of 45 degrees as in Fig. 260-*B*.

The area of opening of the flat-seated valve is:

$$a_F = \pi dh \qquad (12)$$

The area through a conical seat is:

$$a_C = \pi d_1 h_1 = \pi d \cdot \frac{h}{\sqrt{2}} + \pi \frac{h^2}{2\sqrt{2}} \tag{13}$$

The subscripts F and C denote flat and conical seats respectively.

Let:

$$r = \frac{V_C}{V_F} = \frac{a_F}{a_C} = \frac{1.414}{1 + 0.5\,\frac{h}{d}} \tag{14}$$

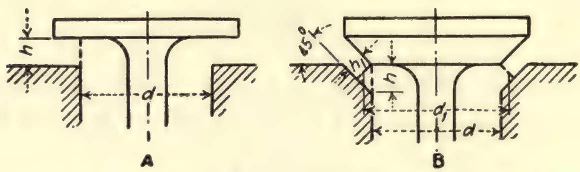


FIG. 260.—Single-ported poppet valves.

In Table 63 are values of r for the complete range of h/d found in practice.

TABLE 63

h/d........	0.040	0.060	0.080	0.100	0.120	0.140	0.160	0.180	0.200	0.220	0.240	0.250
r..........	1.386	1.373	1.360	1.345	1.333	1.321	1.310	1.297	1.285	1.273	1.262	1.257

When $h/d = 0.12$, near the center of the range, $r = 4/3$. For this value (14) gives:

$$a_C = \frac{a_F}{r} = \frac{3}{4} a_F = \frac{3}{4}\pi dh \tag{15}$$

It may be assumed that this gives the area of opening for all poppet valves. It is on the side of safety for all values of h/d greater than 0.12, and these values are most common in modern engines. The error is slight for smaller values. The theoretical opening is greater in flat-seated valves, but on account of the obstruction to flow offered by sharp corners, it is doubtful if the capacity is greater than for conical valves having the same lift. The conical seat is usually much narrower than shown in Fig. 260, the opening approaching that of the flat seat for high lifts. The lift of a flat-seated valve to give full opening of the port is ¼ the diameter of the port, neglecting the stem. For a conical seat it is higher, but $d/4$ is the maximum value of h given in treatises on internal-combustion engines, and this is seldom used.

Dropping the subscript in (15) and substituting in (2) gives:

$$\frac{3}{4}\pi dhV = \frac{\pi D^2}{4} \cdot S.$$

Dividing through by d^2 gives:

$$3V\frac{h}{d} = S\left(\frac{D}{d}\right)^2.$$

Let $h/d = m$ and $d/D = q$; then:

$$\frac{V}{S} = \frac{1}{3mq^2} \tag{16}$$

From (16) Table 64 has been calculated. This table will be found useful in preliminary calculations, and for any assumed or measured values of m and q the value of V/S is seen. This factor multiplied by S gives the nominal gas velocity.

TABLE 64

q	Values of V/S for $m =$									
	0.06	0.08	0.10	0.12	0.14	0.16	0.18	0.20	0.22	0.25
0.25	89.00	66.60	48.00	44.50	38.00	33.30	29.40	26.70	24.30	21.40
0.30	61.80	46.30	37.00	30.90	26.50	23.10	20.60	18.50	16.80	14.80
0.35	45.40	34.00	24.50	22.70	19.40	17.00	15.10	13.70	12.40	10.90
0.40	34.70	26.00	18.80	17.40	14.90	13.00	11.60	10.40	9.44	8.34
0.45	27.40	20.60	14.80	13.70	11.80	10.30	9.14	8.23	7.49	6.59
0.50	22.20	16.70	12.00	11.10	9.50	8.34	7.40	6.66	6.06	5.34
0.55	18.40	13.80	11.00	9.18	7.85	6.89	6.12	5.50	5.01	4.40
0.60	15.50	11.60	9.26	7.71	6.60	5.78	5.14	4.63	4.20	3.70
0.65	13.20	9.88	7.90	6.57	5.63	4.93	4.38	3.94	3.59	3.16
0.70	11.40	8.52	6.80	5.66	4.85	4.25	3.78	3.40	3.09	2.72
0.75	9.88	7.42	5.92	4.94	4.22	3.70	3.29	2.96	2.69	2.37
0.80	8.70	6.52	5.21	4.34	3.73	3.25	2.89	2.60	2.37	2.09
0.85	7.70	5.78	4.61	3.84	3.29	2.88	2.56	2.30	2.01	
0.90	6.86	5.15	4.12	3.43	2.94	2.57	2.29	2.05		
0.95	6.15	4.62	3.69	3.08	2.64	2.31	2.05			
1.00	5.55	4.17	3.33	2.78	2.38	2.08				

Various values of V are given by different authorities. Güldner recommends 4500 ft. per min., with 6000 as a maximum under favorable conditions; but these values do not seem feasible for modern high-speed engines. It is true that the lower the value of V/S can be kept the greater will be the capacity of the engine, therefore as large values of m and q should be selected as is practicable. A few data of representative engines are given in Table 65.

The dimensions of the airplane engine were scaled from a drawing and may not be strictly correct. If the piston speed were 2000 ft. per min., V would be 12,900. The low value of V/S in airplane engines is no doubt an important factor in enabling them to obtain as high an m.e.p. as 130 lb.

TABLE 65

Engine	Valves	m	q	V/S	S	V
4-cylinder truck	inlet	0.156	0.448	10.60	1,196	12,600
4-cylinder truck	exh.	0.171	0.448	9.70	1,196	11,530
4-cylinder auto	inlet	0.170	0.440	10.10	1,660	16,800
4-cylinder auto	exh.	0.208	0.440	8.25	1,660	13,700
6-cylinder auto	both	0.200	0.433	8.95	1,565	14,000
6-cylinder auto	inlet	0.182	0.440	9.42	1,750	16,500
6-cylinder auto	exh.	0.222	0.440	7.70	1,750	13,500
8-cylinder airplane	both	0.227	0.477	6.45	?	?

Double-ported poppet valves are used for superheated-steam engines and uniflow engines. The port area may be taken twice that of the single valve, or:

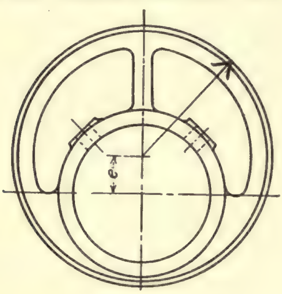
$$a = \frac{3}{2}\pi dh \tag{17}$$

Also:

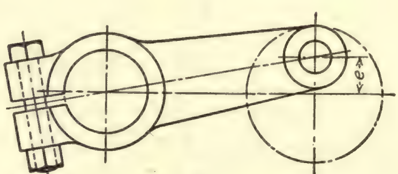
$$\frac{V}{S} = \frac{1}{6mq^2}$$

Table 64 may be used by taking one-half the tabular value.

143. Classification.—The term valve gears is practically always as-



A



B

FIG. 261.

sociated with reciprocating engines, as in these there is always a complete cycle of operations. This is not true of the steam turbine; the gear is not in time relation with the wheels, but is closely connected with the governor, so is treated in Chap. XIX.

A classification of the gears of reciprocating engines is difficult and not very satisfactory, but lists will be given for steam and internal-combustion engines; by dividing into a number of headings a general idea of methods, valves and gears may be had, and these terms will be used in paragraphs which follow.

By the term *eccentric,* a sort of enlarged crank pin is usually meant,

the diameter of which is such that the shaft diameter lies within it; this is so for the reason that the eccentric is a casting which surrounds the shaft and is fastened to it by key or set screw or both. This is shown in Fig. 261-*A*. The center of the eccentric often lies within the shaft circle as shown. On certain locomotive valve gears and some shaft governors, the gear is not driven by such an eccentric, but by a small pin as shown in Fig. 261-*B*. This is also known as the eccentric and will be so included when the term is used in this book.

(1) *Steam engines.*

- Valve system
  - Single valve
  - Double valves (expansion valve)
  - Binary or separate valves
  - Uniflow
- Gear drive
  - nonreversing
    - Eccentric
    - Eccentric and wrist plate
    - Eccentric and cam
  - reversing
    - Eccentric
    - Eccentric and crosshead
    - Connecting rod
- Valve control
  - Shifting eccentric
  - Swinging eccentric
  - Releasing gear
  - Reversing link motion
- Valves
  - Flat
    - Unbalanced
      - Plain D-valve
      - Gridiron valve
    - Balanced. Single- to triple-ported
  - Piston. Balanced, single- or double-ported
  - Rocking. Unbalanced, single- to quadruple-ported
  - Poppet. Balanced, double-ported
  - Piston acting as valve

(2) *Internal-combustion engines.*

- Valve system
  - 4-cycle. Mechanically operated inlet and exhaust
  - 4-cycle. Automatic inlet valve
  - 4-cycle. Sleeve valve
  - 2-cycle. 2-port and 3-port
- Gear drive
  - Cam
  - Cam and eccentric
  - Eccentric
- Valve control
  - Constant gear motion with throttling governor
    - Inlet valve
    - Fuel valve
  - Releasing gear for inlet or fuel valve
  - Hit-and-miss
  - Fuel valve or pump control
- Valves
  - Poppet
  - Sleeve
  - Piston acting as valve

144. Single-valve Gear.—This is the simplest form of gear and it sometimes seems the most difficult to understand, as all events are accomplished for both ends of the cylinder by a single valve. The subject is usually approached by a consideration of the valve without lap or lead, and with no angular advance of the eccentric; these terms will be explained presently. Angularity of connecting rod and eccentric rod will also be neglected, it being assumed that the Scotch yoke is used for each; then the displacement of the valve and piston is measured by the projection upon the line of stroke of the eccentric center and crank pin center respectively.

A sectional diagram is shown in Fig. 262 of such a gear. A general description of steam engine operation is given in Chap. III, in which the cycle is traced through for a 4-valve engine, so it will not be necessary to go

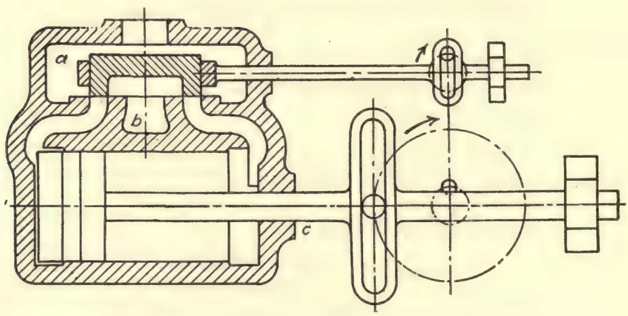


FIG. 262.

into details here. In Fig. 262, *a* is the steam chest and *b* the exhaust passage leading from the cylinder. The crank and eccentric circles are shown separately to avoid confusion.

The piston is at the head end of the stroke and the crank at the head-end dead center. The eccentric is at right angles with the crank, causing the valve to be in the center of its travel, just covering both ports. As the eccentric leads the crank, a movement in a clockwise direction will open the head-end port to the steam chest *a*, and the crank-end port to the exhaust passage *b*. If the movement is continued until the crank is at the crank-end dead center, the valve will again be at its central position with both ports closed. Then admission, cut-off, release and compression all occur at the end of the stroke and the indicator diagram is a rectangle.

The arrangement just described in which the high-pressure steam is in *a* on the outside of the valve, gives what is known as *outside admission.* If steam entered at *b* (necessitating some device to keep the valve on the

seat) and left past the outer edges of the valve, entering a and thence to the atmosphere, the arrangement is known as *inside admission;* then to open the valve at the head end to high-pressure steam, the crank and eccentric must rotate in a counterclockwise direction, and this is the principle of operation of the steering-gear engine used on steam ships.

Lap, Lead and Angular Advance. Still assuming the crank to be at the head-end dead center and that there is outside admission, let us arbitrarily add *steam lap S* and *exhaust lap E* to the valve as in Fig. 263-*A*, and study the effect. The port is now closed, and the valve must be moved to the right the distance S (the steam lap) before it begins to open, and the

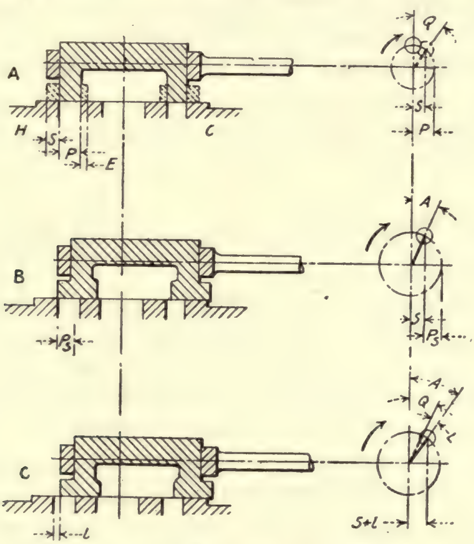


FIG. 263.

eccentric must move through the angle Q (the *lap angle*) to bring the valve to the opening position, or admission. If the eccentric and crank are still at right angles as in Fig. 262, the crank would travel the angle Q from dead center before admission would occur, which is not permissible. It then becomes necessary to advance the eccentric through the angle Q while the crank is still at the head-end dead center; this angle is now denoted by A, as it is the *angular advance*. In some books the angular advance is the angle between crank and eccentric measured from the crank in the direction of motion. This seems logical, but it is American practice to consider angular advance the angle between eccentric and crank minus 90 degrees, and it will be so considered here.

If the eccentric radius were such as to just open the valve to steam

before steam lap were added, the opening would now be $P-S$, in which P is the width of the port. It is usual to determine this opening as in Par. 142; then denoting the steam port opening by P_S, the eccentric radius must be made equal to:

$$\frac{T}{2} = S + P_S \tag{18}$$

where T is the valve travel, which is twice the eccentric radius when the eccentric drives the valve directly as assumed in this discussion. Formula (18) is correct in any event, as it contains only valve measurements.

Admission position is shown in Fig. 263-*B*. As explained in Chap. III, it is customary to open the valve to steam a little before the end of the stroke is reached; the amount the valve is open when the crank is on dead center is called *lead*, and is denoted by l. The angle the eccentric and crank must pass through to move the valve the distance l is called the *lead angle. The angular advance is the algebraic sum of the lap angle and lead angle*; or:

$$A = Q + L \tag{19}$$

Lead is shown in Fig. 263-*C*, the crank (not shown) being at the head-end dead center.

It now remains to follow the crank from admission through a complete revolution, tracing events and determining the indicator diagram produced. This is done in Fig. 264 for the head-end port. Arrows show direction of rotation, direction of piston travel (on indicator diagrams) and direction of valve travel. Full valve sections are shown at the top with valve in central position (the position when lap is measured) and at the extreme position with head-end steam port and crank-end exhaust port open. In the crank-eccentric diagram, crank and eccentric are shown connected by a light line to avoid confusion. The center line of the valve is directly under the eccentric center in each position.

The following description accompanies Fig. 264:

Position I. Admission.—Valve displacement = S = steam lap. Steam edge of port just opening.

Position II. Dead Center.—Valve displacement = $S + l$ = steam lap + lead. Valve open amount of lead (l).

Position III. Maximum Steam Opening.—Valve displacement = $T/2$ = half of valve travel. Valve open to steam $T/2 - S$.

Position IV. Cut-off.—Valve displacement = S = steam lap. Steam edge of valve just closing.

Position V. Valve Central.—Valve displacement = zero. Steam lap S and exhaust lap E are measured in this position.

Position VI. Release.—Valve displacement = E = exhaust lap. Exhaust edge of valve just opening.

Position VII. Maximum Exhaust Opening.—Valve displacement $= T/2 =$ half travel of valve. Valve open to exhaust $T/2 - E$.

Position VIII. Compression.—Valve displacement $= E =$ exhaust lap. Exhaust edge of valve just closing.

From the valve section at the top of Fig. 264 the following equations are derived:

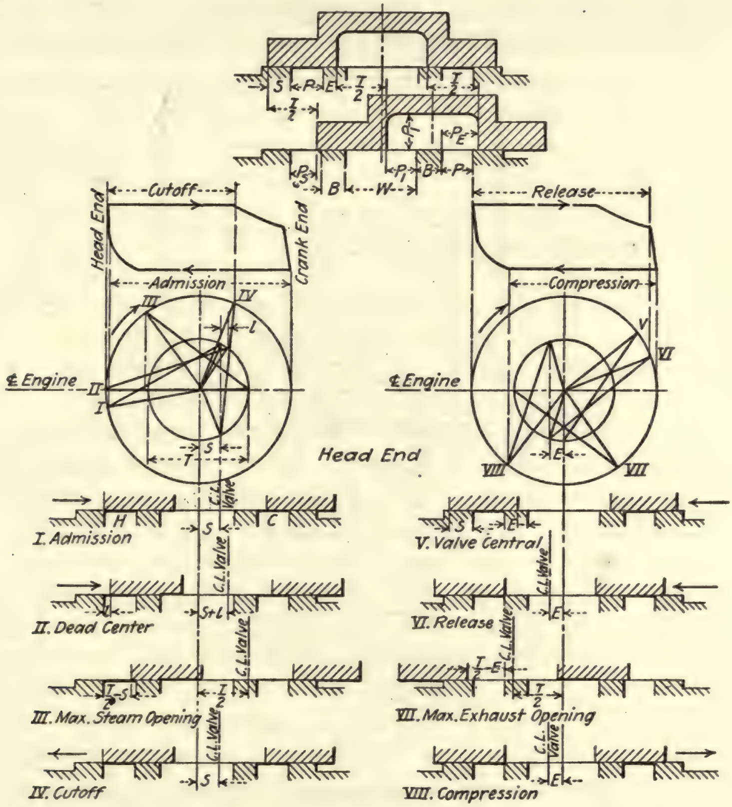


FIG. 264.

$$\frac{T}{2} = S + P_S = E + P_E \tag{20}$$

$$P_E \gtreqqless P \quad \text{(usually)} \tag{21}$$

$$B = \frac{P}{2} + \frac{3''}{8} \tag{22}$$

Formula (22) is arbitrary but may be used as a guide.

For minimum value of W (as $P_1 \gtreqless P$):

$$E + \frac{T}{2} + P = B + W$$

or:

$$W \gtreqless \frac{T}{2} + P + E - B \tag{23}$$

Turning the indicator- and valve-section diagrams end for end, and the crank-eccentric diagrams through an angle of 180 degrees would show

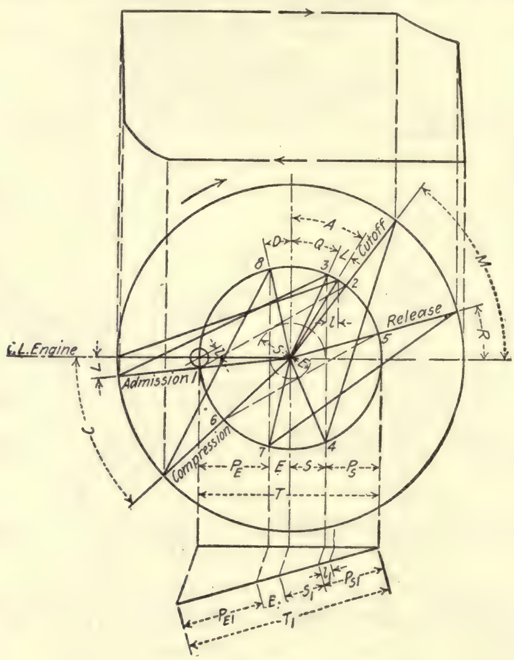


FIG. 265.

the crank-end events, but this is unnecessary when the angularity of the connecting rod is neglected.

When familiarity with Fig. 264 is attained the crank-eccentric diagram may be used alone to show any valve setting.

In following through the events, valve dimensions were assumed; in practice the events of the stroke are assumed, or it may be said that the indicator diagram is designed. In Fig. 265 a crank-eccentric diagram is shown for the head end, for an indicator diagram drawn above it. Any crank circle may be taken, angles and ratios being independent of this. A valve travel may be assumed, then transferred as shown to any

other value by graphical proportion when one dimension (usually P_S) is known. It is necessary with this diagram to assume a lead as a certain fraction of the port opening, as the actual valve travel may be different from that assumed.

The following procedure may be used: Crank positions are found by projection from the desired indicator diagram as shown. The angle between crank positions for admission and cut-off is obviously equal to the angle between eccentric positions for the same events. The projection of these eccentric positions upon the horizontal center line of the valve travel circle (the eccentric circle) must be at the same point, as the valve must be in the same position for both events. Then connecting the points where the valve-travel circle cuts the crank at 1 and 2, and swinging this line around tangent to an arc of radius S into position 3–4 parallel to the vertical center line as shown, the eccentric positions for admission and cut-off, the steam lap-angle Q and the steam lap S are determined.

The exhaust events may be first assumed, the line 5–6 being swung around to position 7–8 tangent to the arc of radius E. The angular advance may thus be determined by admission and cut-off (the steam events), or by release and compression (the exhaust events). If determined from the steam events, the exhaust lap is found by adjusting a proper relation between release and compression for the angular relation between crank and eccentric so found. If determined from the exhaust events (which is unusual except for engines with a releasing gear), the steam lap would be found from the best relation of admission to cut-off possible with this angular relation. Sometimes, especially in locomotives, exhaust lap is zero or may even be negative, when the valve is open to exhaust at both ends when in the central position; it is then known as *exhaust clearance.*

Effect of Rod Angularity.—In Fig. 266-*A* for the Scotch yoke, the horizontal distance between crank pin 1 and crosshead pin 2 is the same for all positions of the crank pin. Let this distance L be the length of the connecting rod of a slider-crank mechanism; then for the same angular movement of the crank from the head-end dead center, the crosshead pin of Fig. 266-*A* moves the distance a, while that of Fig. 266-*B* moves the distance b. This is due to the angularity of the rod and it is obvious that the crosshead and piston of Fig. 266-*B* will always be nearer to the crank than that of Fig. 266-*A* except in the dead-center positions.

In valve diagrams it is convenient to consider piston displacement in connection with the crank circle. Then in Fig. 266-*B*, with the center of the crosshead pin as a center, draw an arc of radius L cutting the center

line of engine; this marks off the distance b which is the piston displacement from the head end of the stroke. Thus for any position of the crank pin the corresponding piston position may be shown on the crank pin circle diameter. The same is true for valve displacement and the eccentric circle, but the maximum angularity of the eccentric rod is usually so small that it is neglected.

It is usual to neglect angularity in drawing valve diagrams, at least for preliminary work; the events used are then called *nominal* cut-off, compression, etc. To find the actual events, the method just given is used and this is shown in Fig. 267. It will be noticed that all events of

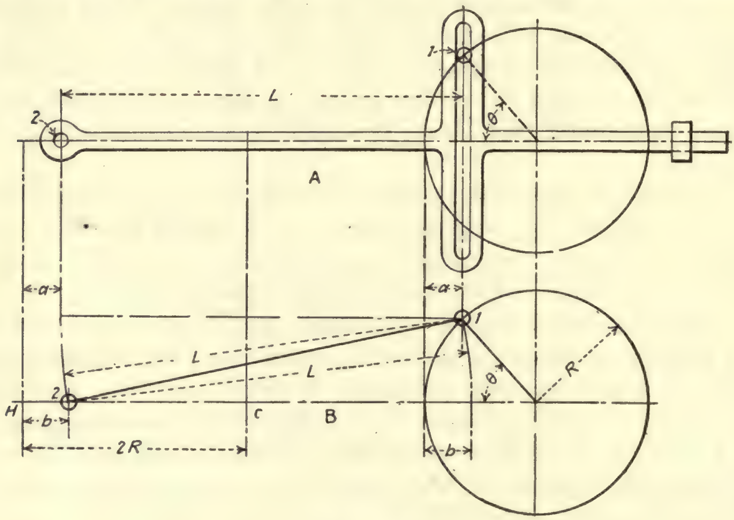


Fig. 266.

the stroke from head to crank end occur later in the stroke than the nominal events, while those from crank to head end occur earlier.

Actual indicator diagrams are as shown in Fig. 267, if, with the piston at the head end of the stroke, the pencil of the indicator is to the right of the paper clips as at A, Fig. 268, the drum moving as indicated by the arrow as the cord is unwound; if for this piston position the arrangement is as at B, Fig. 268, the indicator diagrams will be changed end for end.

The diagram of Fig. 265 has been given because it shows the actual relation of crank and eccentric, and the principles of valve motion may more clearly be seen than with any other diagram. Moreover, it is general in its application and may be used for any eccentric-driven gear in conjunction with other diagrams showing the motion of wrist plate, linkage or cams. For the single valve it has limitations except as the

trial-and-error method is used; this is also true of the Reuleaux and Zeuner diagrams. Therefore no further use will be made of this diagram for the single valve except to explain principles, but a diagram having

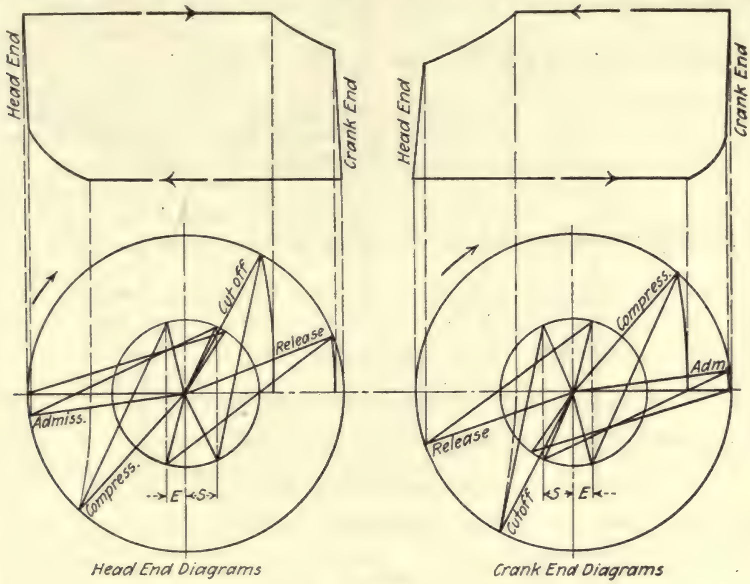


Fig. 267.

all the advantages and none of the disadvantages of the other diagrams for the single valve will now be explained.

The Bilgram Diagram.—Starting with the crank at the head-end dead center and with angular advance A, Fig. 269, move crank and eccentric through angle θ. Distance 1–2, perpendicular to vertical center line, represents valve displacement from central position by the method of Fig. 265. Produce the new crank position as shown, forming angle θ below the horizontal center line and to the right of the vertical center line. Now lay off angle A (the angular advance) above the horizontal center line as shown by the heavy line. The intersection of this line with the valve-travel circle is the eccentric center of the Bilgram diagram, being a fixed point for any given valve setting. Drop the perpendicular 3–4 to the new position of the crank produced. Then triangles 1–2–0 and 3–4–0 are equal, as each contains equal sides 1–0 and 3–0 (being radii of the same circle), one right angle, and the angle $A + \theta$. Then line 3–4 equals line 1–2, the displacement of the valve from the central position. This is true for all

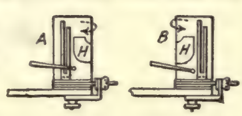


Fig. 268.

positions of the crank, the perpendicular 3–4 sometimes falling on the center line of the crank and sometimes on this line produced beyond the center of the circle.

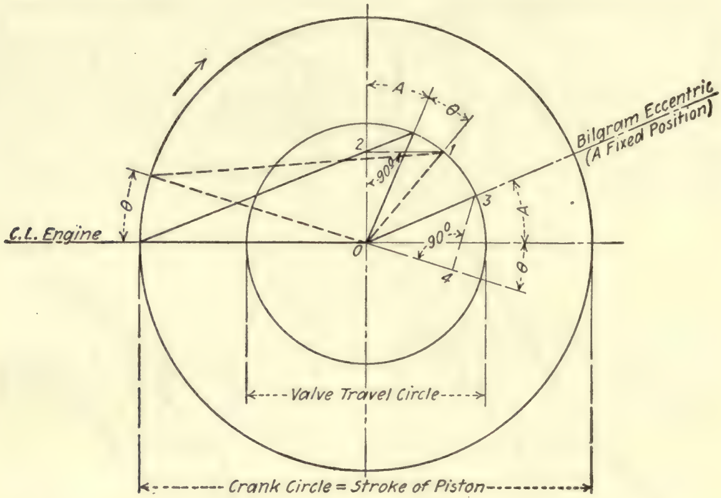


FIG. 269

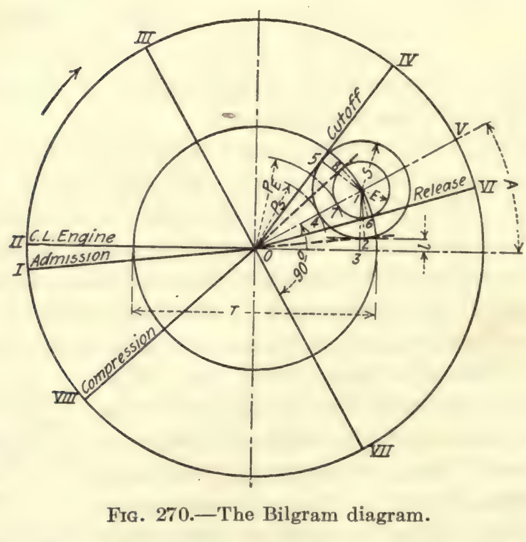


FIG. 270.—The Bilgram diagram.

For convenience of application the *lap circles* are drawn about the eccentric center of the Bilgram diagram, the radius of the circle equalling the lap. This is shown in Fig. 270, drawn with the same data as Fig.

264, with which it may be compared. With the same numbers for crank positions the following description will make the diagram clear:

I. *Admission.*—Valve displacement (1–2) = S = steam lap. Steam edge of valve just about to open.

II. *Dead Center.*—Valve displacement (1–3) = $S + l$ = steam lap + lead. Valve open amount of lead l.

III. *Maximum Steam Opening.*—Valve displacement (1–0) = $T/2$ = half of valve travel. Valve open to steam (0–4) = $P_S = T/2 - S$.

IV. *Cut-off.*—Valve displacement (1–5) = S = steam lap. Steam edge of valve just closing.

V. *Valve Central.*—Valve displacement = zero.

VI. *Release.*—Valve displacement (1–6) = E = exhaust lap. Exhaust edge of valve just opening.

VII. *Maximum Exhaust Opening.*—Valve displacement (1–0) = $T/2$ = half travel of valve. Valve open to exhaust (0–7) = $P_E = T/2 - E$.

VIII. *Compression.*—Valve displacement (1–8) = E = exhaust lap. Exhaust edge of valve just closing.

Fig. 270 is the head-end diagram; for the crank-end diagram the location of the lap circles and the crank positions for the various events are diametrically opposite.

In applying the diagram the cut-off, maximum steam port opening P_S and lead l are usually assumed. The cut-off position of the crank, lead line and arc of radius P_S are drawn as shown; then the steam lap circle is drawn so as to come tangent to these three lines, fixing the center of eccentric, angular advance and valve travel. The compression is the exhaust event usually assumed; producing this line as shown dotted, the exhaust lap circle is drawn tangent to it from the eccentric center, then the release position is tangent to the other side. Should release and compression exchange positions, release would occur before compression (in fraction of stroke); exhaust lap would then be negative (the valve would be open to exhaust at both ends when valve is at center of travel, giving exhaust clearance) and the circle would be a dotted line. The effect of angularity of connecting rod is found as in Fig. 267.

Double-ported Valves.—There are various designs of multi-ported valves, but the principle of all is to increase the valve opening with a given travel. The principle of operation of a double-ported, balanced slide valve is shown in Fig. 271. An adjustable balance plate (or pressure plate) has depressions to match the ports, the pressure being kept from all portions of the valve not opposite the ports by the valve seat and balance plate. At A the valve is displaced to the right by the amount of the steam lap and is just about to open the head-end port. At B it has moved from the edge of the port a distance equal to the width of the

auxiliary port through the valve; up to this time the opening has been double that indicated by the movement, as steam enters at two edges, but any further movement up to the position shown at C will not increase the opening, as the port through the valve is closed as fast as the cylinder port is opened. Sometimes the port opening equals the width of the auxiliary port before the valve reaches the position B (where the auxiliary port begins to close); then the opening is as for a single port till position B is reached and constant port opening begins. The valve is at the

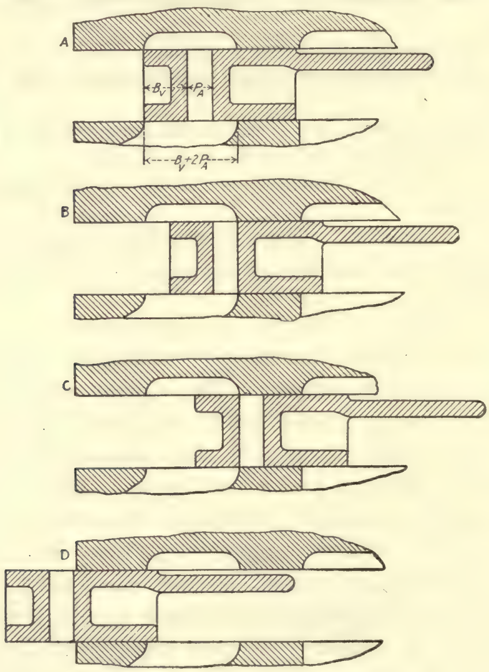


FIG. 271.—Double-ported slide valve.

extreme left of its travel at D, uncovering the exhaust port; a double-ported effect at the exhaust edge is not necessary with a single valve as the exhaust lap is small and there is usually a wide opening.

Some double-ported valves give a double opening throughout their entire travel, as the Corliss valve of Fig. 259.

Shifting and Swinging Eccentric.—In Chap. XIX, illustrations are given of governors in which the angular position and radius of the eccentric are changed when the load changes. If the eccentric moves in a straight line it is called a shifting eccentric, and if it swings from a pivot in the arc of a circle, a swinging eccentric.

The diagram of Fig. 265 will first be used to show the changes, which are given for different methods in Fig. 272. At *A* is shown an older style in which the eccentric is rotated about the shaft. The angular advance is increased with an increasing lead, but the valve travel is not changed. Full lines show the setting for maximum cut-off and dotted lines for a shorter cut-off.

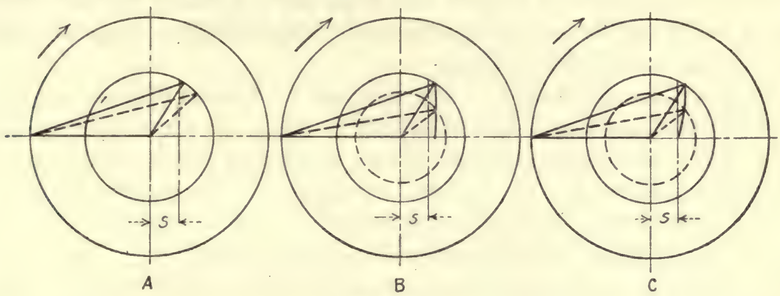


FIG. 272.

A shifting eccentric is shown at *B* in which both valve travel and angular advance are changed, as is also the case with the swinging eccentric shown at *C*. The pivot for Fig. 272-*C* was located at the crank side of the vertical center line, but it is sometimes at the right. The pivot was located to give zero lead when the angular advance is 90 degrees. The lead is constant for *B*.

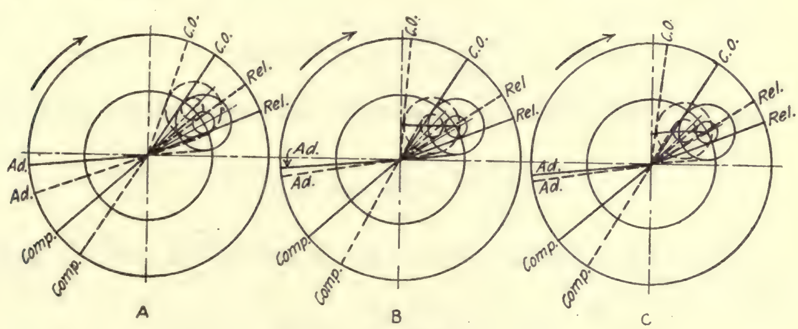


FIG. 273.

These diagrams are not convenient for the actual design of the changing eccentrics, so the same three cases are shown for the Bilgram diagram in Fig. 273. Crank positions for all events of the stroke (for the head end) are shown by the full lines, and for the shortened cut-off by the dotted lines; the decreased valve-travel circle for the short cut-off is omitted as it serves no useful purpose. The path of the eccen-

tric change is drawn as though the crank pin were at the bottom of the circle and rotates counterclockwise.

It will be noticed that the maximum steam port opening P_S (see Fig. 265) has decreased considerably for the shorter cut-off of B and C; it may easily be shown that for one-quarter cut-off the opening would be extremely small unless the valve travel and steam lap were very large. It is for this reason that double- and triple-ported valves are used, giving double or treble the opening indicated by the Bilgram diagram, which gives the *nominal* port opening.

Rectangular valve diagrams are diagrams in which ordinates represent valve displacement, and abscissas either piston or crank displacement; if the latter, the diagram is plotted on a rectified crank circle. As displacement is the sum of lap and port opening, such diagrams may be used to show port opening during the cycle, and may show the point on piston path or crank circle at which the valve event occurs.

To illustrate a rectangular diagram on the piston path, together with a double-ported valve, shifting eccentric and some of the principles of Par. 142, the design of a high-speed engine will be assumed with data as follows: Cylinder diameter 12 in., stroke 18 in. and r.p.m. 200; initial gage steam pressure 125 lb., back pressure 15 lb. absolute and clearance 6 per cent.

The nominal steam velocities from (3) and (4) are: $V_S = 7250$ and $V_E = 5180$. The ports in the cylinder must be determined from V_E; then (2) gives:

$$a_E = \frac{113 \times 600}{5180} = 13.1 \text{ sq. in.}$$

Assuming the length of the port equal to the cylinder diameter, the width is:

$$w_E \frac{13.1}{12} = 1.09, \text{ say } 1\tfrac{1}{8}''$$

The maximum steam port opening will be determined from V_S, which from (2) gives:

$$a_S = \frac{113 \times 600}{7250} = 9.38$$

and

$$P_S = \frac{9.38}{12} = 0.78.$$

On account of the double-ported valve and the desirability of having sufficient port opening at short cut-offs, assume the nominal port opening which will be used for the Bilgram diagram at maximum cut-off to be

equal to w_E or $1\frac{1}{8}$ in. Also assume the maximum cut-off to be $\frac{3}{4}$ stroke and the nominal lead (that used on the Bilgram diagram) to be constant

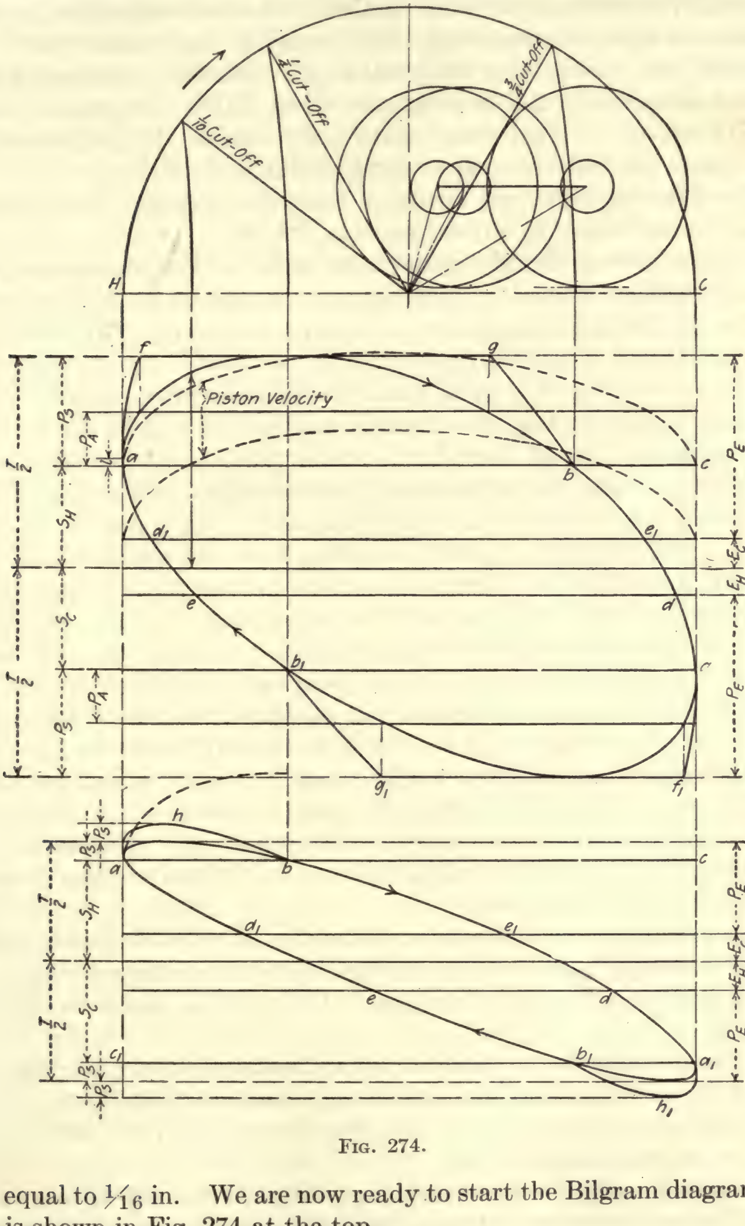


Fig. 274.

and equal to $\frac{1}{16}$ in. We are now ready to start the Bilgram diagram and this is shown in Fig. 274 at the top.

We must assume compression or release to find the exhaust lap. This is rather arbitrary and is done in various ways. It is sometimes determined by assuming that nominal compression raises the compression pressure to initial pressure when the cut-off is very short. With the clearance and compression assumed, initial pressure is reached with a nominal compression of 0.56 stroke (see Chap. XII). Let us assume that with 0.1 cut-off, nominal compression is 0.5 stroke. Laying this off on the Bilgram diagram the exhaust lap is found as shown.

The following data are obtained from the diagram: Valve travel, $4\frac{3}{8}$ in.; steam lap, $1\frac{1}{16}$ in.; exhaust lap, $\frac{9}{32}$ in.

The displacement curve on the piston path is called the valve ellipse; this is sometimes drawn by neglecting the angularity of the connecting rod, but this is taken into account in Fig. 274, the rod being 5 cranks long. The crank circle is usually divided into a number of equal parts (they need not be equal but it is usually more convenient) from which the crank positions are drawn; the correct piston position is found by drawing the arc to the center line as shown for the crank position marked $\frac{1}{10}$ cut-off, and this is produced to the center of the valve ellipse (not a true ellipse) from which valve displacements are measured. The lap lines are then drawn in, the subscripts H and C denoting head and crank end; should exhaust lap be negative it is drawn on the same side of the center as the steam lap, as it is on the same side of its respective port edge. Ordinates measured from these lap lines to the ellipse give valve openings for a single-ported valve at any point; it is then easy to find the part of the stroke at which the valve is opened and closed. The ellipse is tangent at the ends at the ordinate $S_H + l$. The upper valve ellipse is for $\frac{3}{4}$ cut-off (the maximum) and the lower for $\frac{1}{4}$ cut-off. In both ellipses head-end cut-off occurs at b, head-end release at d and head-end compression at e; the subscript 1 denotes the same for the crank end. Admission is so near the ends of the stroke that it may not be well shown in this diagram unless drawn to a large scale.

The diagram may be easily modified for the double-ported valve without changing in any way the timing of the events; the only difference is the increased steam valve opening. The valve is shown to scale in Fig. 271.

There are different methods of proportioning the auxiliary port P_A through the valve; in this case it has been taken as one-half of P_S. The port in the cylinder at the valve face must be equal to $B_V + 2P_A$ $(= B_V + P_S)$ as shown in Fig. 271.

To show the effect of the double port draw lines on the ellipses a distance P_A from the steam lap lines. During this displacement the opening

is twice that due to the displacement, the piston travelling up to the point where the line crosses the ellipse; after this the opening is constant as explained in connection with Fig. 271. When the ellipse again crosses this line the valve begins to close again, closing at *b* as before the double port was added. The opening diagram now has the form *afgb*. For the one-quarter cut-off P_S is less than P_A, so that the double opening obtains all the time the valve is open as shown at *h*.

Piston-velocity curves drawn with a crank-circle radius equal to P_S for the ¾ cut-off are shown plotted on the steam and exhaust lap lines for the long cut-off in dotted lines, and for the short cut-off on the steam-lap line as far as the valve is open. It will be seen that in all cases the valve-opening curve rises quicker than the velocity curve at

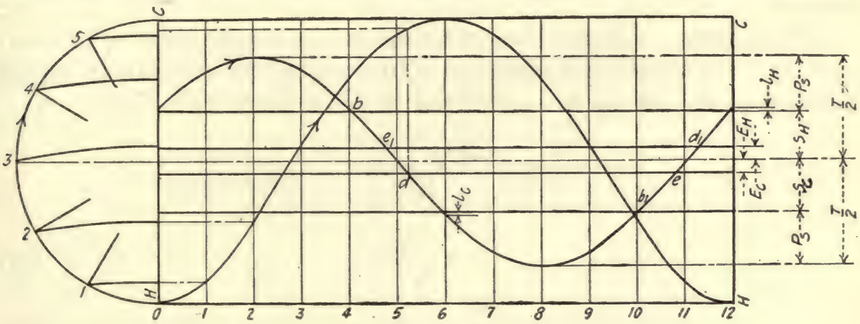


FIG. 275.

first, and for the long cut-off lies above till maximum steam port opening has been reached. The exhaust opening curve is above it for three-quarters of the stroke. For the short cut-off, the velocity line is crossed early by the valve-opening line. The effect upon the steam pressure depends upon the value of nominal steam velocity *V* assumed in determining the ports, but it is obvious that for any type of gear in which the valve closes gradually, the ratio of port opening to piston velocity decreases very rapidly beyond a certain point, and the relative steam velocity increases; if this velocity is very high the pressure head increases and *wire-drawing* occurs—a very common experience. Gradual closing of the exhaust port gives the effect of an early compression and is not objectionable, as allowance may be made for it.

The method of plotting a piston displacement curve on a rectified crank circle, accounting for angularity of the connecting rod, is shown in Fig. 92, Chap. XIII This forms part of a rectangular valve diagram which is shown in Fig. 275. The valve displacement curve may be plotted in the same way, although the angularity of the eccentric rod is usually

neglected. By placing this curve in correct phase relation with the piston displacement curve the crank position for any given event may be determined, from whence the piston position may be found. This is traced through for head-end release by dotted lines, the letters denoting the same as in Fig. 274. Fig. 275 is for the single-ported valve with $\frac{3}{4}$ cut-off in Fig. 274.

This diagram is very valuable for complicated gears difficult to handle in any other way. One or more valve curves may be used, drawn on separate pieces of tracing cloth so that their relative positions may be changed. It is sometimes convenient to draw the lap lines parallel to the curves, especially when an auxiliary valve is used. This diagram will be referred to in connection with sleeve valves for internal-combustion engines.

Valve Stems.—Let d be the diameter of the stem, a the unbalanced area of the valve, μ the coefficient of friction, p the unbalanced steam presures in lb. per sq. in. and S the stress in the stem. Then:

$$\frac{\pi d^2}{4} S = \mu pa$$

or:

$$d = \sqrt{\frac{4\mu pa}{\pi S}} \qquad (24)$$

For practical results, a high coefficient of friction and low stress must be used. Assume $\mu = 0.25$, $S = 5000$ and $p = 125$.

145. Corliss Releasing Gear.—It is probable that, strictly speaking, the Corliss gear is a releasing gear with rocking valves and a wrist plate operated by a single eccentric; however, the name has been applied to engines with two eccentrics and a double wrist plate, and even when a steam wrist plate is omitted. It is also applied to nonreleasing gears with shaft governor and swinging eccentric, although this usage is opposed by some engineers.

In Fig. 276-*A*, a valve is shown at the extreme left of its travel, fully opening the port at the right; as it uncovers the port at the left it travels the distance T. At B, the central arm of a double bell crank travels the same horizontal distance. It is so connected to the two valves that it begins to open the port at the same point in its horizontal projection that A does, and opens the valve the same width at the end of its travel. The distance traveled by the valve is T_1, much less than T. The valve starts to open slowly due to the dead-center effect of the bell crank arm, but opens quickly when the port edge is reached. This is the effect of the wrist plate. If another bell crank is used as at C, and

the movement is made from the full to dotted position, this effect is still more marked due to the more rapid increase of angle α as the valve moves to the right. This effect is also obtained in angle δ of the main bell crank. The relative positions of the bell cranks and the change of angularity of the rod connecting them has influence, and advantage is taken of these principles in different ways in Corliss and other gears.

In designing the Corliss gear two diagrams are employed, one for the wrist plate and valve levers and another for the crank and eccentric. The latter is sometimes omitted, the laps being taken from a table, the values of which are usually for the old, slow-speed type with very late compression and are not generally applicable. Some idea of these diagrams is given in Chap. III.

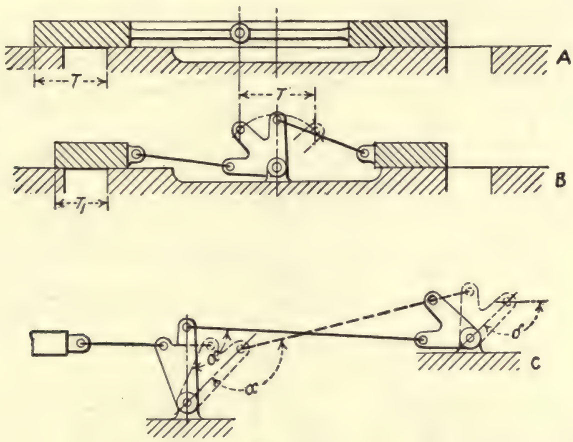


Fig. 276.

The crank-eccentric diagram is that of Fig. 265. The cut-off is effected by the governor and trip mechanism, so the cut-off may be neglected in the diagram; it may be found as in Fig. 265, neglecting lead, and gives the maximum cut-off should the load be too great for the engine, and the governor fail to trip the gear. The distances S and E will be used for convenience but are not the laps; starting with the wrist plate at the center of its travel, they give the positions of the eccentrics which will move the valves by the amount of their laps (lap plus lead in the case of the steam valves).

In most diagrams given in text-books, very late compression is assumed, giving a small angular advance; the resulting maximum cut-off under governor control is therefore about 0.375 to 0.4 stroke and presents no difficulties for reasonable overloads. In laying out the diagram

in this paragraph, the indicator diagram of the Corliss example (20 × 48 − 100) of Chap. XII is used. The problem calls for a long-range cut-off, but the single-eccentric layout will be first taken up. This shows the limit of capacity when a rather early compression (0.8 stroke) is used, and shows why so late a compression and release as are practicable are generally used; also, in conjunction with the diagrams of Chap. XIII, why double eccentrics with separate steam and exhaust wrist plates were

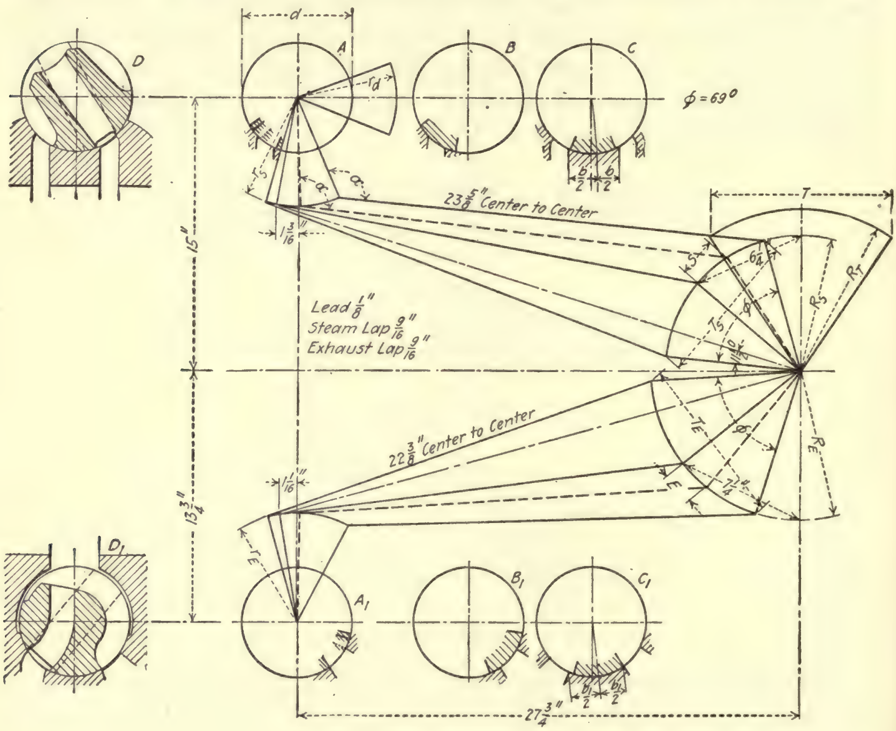


Fig. 277.

used on the low-pressure cylinders of compound Corliss engines even though the high-pressure cut-off gave ample overload capacity with a single wrist plate. The single-wrist-plate diagram is shown in Fig. 277 and the crank-eccentric diagram in Fig. 278. These are for the head end, and as the gear is so adjustable and cut-off depends upon governor connections, angularity of the connecting rod is neglected in this paragraph.

Procedure.—The diagram for wrist plate and eccentric may be first drawn. Until considerable experience is had, this may require a number of trials, but for a given compression and release, formulas may be made

which greatly reduce labor. The method by which Figs. 277 and 278 were drawn has been used by the author for several years with satisfaction and dispatch. The dimensions given are those practically required

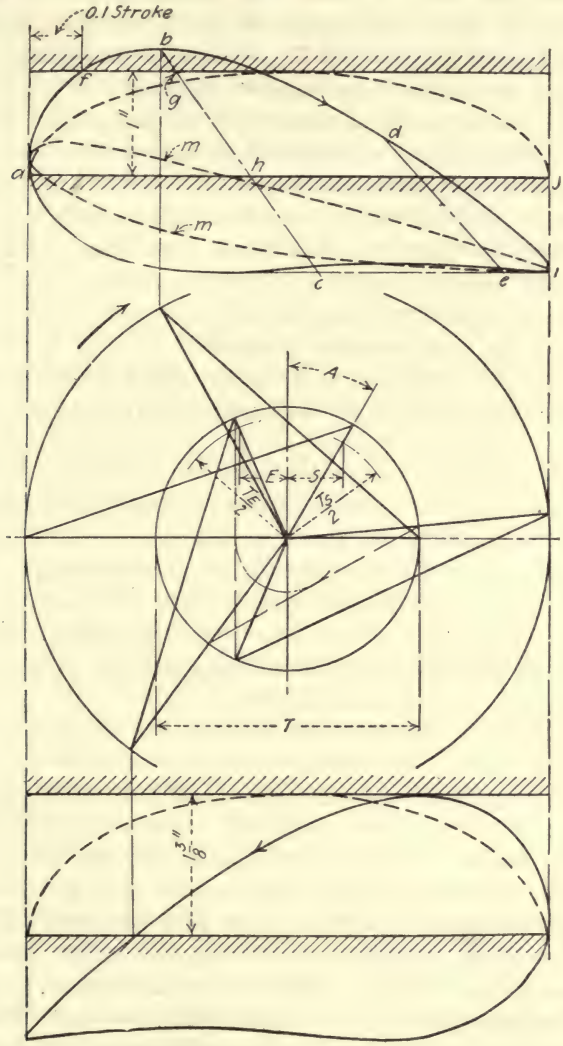


Fig. 278.

with the exception of the angles, which are omitted. For standard gears already designed, and usually intended for several cylinder diameters, r_S and r_E are fixed. For new work their value is more or less

arbitrary, the minimum depending upon the detail design of the mechanism. In Fig. 277 they are taken equal to d, the valve diameter, which in this case was computed by the formulas of Par. 142. R_S was taken as $r_S + 1.5$ in. and $R_E = 1.1R_S$. The radius R_T of the main wrist-plate pin depends upon the design of the reach-rod releasing mechanism, but its horizontal travel is T. Draw arcs of radii r_S and r_E from valve centers; from center of wrist plate draw arcs of radii R_S, R_E and R_T. Draw lines from center of wrist plate tangent to arcs of r_S and r_E. For new work draw valve levers normal to these lines. For old designs the levers must be so located that the travel of the dashpot pin at radius r_A will be equally divided by the horizontal center line, but the normal line may be used tentatively. Now locate a point on arc R_S below the tangent line, and on arc R_E above the tangent line. This is arbitrary, but is made about 11.5 degrees in Fig. 277. Old practice made this point on the line and for very late compression this is permissible; but the method shown reduces the travel when the valve is closed. It necessitates a greater angle of wrist-plate motion ϕ, which is 69 degrees in this design.

Now connect this point with the valve lever position already found; locate other extreme wrist-plate positions by angle ϕ, which is arbitrary and found by trial; and bisect, giving the position when the wrist plate is at the center of its travel. With the length of connecting link found for the first extreme position laid off with a beam compass, find the valve lever positions to correspond with the other wrist-plate positions.

For single-ported valves the dimensions would be different, but these are not much used.

Draw in one of the ports in any position, the location of the second port being found later. Show the steam valve edge dotted in the lead position which may be taken from $\frac{1}{16}$ to $\frac{1}{8}$ of the port width; the latter was assumed.

Now draw the crank-eccentric diagram for the compression and release desired; the latter is taken arbitrarily and not far above the center line as shown; in Fig. 278 it is $\frac{1}{4}$ in. on a 5 in. circle. The angular advance may be found as described for Fig. 265. Now draw the crank on the head-end dead center. The eccentric circle may be made equal to T if this is greater than T_E; its value is unimportant. For the steam valve, cut the angular advance position with arc of radius $T_S/2$; this gives the virtual eccentric radius to give the steam wrist-plate arm the travel T_S. Lay off S on Fig. 277, giving the position of the wrist-plate pin when the crank is on the dead center. Find the corresponding valve lever position; this is shown dotted. This position corresponds with

the lead position of the valve; taking this distance along the valve circle on a divider, the three positions (two extreme and central) of the valve may be found. The position corresponding to the central position of the wrist plate gives the steam lap.

It will be noticed that when the valve is wide open there is some over-travel; this gives a little quicker valve opening.

At B draw the valve edge in its extreme position; this is some greater than that indicated at A due to the wrist-plate arm crossing its dead center, and besides this some clearance must be allowed the latch hook; but this is usually small and may be overlooked if ample allowances are made. When in position B, the back edge of the valve must have sufficient seal to prevent leakage; this was made ⅝ in. in Fig. 277, but may be greater for large engines and less for smaller engines. This determines one valve face. Now move this face to its extreme open position as at C. It is well to allow a small amount for adjustment, and discrepancies in the cylinder casting as shown. This determines the bridge width. Draw a center line through the bridge as shown and locate on the vertical center line of the valve chamber as at D. The valve design may be now completed.

The exhaust valve may be treated in the same way, the dotted position corresponding to compression, when the valve is "line-and-line" (just closing). Dimensions may be determined by B_1, C_1 and D_1 as for the steam valve. The exhaust valve is sometimes made symmetrical as with the steam valve.

The dotted lines across the valves in D and D_1 show the slot for the tee-head of the valve stem; it is located so that it is vertical when the valve edge is halfway between line-and-line and full closed positions, thus allowing for wear.

Valve-opening diagrams for steam and exhaust valves are shown above and below the crank-eccentric diagram respectively in Fig. 278. The steam valves open quickly, the more so because of the over-travel. A piston velocity curve is shown dotted, but the opening curve is above it until past the point of maximum cut-off. The theoretical point of maximum cut-off is found by placing the eccentric to the extreme right of its travel; this places the wrist plate in its extreme position, and if cut-off does not occur before this time the valve is not unhooked and the opening curve is that shown by the full line. But past the point when the eccentric and wrist plate are at the extreme right the governor does not control, so that this gives the limit of valve release. However, it takes some appreciable time for the dashpot to close the valve, so that the real cut-off is later.

For the early compression assumed, the angular advance is so great that the maximum cut-off under governor control is but $\frac{1}{4}$ stroke; this is at *b*, the valve being released at this point. Furman says that a good dashpot will close the valve in $\frac{1}{16}$ sec., which for 100 r.p.m. would be about 0.1 of a revolution. Assuming the valve to be entirely closed in this time, the piston would travel the horizontal distance between *b* and *c*; further assuming the closing curve between these two points to be a straight line, the actual displacement curve would be *abcic* and the opening of the port would follow the line *afghjh*. It will be noticed that the port is wide open at 0.1 stroke. The actual closing for maximum cut-off begins at about 0.29 and ends at 0.43 stroke; a relatively slower dash-pot would give a later cut-off, and if the speed were high, would show signs of wire-drawing.

In Fig. 277, the exhaust valve shows no over-travel but the opening is quick; it is not likely that the opening would be less than shown as the compression is not apt to be earlier than assumed, but if some over-travel is desired, R_E may be increased, thus increasing the angle of valve travel. The displacement curve needs no modification as there is no releasing mechanism for the exhaust valve.

Long-range Cut-off.—These gears were formerly furnished with two eccentrics and a double wrist plate, the steam and exhaust portions working independently. For the design considered the exhaust wrist plate may be exactly as determined, so will not be redrawn. A wrist plate is sometimes used for the steam valves, but the travel of the valve from the central wrist-plate position to full closed position is nearly the same as to the full open position, therefore, the wrist-plate pin must be located nearer the top and the angular motion much reduced. This led to abandoning the wrist plate for the steam valve and this will be assumed; then in Fig. 279 the wrist plate is not shown. Fig. 280 is the crank-eccentric diagram.

As in this problem a $\frac{3}{4}$ cut-off under governor control is desired, the crank must be in this position as the wrist plate and eccentric are at the extreme right; moving the crank back to head-end dead center brings the eccentric back of the vertical center, giving negative angular advance. As the valve is open the amount of the lead in this position, it is obvious that when the wrist plate is at the center of its travel the valve is open, giving negative steam lap, or steam clearance. The design is completed at *A*, *B*, *C* and *D* as before, taking *S* on the valve lever, which travels equally either side of the vertical center; a rod connects this with the crank-end lever which is longer, and receives the reach rod. The bridge is made the same as for the short-cut-off gear but the valve faces are less.

The displacement diagram is practically the same as for a single-valve engine, but due to a negative angular advance, is tilted in the opposite direction; if the gear fails to trip, cut-off will not occur until well on the return stroke.

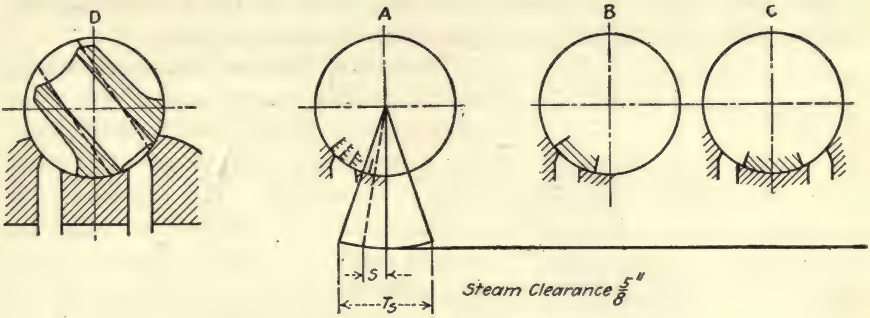


FIG. 279.

Assuming the valve to close in 0.1 of a revolution as before, release of the gear would occur at *b* to give a ¾ cut-off; the actual maximum cut-off would therefore be much greater, nearly full stroke. The actual opening curve would be *afghjh*. The opening is not quite as quick as

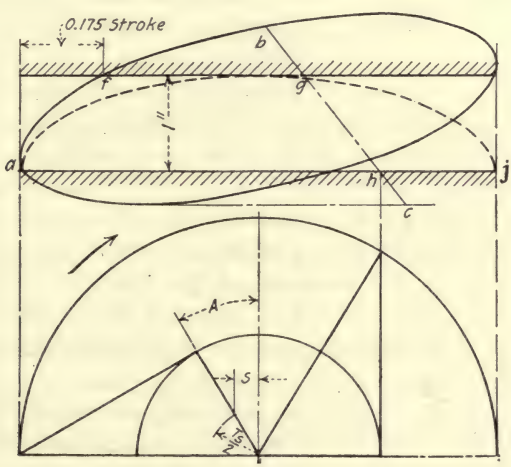


FIG. 280.

before, but while there is a large over-travel, the maximum valve travel is not as great as with the short-cut-off gear, the travel when the valve is closed and under pressure being much less. The large over-travel is necessary for a quick port opening and may even be increased if desired.

Releasing Gear Operation.—There are various designs of releasing gears used by different builders, but the principle is practically the same in all, with one exception (see Nordberg Bulletin). A simple sketch (not an actual design) will be used to illustrate the principle and this is given in Fig. 281, which is similar to the gear used on the Fishkill Corliss engine. This type is used as the operation is clearly seen.

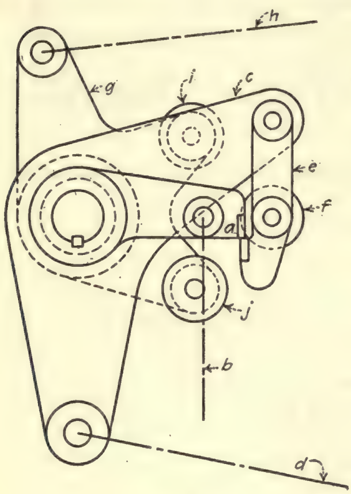


FIG. 281.

The valve lever or steam crank a is keyed to the valve stem. To it is attached the dashpot rod b which pulls the lever down and closes the valve when it is released. The bell crank (or steam rocker) c receives a constant motion from rod d which connects with the wrist plate. At the upper end of the bell crank is hung the latch e, having a hardened steel block at its lower end, which engages with a like block on the end of the valve lever. The latch is secured to a pin which passes through a hub at the end of the bell crank, and at the other end is attached a lever, sometimes called the knock-off lever, which carries the roller f; if this roller is forced away from the valve stem the latch is unhooked. The cam collar g, sometimes called the governor toe collar, moves independently of the other mechanism and is attached to the governor bell crank by the rod h. The cam collar carries two rollers, i being the knock-off cam and j the safety cam; as bell crank c oscillates, the latch roller f may come in contact with one of the cam rollers, depending on the position of the governor when the latch is unhooked. For simplicity it will be assumed that the latch is unhooked when the centers of the rollers are on the same radial line from the center of the valve stem.

Fig. 282 is a diagram showing governor and cam roller in the three important positions. A is the safety position, B the starting position and C the zero cut-off position. These will be taken up separately. The latch roller has a constant oscillation through angle θ.

The safety position A. The governor is in its lowest position. The latch is held away from the valve lever by the safety cam and will not "hook up." The valves remain shut and the engine will not start, or if running, will stop. To start the engine the governor must be lifted

so that it rests on the high part of the safety collar, or more safely, on a safety pawl which drops out of the way when the engine gets up speed and the governor lifts.

The starting position B. The valves will hook up and will not unhook, as both cut-off and safety cams are out of reach of the latch roller. Cut-off will be late, depending on the angular advance. When the engine speeds up and the governor begins to take control, it lifts from the safety pawl (shown by separate sketch), which drops out of the way, allowing

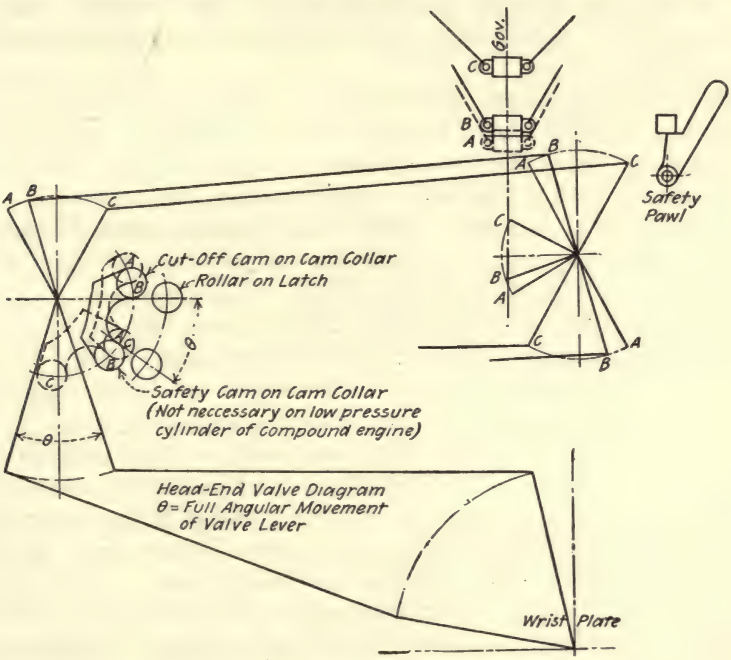


FIG. 282.

the governor to sink to its safety position should the belt break. The so-called safety collar which must be turned to the safety position by hand after the engine is started is more convenient when stopping the engine, but is only a partial safety device.

Zero cut-off position C. The latch is held away from the valve lever by the knock-off cam and will not hook up. This is a limiting position and is never reached unless the governor "hunts," as the cut-off must at least be long enough to carry the friction load.

The governor in action gives any cut-off between zero and the maximum cut-off under governor control. Actual cam positions may be found

for a given cut-off which is to be the same at head and crank ends. For equal cut-offs, the knock-off and safety cams would be closer together at the head than at the crank end if it were necessary to have the safety cam in contact at the extreme lowest position of the latch roller; for, as the piston moves farther on its stroke from head to crank end than on the return stroke for the same movement of the eccentric, cut-off would occur later in the head-end than in the crank-end stroke if latch roller and knock-off cam bore the same relation at both ends of the cylinder. Then to equalize a given grade of cut-off, the head-end cam must be nearer to the latch roller, or the crank-end cam farther away, or both.

The safety cam should always keep the latch from hooking up when the governor is in the lowest position, but the engagement with the latch roller need not be at the lowest position; the valve lever may move a certain distance before the valve begins to open, and the release of the latch may occur at any point within this distance, which is much less for the long-range cut-off than for the single-wrist-plate gear as may be seen by comparing Figs. 277 and 279.

Comparing the long- and short-range gears, the same governor motion (and consequently the same speed fluctuation) is used for both; then for the same range of load it is obvious that regulation will be closer for the long-range gear; this is true of any type of valve gear.

In *cross-compound* engines the low-pressure cut-off may either be adjusted by hand to any fixed position, or be under the control of the governor. In the latter case the receiver pressure is kept more nearly constant under change of load, avoiding very low pressures with light loads and excessively high pressures with heavy loads. When the low-pressure cut-off is controlled by the governor, the fulcrum pin of the controlling lever is lengthened into a *cross shaft* having one bearing on the governor stand, which is usually located on the high-pressure engine frame, and the other on a stand occupying a similar position on the low-pressure frame and sometimes called the "dummy." A lever is placed on the low-pressure end of the shaft, with rods connecting it with the cam collars of the low-pressure gear. This lever is sometimes constructed so that it may be adjusted while the engine is running, dividing the load between the two cylinders as desired. It is sometimes arranged so that the governor may be temporarily disconnected and hand adjustment used for starting as explained in Chap. XIII.

Figs. 247 and 248, Chap. XIX show safety devices used in connection with Corliss and other trip gears, and are explained in the text.

Corliss Valve Stems.—With certain assumptions the following formula

has been derived, where D is the cylinder diameter, d the valve diameter and d_1 the diameter of the stem at the smallest part.

$$d_1 = 0.03\sqrt[3]{pd^2 D} \qquad (25)$$

The maximum steam pressure p may be taken as 100 lb. per sq. in. for low-pressure cylinders to give practical results.

146. High-speed Corliss Gears.—This term is commonly applied to non-releasing gears when rocking valves and a shaft governor are employed. The diagrams are much the same as for the releasing gear, these giving the maximum cut-off, which may be about ¾ stroke; then for the shorter cut-offs the eccentric swings or shifts, giving smaller valve travel and greater angular advance. The steam distribution is practically the same as for a single valve except that the opening is a little quicker. Assuming the same diagram as Figs. 277 and 278, the curve m is the valve opening curve for ¼ cut-off; it indicates that greater over-travel for the maximum cut-off would be desirable.

147. McIntosh and Seymour Gear.—Formerly the gridiron-valve engine built by McIntosh and Seymour had expansion

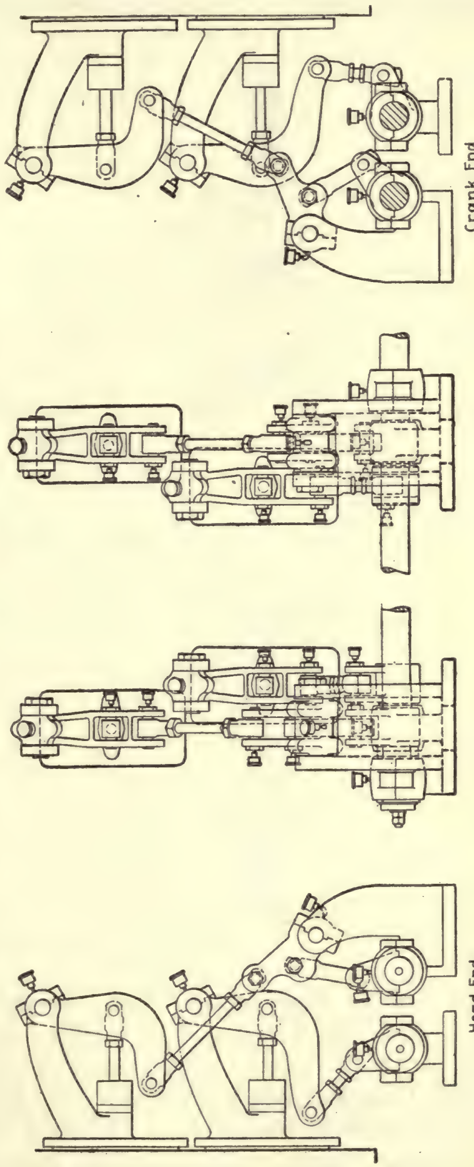


Fig. 283.—McIntosh & Seymour valve gear.

valves on the steam valves, but in the Type *F* engines these are omitted, greatly simplifying the gear and still giving good results. End and side elevations of this gear are shown in Fig. 283. The motion for the steam valve is equivalent to one wrist plate driving another. The valve movement is very small. The mechanism is operated from rocking lay shafts

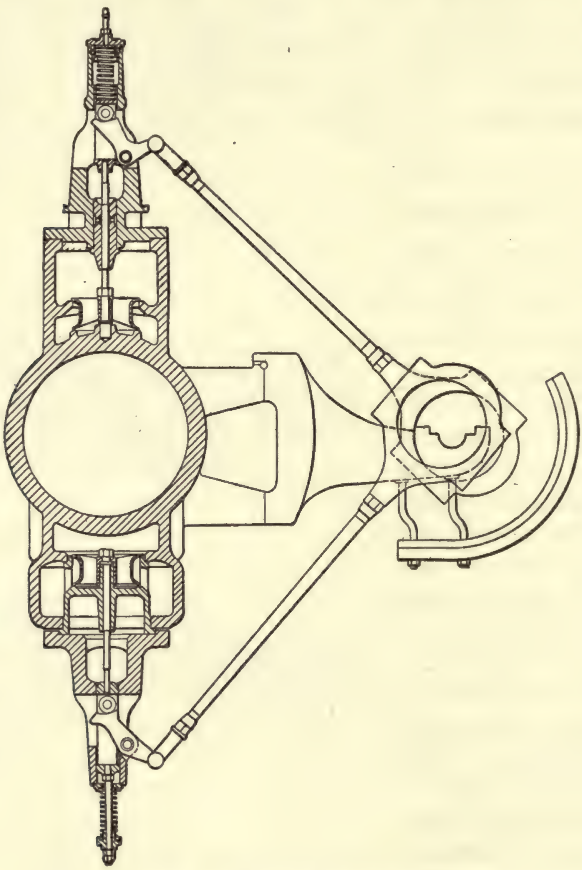


FIG. 284.—Lentz valve gear.

which receive their motion from eccentrics. The steam eccentric is attached to the governor and is offset in its connection with the lay shaft for the purpose of obtaining equal cut-offs.

148. Cam and Eccentric Gears.—In these gears the cam is used in place of the wrist plate and for the same purpose. They are used only with poppet valves, their advantage being that they are free to move

after the valve is seated without interfering with it. The crank-eccentric diagram of Fig. 265 may be used and the cam designed to give as quick an opening as practicable. As with the Corliss gear, if the lead is ignored in drawing the crank-eccentric diagram, the cut-off will be some longer than the diagram indicates; a slightly shorter cut-off may be taken to offset this if desired.

Lentz Gear.—The eccentrics of this gear are on a lay shaft driven by miter gears. The steam eccentrics (one for each end) are connected with the governor, being shifting eccentrics with constant lead. An end section showing the general arrangement is given in Fig. 284. In laying out the crank-eccentric diagram for the steam valves, the line of stroke would be assumed as a line connecting the lay shaft center with the mean position of the cam arm; the dead-center crank position for corresponding ends of the cylinder would lie along this line toward the cam. For the exhaust valves, the dead-center position would be on the side opposite the cam. If the engine is right hand and runs over, the rotation for Fig. 284 is counterclockwise.

149. Reversing Gears.—Space will not permit an extended treatment of this subject, but a brief description of a few of the most important gears will be given, with some explanation of the principles involved; for a description of other types, or a fuller treatment of any given type the reader is referred to the works named at the end of the chapter. Most of the illustrations are diagrammatic.

Reversing gears are usually associated with the locomotive and ship engine; they are also used for hoisting, rolling mills and for some other stationary engines. They are sometimes used in connection with the Corliss gear, in which case their function is reversing only; but in most cases they are also used for varying the cut-off. This latter function was probably discovered accidentally. As first applied to locomotives, reversing gears consisted of two eccentrics, each with its own eccentric rod with sort of a crab-claw hook on the other end; either one of these could be hooked onto the rocker pin by a double bell crank operated from the cab; this gave full gear forward or backward. It is obvious that the reversing operation could not well be done when the locomotive was under much motion, and it was probably to obviate this defect that the sliding link of the Stephenson gear and the sliding block of the Walschaert gear were devised, both of which were invented about the year 1844.

Most reversing gears give an effect similar to a single shifting or swinging eccentric, and when the path of change can be approximately determined, the Bilgram diagram may be used for preliminary work if a single valve is employed.

Stephenson Reversing Gear.—This is perhaps the best known and the most difficult to lay out, as there are many variables which effect the results. Fig. 285 shows a link which has been used on stationary engines; the proportional figures are based upon the diameter d of the saddle pin, which may be made 0.1 the cylinder diameter (D_S, Chaps. XII and XIII) for unbalanced valves and ¾ of this for balanced valves. A is the saddle with pin forged integral, by which the link is lifted and lowered; while B is the link block which is drilled and counterbored for the pin which is fitted to rocker or valve-rod crosshead. For solid-ended link as shown, one flange of the link block is loose, and fastened to the block with rivets or countersunk screws.

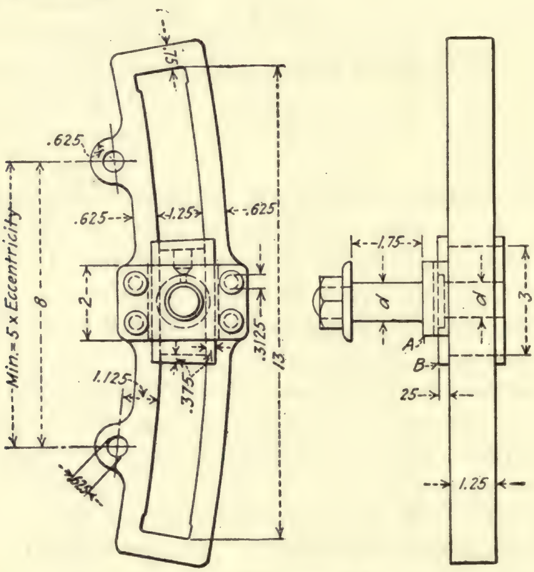


FIG. 285.—Stephenson link.

Fig. 286 shows the gear in neutral position (mid-gear) at A, forward gear at B and backward gear (reverse) at C, as arranged for a locomotive with outside admission and the link block attached directly to the valve-rod crosshead. Should there be inside admission, or a rocker which reverses the valve motion, the crank would be on the opposite dead center. As it is locomotive drafting room practice to show the cylinder at the right, this has been assumed in Fig. 286, although for all previous work it has been at the left.

Equivalent Eccentric.—The link radius is usually equal to the distance from the shaft center to the center of the link block at mid travel. In

Fig. 287 assume the link to be straight and the eccentric rods so long that their angularity has no effect; further assume that the valve-rod cross-head is raised and lowered instead of the link. Let the valve rod be

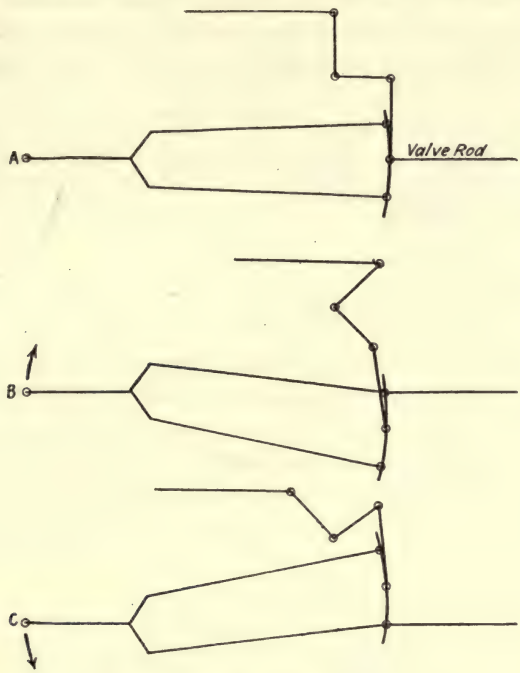


Fig. 286.

moved to position B, $\frac{1}{4}$ of the distance to the lower end of the link. Now let the crank and eccentric be rotated clockwise through a small angle, moving the end of the link the distance a; the valve rod would then

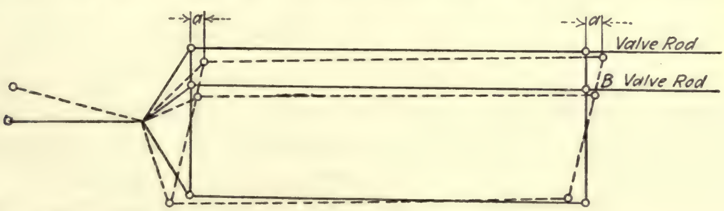


Fig. 287.

be moved the distance $a/2$. If the two eccentrics are connected by a line, it is clear that a point located $\frac{1}{4}$ of the distance from top to bottom of this line would travel a horizontal distance of $a/2$. The movement of

the valve rod due to two eccentrics and a link is then equivalent to that produced by a single eccentric which shifts along this connecting line in the same ratio that the link block is shifted.

Open and Crossed Rods.—If the rods are open when the eccentrics are both on the side toward the link, they are known as open rods. If crossed when in this position they are known as crossed rods; this is shown in

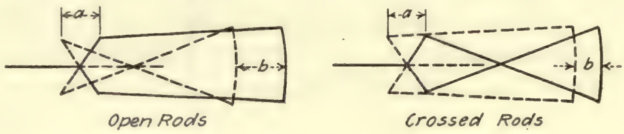


FIG. 288.

Fig. 288. Placing the eccentrics in the opposite position indicated by the dotted lines gives the center of the link its maximum travel b; this is the valve travel for mid-gear, or when the equivalent single eccentric has an angular advance of 90 degrees. From Fig. 288 it is obvious that due to the effect of angularity, b is greater than a for the open rods and less than a for the crossed rods; it may also be seen that when the crank is on the dead center in each case the valve displacement for full gear is $a/2$ and for mid-gear it is $b/2$. Assuming the gear to be so designed that full-

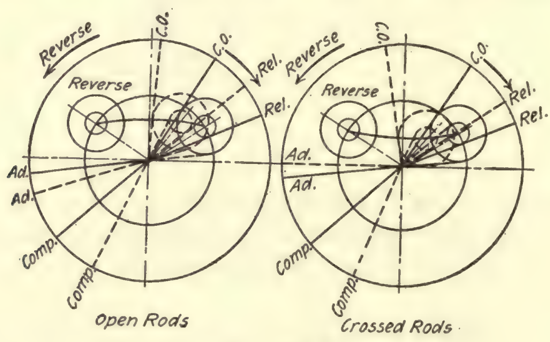


FIG. 289.

and mid-gear lead are the same at both ends of the stroke, this may be shown on the Bilgram diagram as in Fig. 289. The curve of eccentric change is unknown but has been assumed as the arc of a circle.

It will be noticed that for open rods the lead increases from full to mid-gear; this is always used for locomotives and is ideal for the service as it gives a greater port opening when the gear is "notched up" to give short cut-off at high speed. Equalization of lead is also more important at mid-gear than at full gear for the same reason.

Crossed rods give a decreasing lead which may be zero or even negative at mid-gear; this is sometimes used for hoisting engines and tractors when it is desired to control the engines by the reverse lever; if placed in mid-gear no steam is admitted.

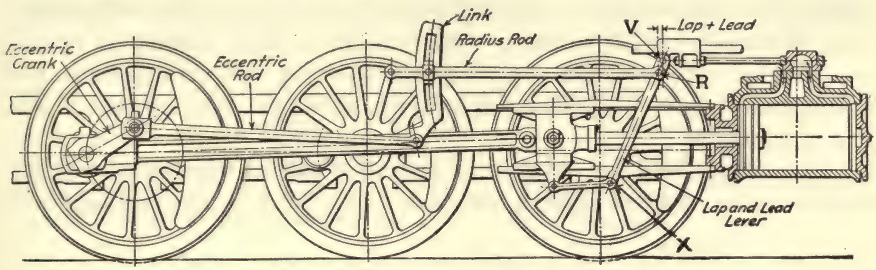


Fig. 290.—Walschaert gear.

The most complete practical treatment of the design of the Stephenson link motion with which the author is acquainted is given in Link and Valve Motion by W. S. Auchincloss.

Walschaert Reversing Gear.—This gear has largely replaced the Stephenson gear in locomotive construction in the United States. This is

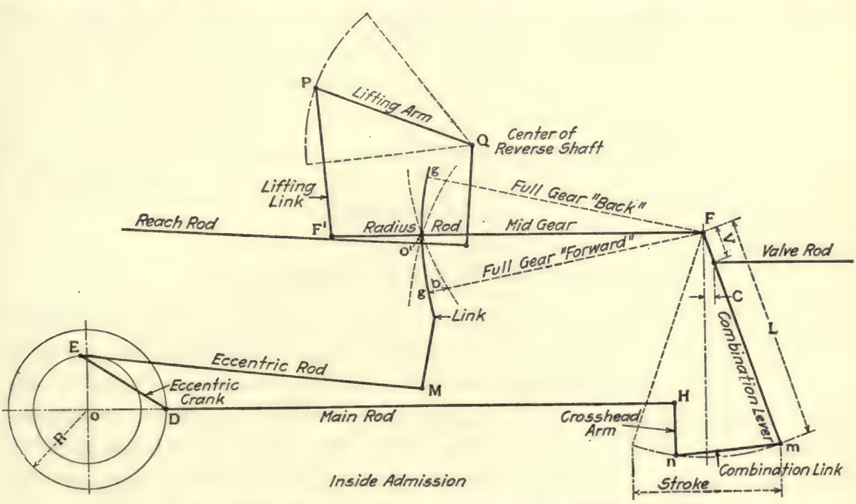


Fig. 291.—Walschaert gear for inside admission.

largely due to constructional purposes, although it possesses some other advantages. Fig. 290 shows a general view of the gear with the crank on crank-end dead center and the radius rod in mid-gear position. This is for outside admission which is used for flat valves. Inside ad-

mission is used with piston valves and requires a different arrangement. The radius rod is dropped to the lowest position for full forward gear; this divides the force required to move the valve between the link trunnions and the eccentric rod. For back gear the radius rod is above the trunnions, which carry the combined loads of radius rod and eccentric rod, which, with the exception of switching engines is for a small proportion of the time. Figs. 291 and 292 are diagrams showing the arrangement for inside and outside admission respectively, with the crank on the head-end dead center.

Equivalent Eccentric.—It may be seen that the eccentric has no angular advance; the effect of angular advance is obtained by the combination

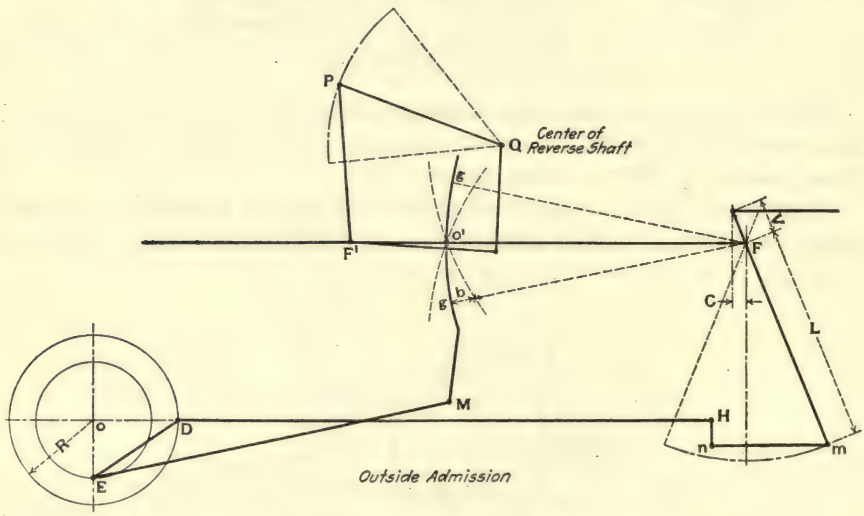


FIG. 292.—Walschaert gear for outside admission.

lever, which, when the radius rod is in mid position moves the valve twice the sum of lap and lead. The method of proportioning this lever is obvious. It is also plain that the effect of the crosshead might be obtained by an eccentric with an angular advance of 90 degrees for an outside-admission gear as shown at *B*, Fig. 293. The eccentric which operates the link is at *A*. Connect these two points by a straight line; on this line a point *C* will be located which depends upon the proportions of the combination lever, which in turn is determined by the lap. Then *C* is the equivalent eccentric which would give the same results as the gear. When the reverse lever is "notched up" the radius of eccentric *A* (which has no angular advance) is virtually decreased—say to *OD*. As the lead is constant, *C* must travel parallel to line *OD* to the new position; at any

rate the ratio AC/AB is constant, so C must move in a vertical line. As C moves to the center line, eccentric A has no effect on it; the angular advance of the equivalent eccentric is 90 degrees and the valve travel is twice the sum of lap and lead. As C crosses the line toward A_1, the gear is reversed. This may be shown on the Bilgram diagram as in Fig. 294.

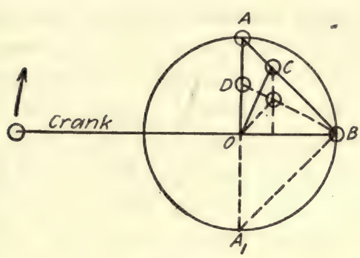


FIG. 293.

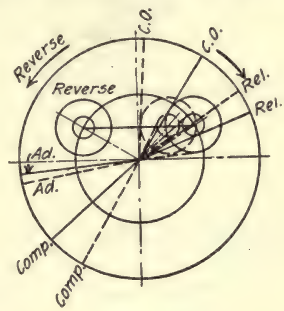


FIG. 294.

Variable lead is sometimes used on passenger and fast freight locomotives equipped with the Walschaert gear. It is obtained at the expense of too great a lead in back gear, but this is not much used on these engines. Maximum mid-gear lead is determined, which may be greater if the lead is to be variable. The eccentric is then set back for forward gear by

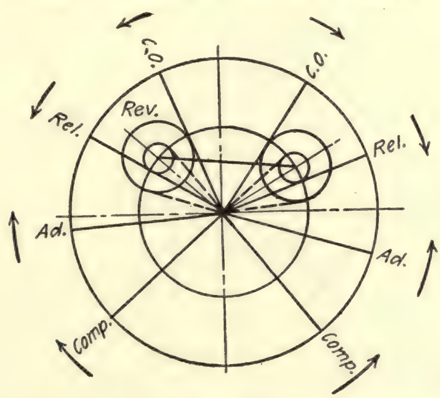


FIG. 295.

swinging the eccentric crank out or in, depending upon whether it leads or follows the crank, thus reducing the full-gear-forward lead and increasing the full-gear-backward lead. This may be shown on a Bilgram diagram as in Fig. 295. This also gives a longer cut-off and later compression in forward gear upon starting, but shorter in back gear.

Hackworth and Marshall Gears.—A simple diagram of the Hackworth gear is shown in Fig. 296. A block slides in the straight link which may be turned through angle A, which reverses the engine. If placed in a vertical position the link has no effect and the valve travels twice the sum of lap and lead. The link and sliding block are equivalent to an eccentric fixed at right angles to the crank (with no angular advance); there is therefore a similarity of principle to the Walschaert gear. In the Marshall gear the link and block are replaced by a swing link as shown in Fig. 297. Figs. 296 and 297 are for outside admission; for inside admission the eccentric must be on the same side of the circle as the crank, or the eccentric rod must be produced beyond the link block and the rod attached to this extension. The Hackworth gear is used on the Case steam tractor. The Marshall gear has been used on reversing Corliss rolling mill engines of large power.

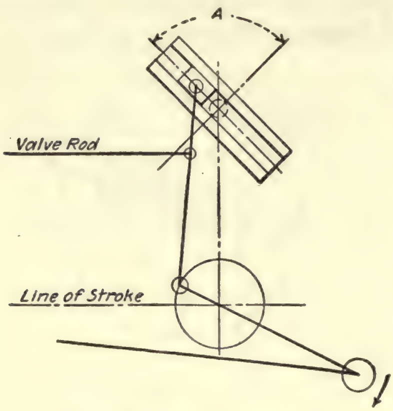


FIG. 296.—Hackworth gear.

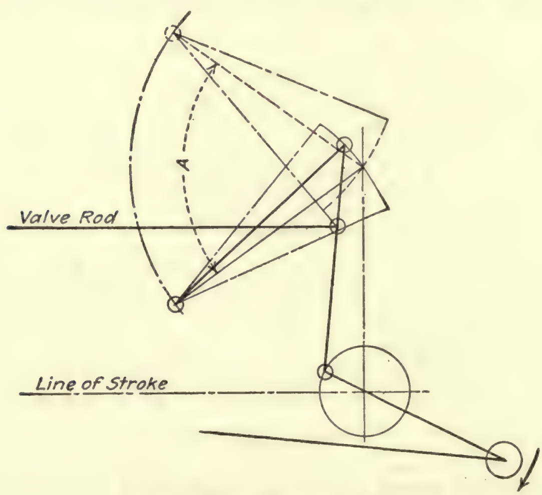


FIG. 297.—Marshall gear.

Joy Reversing Gear.—In this gear no eccentric is used, but the motion is taken from the connecting rod. A link is attached to the rod at A and another to the frame at B; these connect at C. To the link AC

another link is pivoted at D; this is equivalent to the eccentric of the Marshall gear, the remainder of the gear being much the same. The arrangement is equivalent to having the eccentric on the same side of the

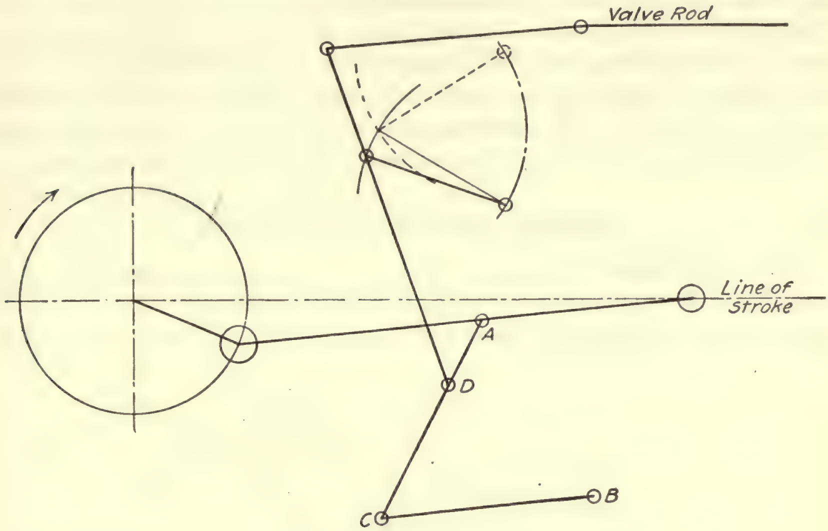


FIG. 298.—Joy gear.

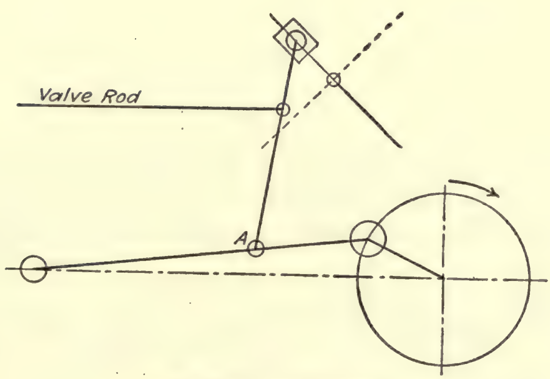


FIG. 299.—Doble gear.

shaft as the crank, therefore for outside admission the reversing link must be between the point D and the valve rod.

The Hackworth, Marshall and Joy gears have constant lead; but variable lead could be obtained as with the Walschaert gear by causing the angle between the center line of action of the eccentric rod and the

line of stroke to be slightly different from 90 degrees. The Bilgram diagram for these gears is the same as for the Walschaert gear.

The Doble reversing gear, used on the Doble steam car, is like the Joy gear with links AC and CB omitted, the eccentric rod being attached at A on the connecting rod. The link block slides in a straight line, being like the Hackworth gear in this respect; it is practically the same as the Hackworth gear with the eccentric replaced by the connecting rod attachment at A, Fig. 299. The valve has inside admission. The Doble engine is a uniflow engine, so that the gear controls the steam valves only.

INTERNAL-COMBUSTION ENGINES

150. Cam Gears.—General arrangements of cam gears may be seen in Chap. V. The design of cams in general may be found in books on mechanism, but application will be made to two cases. The first thing to be

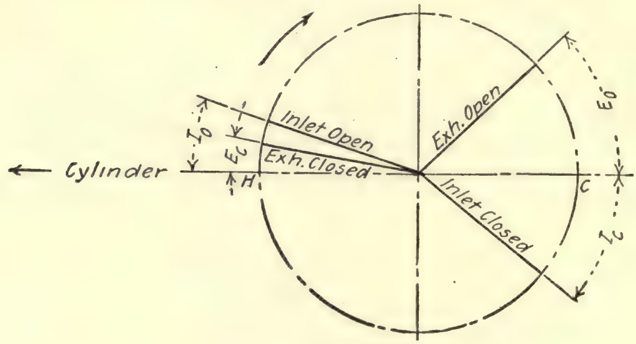


FIG. 300.

determined is the valve timing, and Fig. 300 gives the position of the crank, assuming a horizontal engine. Fig. 301 shows the timing laid off on rectified crank circles, two revolutions being required for the 4-

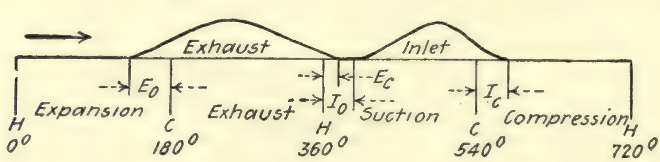


FIG. 301.

cycle engine. The letters H and C denote head and crank ends of the stroke respectively. Fig. 302 shows a timing diagram referred to the cam shaft, being curves of cam rise on a rectified zero circle of the cam.

As the cam makes one revolution per cycle, the angles e and i are one-half the angles E and I respectively. It may be assumed that the cam

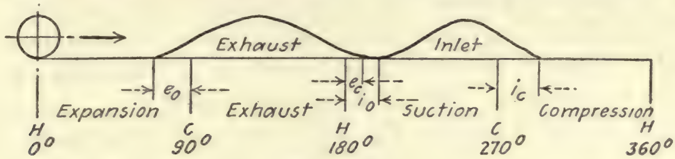


Fig. 302.

roller travels in the direction of the arrow, the letters H and C indicating the corresponding positions of the crank.

Table 66 gives values of E and I taken from various sources. The

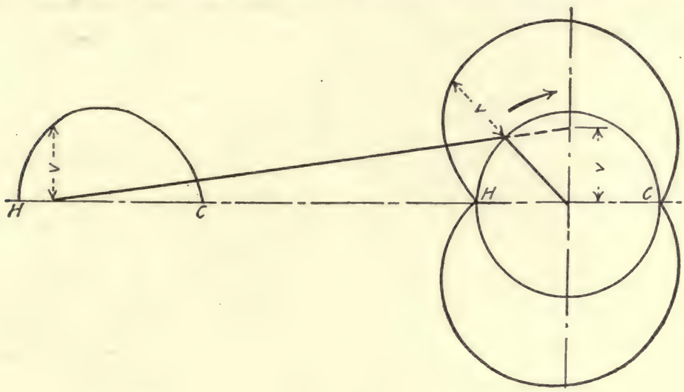


Fig. 303.

Table 66

Case	Eo	Ec	Io	Ic
Roberts' Handbook	45.3	10.3	14.7	35.4
Large, slow-speed	25.0	0.0	5.0	10.0
Small, medium- or slow-speed	25.0	0.0	3.0	4.0
Small, high-speed	35.0	2.0	6.0	15.0
Bruce-Macbeth	45.0	15.0	10.0	45.0
Continental 4-cylinder truck	42.6	8.3	17.9	29.4
Continental 6-cylinder auto	55.0	12.0	12.0	45.0
Racing engine	50.0	10.0	10.0	50.0
Low-speed (*Power*, Oct. 13, 1914)	30.0	5.0	10.0	10.0 to 15.0
High-speed (*Power*, Oct. 13, 1914)	40.0	10.0	15.0	30.0 to 35.0

negative sign indicates that the event occurs after the dead center at which it is theoretically supposed to occur.

The method of drawing the piston velocity curve is given in Chap. XVI; the radius of the crank circle may be taken equal to the rise of the cam roller, giving a curve of proper scale to be transferred to the cam layout for comparison. This is shown in Fig. 303.

Using Fig. 302, as a guide, Fig. 304 is drawn for the exhaust cam with roller. The cam rise is denoted by h, the radius of the base circle of the cam by R and the radius of the roller by r. The following proportions are used in Fig. 304:

$$R = 2h, \; r = 0.6R = 1.2h.$$

It may be assumed that the cam roller rotates clockwise while the cam is stationary. The engine crank passes through positions H and C

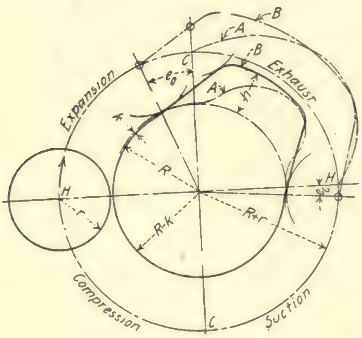


FIG. 304—Exhaust cam with roller.

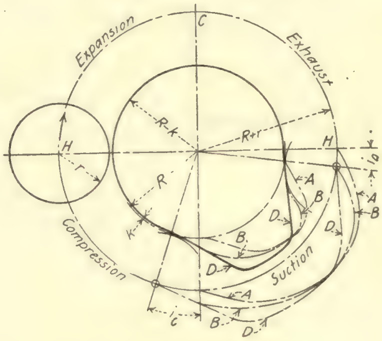


FIG. 305.—Inlet cam with roller.

(Fig. 303) coincident with the passing of the center of the cam roller through positions so marked on Fig. 304. If a separate spring is used to return the push rod and other mechanism transmitting motion to the valve stem, the cam roller rides on the cam body of radius R, the clearance k being between the push rod and valve-stem lever. According to G. W. Meunch in *Power*, Oct. 13, 1914, k should be about $\frac{1}{32}$ in., and somewhat more for large engines; the clearance should be adjusted when the engine is hot. For small engines with very high speed the clearance is sometimes made less.

In drawing the cams the center of the roller is placed at the opening and closing angles as shown; the cam-rise lines are then drawn tangent to the base circle and roller circle as shown. The circular arc forming the crown of the cam is then drawn, and joined to the tangent lines with a radius. This curve may be transferred to the roller-center circle as

shown. The velocity curve may be laid off on this circle, care being taken to place the crank-end of the curve (the low end) at the point marked C. If desired the velocity curve may be transferred to the base circle as shown by heavy dotted line. Curves A are the velocity curves.

It may be seen that the clearance k influences the form of the cam, a small clearance causing the valve to open and close more slowly.

Fig. 305 is the inlet cam drawn in the same manner. Due to the opening of the valve after dead center is passed, the cam curve lies below the velocity curve at first. The cam curve B tends to correct this, but the curve shown at D is the curve used, as the cost of production is less.

A type of cam much used for small engines has a push rod with mush-room head with a hardened steel face. Exhaust and inlet cams are drawn together in Fig. 306. The form of the cam is quite different, and the opening and closing are more rapid. The push rod is held against the cam by a spring, the clearance being between the push rod and valve stem or lever.

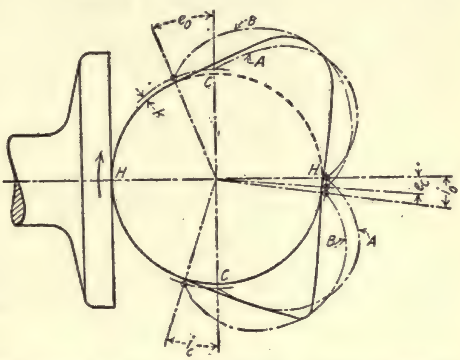


FIG. 306.—Inlet and exhaust cams—mushroom type.

Multi-cylinder Gears.—The usual order of firing for multi-cylinder engines is given in Par. 106, Chap. XVI. There is some difference of opinion regarding the 8-cylinder engine. Considering cylinder No. 1 at the front of the car and the driver's right as R, the usual order of firing is: 1-L, 2-R, 3-L, 1-R, 4-L, 3-R, 2-L and 4-R. *The Gas Engine*, April, 1915, gives for the "Ferro:" 1-L, 3-R, 3-L, 4-R, 4-L, 2-R, 2-L and 1-R; it quotes the reason as follows: "(1) Because two successive impulses acting on one crank pin has substantially the effect of a prolonged impulse with the average thrust in a vertical line, and since all shaft and crank-pin bearings are, or should be split in a horizontal plane, it is right that the thrust should be as nearly as possible at right angles to the path of the split. (2) If the spark is advanced too far, and No. 1 left fires too early after No. 1 right, it is apparent that this will set up no crank-shaft strains whatever, but if No. 4 left followed No. 1 right, a too early ignition means an opposed torque tendency at the opposite ends of the crank shaft."

151. Eccentric-and-cam Gear.—Trial and error must be used in determining the best form of cam and the proper lift. The diagram of Fig. 265 may be employed for the crank and eccentric as seen in Fig. 307. The eccentric circle may be assumed, and the angles determined; then the proper dimensions may be found by trial. The movement of the eccentric rod required to give full valve lift is equal to h when the proper scale is found. H–O is a reference line, or center line of an imaginary crank, revolving at the speed of the lay shaft (½ the speed of the engine shaft), and passing through points marked H and C as the engine crank passes head- and crank-end dead centers respectively, thus locating the eccentric position for any position of the engine crank. The diameter H–H lies on a line joining the lay-shaft center with the center of motion

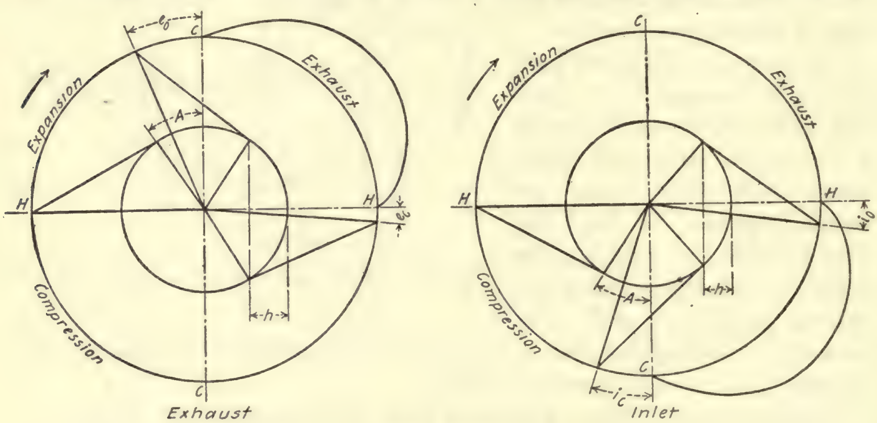


Fig. 307.

of the cam end of the eccentric rod, the side on which h lies being away from the cam if there is tension in the eccentric rod in opening the valve.

152. Valve springs may be calculated by the chart and formulas of Par. 132, Chap. XIX, using P_1 for the spring tension when the valve is seated, P_2 when wide open and m for the valve lift h, or the spring deflection under any circumstances. Further let:

z = weight of valve and other parts to be moved by spring.

N = r.p.m. of engine.

r_T = ratio of time required to close valve, to time required per stroke.

Then the force required to accelerate the valve is:

$$P_1 = \frac{\pi h N}{173{,}664 r_T^2} \tag{26}$$

The force required to hold the valve against suction is:

$$P_1 = \frac{\pi D^2 p}{4} \tag{27}$$

where D is the cylinder diameter in inches and p the maximum suction in lb. per sq. in., and which may be from 5 to 9 lb. The larger of the two values of P_1 should be used. A good value of P_2/P_1 is about 1.15, but this may be increased when it gives a larger number of coils than is convenient.

153. Two-cycle Engines.—The piston forms the valve with this type, and as the ports are opened only near the end of the stroke, the same principles do not apply to timing and port opening. It has been stated that the width (measured along the stroke) should be from 9 to 13 per cent. of the stroke and the exhaust port from 20 to 22 per cent., and each should extend around 90 degrees of the circumference. This was given especially for heavy oil engines, but probably applies as well to gasoline engines.

154. Reversing Gears.—There are various reversing gears used on internal-combustion engines, especially on Diesel engines used for ship

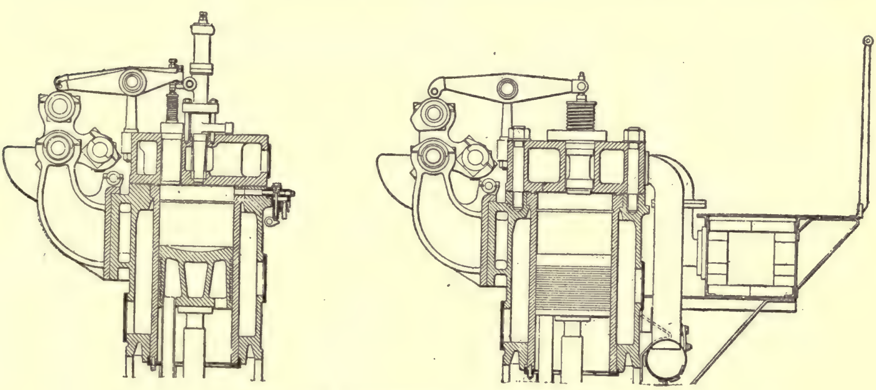


FIG. 308.—Diesel engine reversing gear.

propulsion. Fig. 308 shows one type used on Diesel engines, in which there are two cam shafts, either of which may be in gear.

155. Ignition and Fuel Regulation.—High-tension is largely used. In this type the sparking and timing apparatus is not usually made by the engine builder so space will not be taken to describe it.

With low-tension or make-and-break ignition there are moving parts connected with the engine, so a brief description will be given. This type is used for large gas engines and sometimes for small kerosene engines.

An igniter block from *The Gas Engine* is shown in Fig. 309 which is almost self explanatory. This must be operated by some form of trip mechanism which may be properly timed. Diagrams of two methods are given in Fig. 310. Timing may be changed in the upper method by raising or lowering the roller *a* and this may be under governor control if desired. In the lower figure the cam may be advanced or retarded. The crank *b* and cam *c* run at half the engine speed.

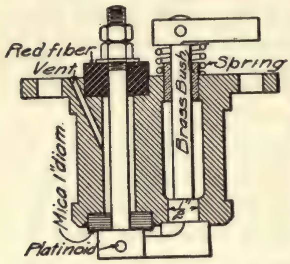


FIG. 309.—Igniter block.

Fuel regulation of gas engines and light-oil engines is commonly done by throttling; for the latter the carbureter is used, and this is not usually designed or manufactured by the engine builder. The subjects of ignition and carburetion are too extensive to attempt a treatment in a book of this kind and are omitted. For gas engines the regulation usually depends upon a mixing valve controlled by the governor and this is shown in Chap. XIX.

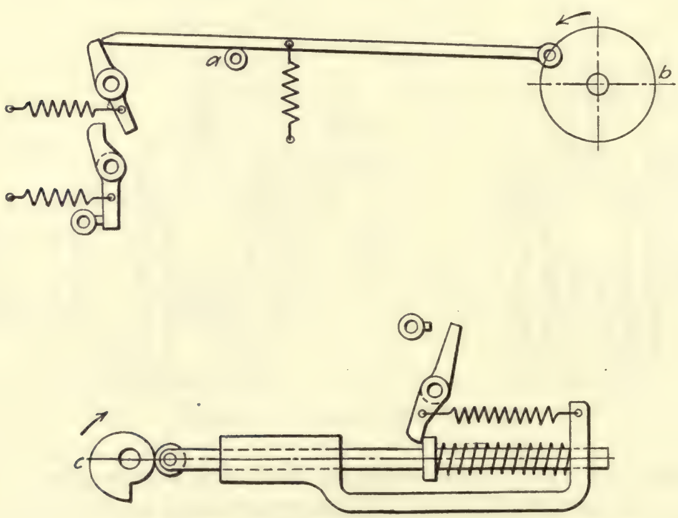


FIG. 310.

The Jacobson Gas Engine Co. use a form of trip gear and this is shown in Fig. 311.

Oil fuel regulation is accomplished in several ways. Figs. 312 to 314, taken from Journal of A.S.M.E. show three methods used with Diesel

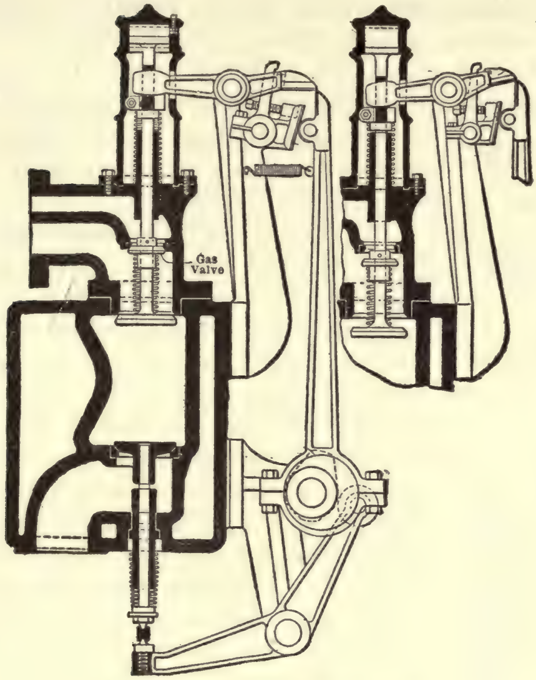


FIG. 311.—Jacobson releasing gear.

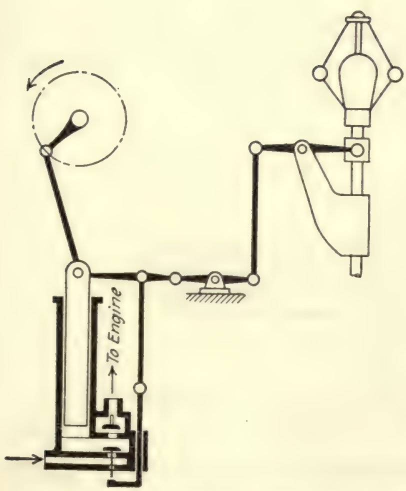


FIG. 312.—By-passing to suction chamber.

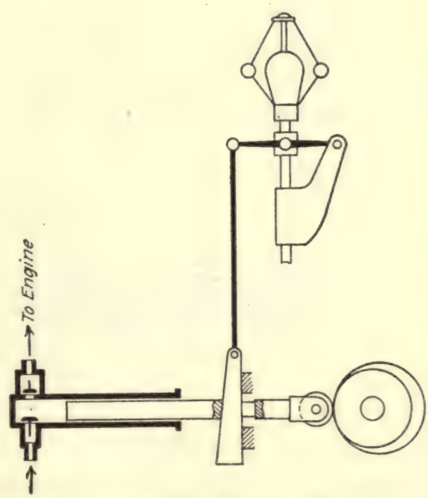


FIG. 313.—Controlling length of suction stroke.

engines. In all cases the governor rises for light loads when less fuel is required; the principle of operation may be easily seen.

A regular Diesel spray nozzle is shown in Fig. 315. Oil is delivered under pressure between perforated plates by a small passage; when the fuel valve is opened, high-pressure air enters at A and sprays the oil into the cylinder.

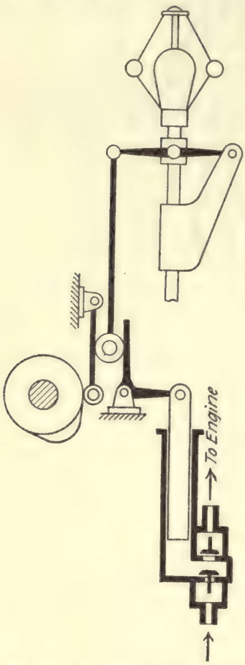


FIG. 314.—Changing stroke of pump.

An open nozzle for a Diesel engine is shown in Fig. 316. Oil is pumped in during the suction stroke against no pressure; it is then sprayed into the cylinder when the fuel valve is opened. It is claimed by some that this type of nozzle will not foul as easily as the closed nozzle.

156. The Sleeve Motor Gear.—The best diagram for designing this gear is doubtless the rectangular diagram on the rectified crank circle. Two revolutions of the crank are required. The author does not have the measurements for this engine, so the diagram of Fig. 317 is only assumed. The piston displacement curve is given above, assuming a connecting rod 4 cranks long. The valve-displacement curves neglect the angularity of the eccentric rods, but this may easily be accounted for if desired. Considerable "cut-and-try" is necessary, and it is best to draw the valve curves separately on tracing cloth. Valve-opening curves are compared with piston velocity curves (dotted), laid off on a rectified crank circle.

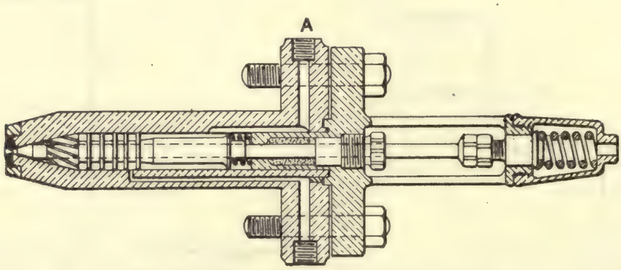


FIG. 315.—Diesel fuel nozzle and valve.

Different arrangements may be made to give the same valve timing; by trial, an arrangement may be obtained which will give the best results,

and this has doubtless been done in the Willys-Overland engines although this method may not have been used. Fig. 317 is given to show its utility in problems of this kind.

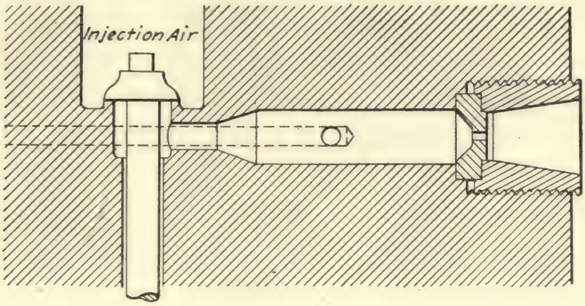


FIG. 316.—Open type of fuel nozzle.

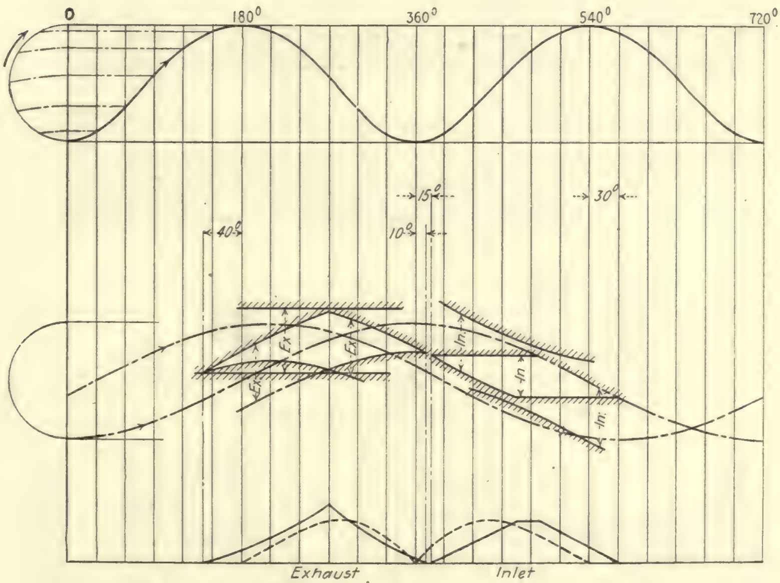


FIG. 317.—Rectangular diagram for sleeve motor.

157. Details from Practice.—Some idea of valve gears may be had from Chaps. III and V, and their connection with the governor in Chap. XIX, but a few details will be given in this paragraph.

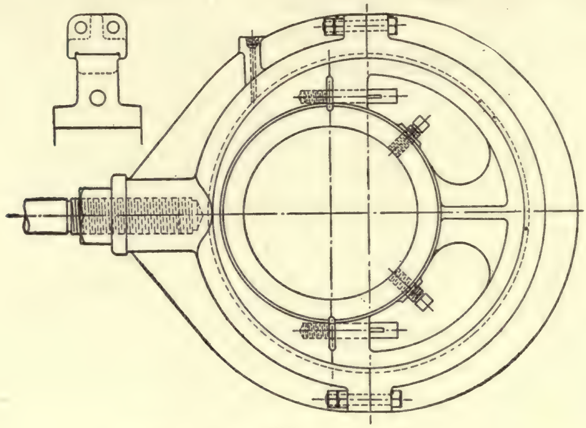


FIG. 318.—Eccentric and strap.

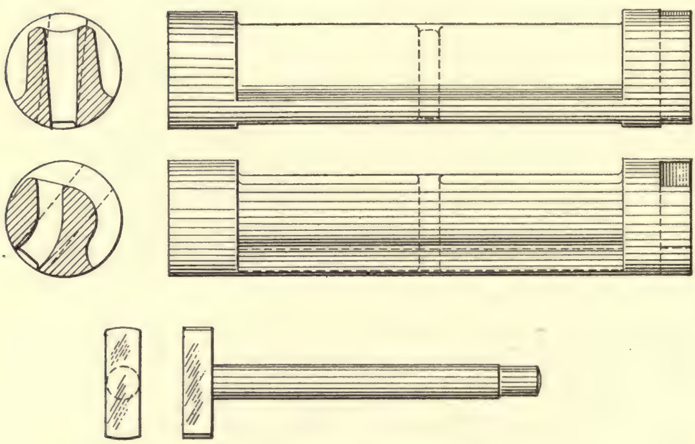


FIG. 319.—Corliss valves and stems.

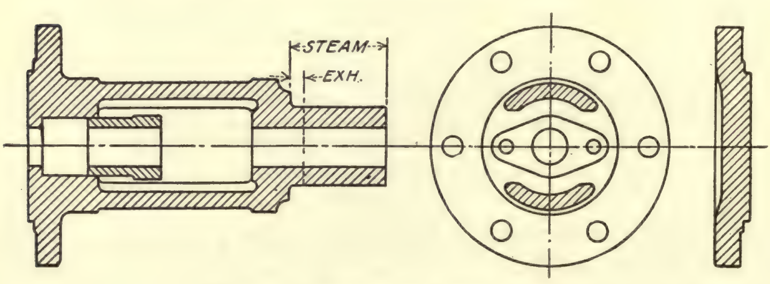


FIG. 320.—Bonnets for Corliss valves.

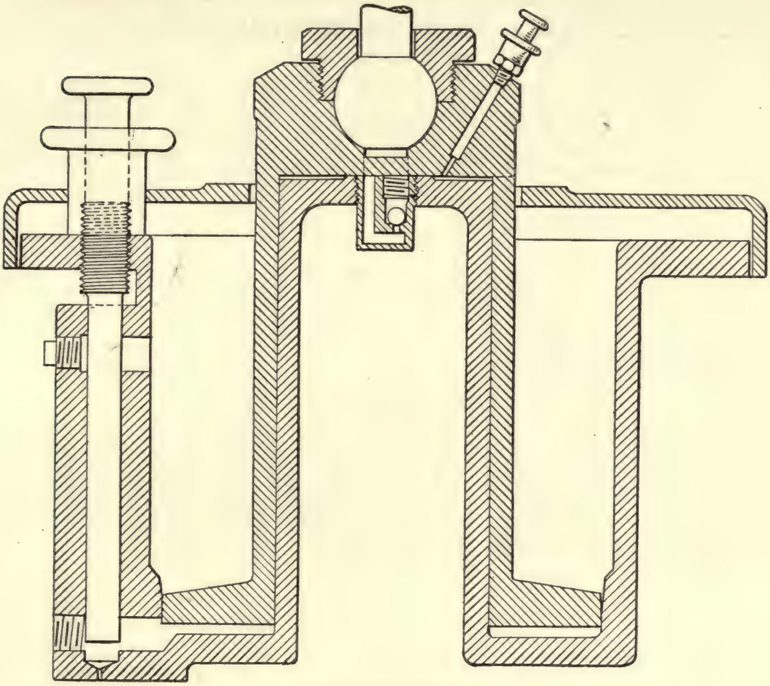


FIG. 321.—Bass-Corliss dashpot.

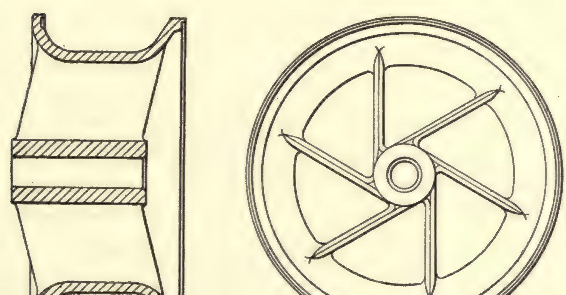


FIG. 322.—Lentz poppet valve.

Fig. 318 is a simple fixed eccentric; the eccentric is split for convenience in placing on the shaft, and the strap is necessarily split for placing over the eccentric, and it also serves for making adjustment.

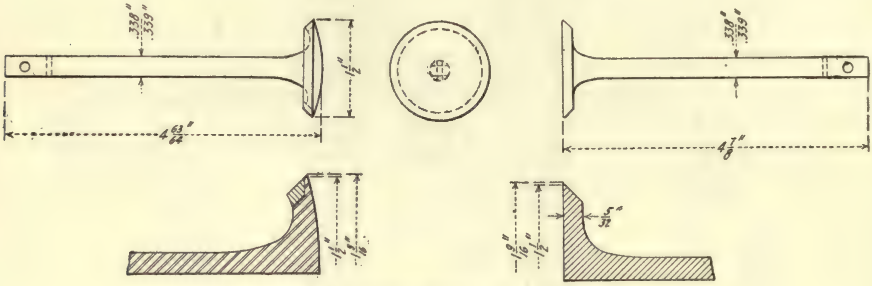


FIG. 323.—Franklin automobile engine valves.

Fig. 319 shows the valves and stems for the 20 in. Corliss engine for which the diagram was designed, and Fig. 320 shows the bonnets.

Fig. 321 is the dashpot used by the Bass Foundry and Machine Co. It is adjustable for vacuum and cushion.

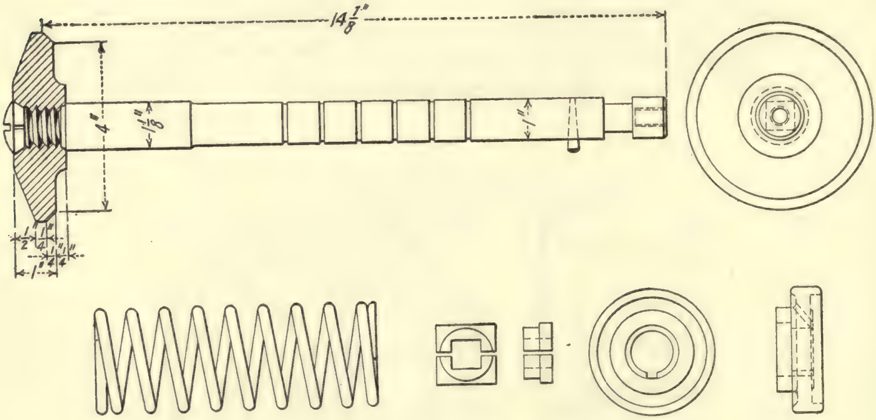


FIG. 324.—Bruce-Macbeth gas engine valves.

Fig. 322 shows a poppet valve used on the Lentz superheated-steam engine.

Fig. 323 shows the valves of the Franklin Automobile Engine, and Fig. 324 and Fig. 325 the valves and cages respectively of the Bruce-Macbeth gas engine.

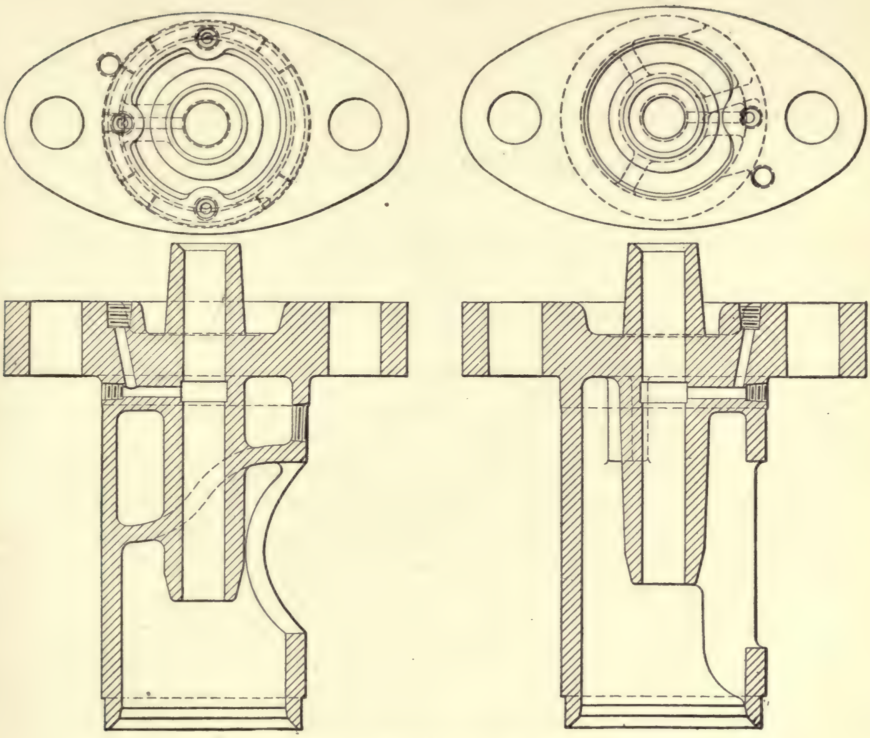


Fig. 325.—Bruce-Macbeth valve cages.

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Valve Gears.................................... W. E. Dalby.
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Valve and Link Motion.......................... W. S. Auchincloss.
Handbook and bulletins of American Locomotive Company.

PART VI—MACHINE DESIGN

CHAPTER XXI

GENERAL CONSIDERATIONS

158. Introduction.—In this chapter are discussions of a few of the fundamentals of machine design, being introductory to the chapters which follow, obviating the necessity of interruption in the way of derivation of fundamental formulas before they may be applied to specific cases.

This is not intended as a treatment of the mechanics of materials or applied mechanics; on the other hand, some knowledge of these subjects is assumed, but in the absence of such knowledge the working formulas given may be used without the ability to follow their derivation.

In the use of all formulas in machine design it is assumed that all material is homogeneous, isotropic and free from internal strains, a condition of things never fully realized.

In this chapter, as each paragraph deals with a separate subject, the notation does not apply outside the paragraph in which it is used, although as far as practicable it is kept uniform.

159. Rational machine design consists in so distributing the material used in machine parts that economy in construction, effectiveness, safety and durability may result. This does not of necessity imply the use of rational or theoretical formulas, but a rational application of the formulas which most correctly express the behavior of materials when subjected to the loading under consideration. Indeed, we may go a step farther in the case of machine parts in which the acting force is indeterminate, or the shape of the section such that a correct estimate of stress relations is impossible; and for which empirical formulas are sometimes used which have been found by long experience to give proper strength and rigidity. This method, when used by experienced and skillful designers, can hardly be called irrational. However, whenever possible, a more rigid analytical treatment is more satisfactory, even though all factors involved are not accurately known; this results in working formulas of a more general character; then numerial values may be assumed for the factors which are constant under a given set of conditions (these to be based upon ex-

perience or the best judgment of the designer), and the formulas may often be reduced to a single constant with perhaps one or two variables. In this simple form the constant is readily compared with that obtained from successful machines of the same class, operating under similar conditions; but in case of disagreement, judgment must not be too hasty in favor of the existing machine.

In all machine design, system and uniformity of practice should be maintained From time to time, in the light of new knowledge gained by experience or from the published investigations of experts, it may be necessary to change values, but the indiscriminate and capricious varying of constants by designers is as unsatisfactory as it is unscientific, and leaves no reference point from which to measure progress.

The determination of proper bearing pressures and working stresses is of paramount importance in rational design. Bearing pressures depend entirely upon judgment and experience; working stresses or unit loads, with which this discussion is concerned, have been determined partly by experience and partly from laws deduced from tests of materials.

By working stresses and unit loads are meant those values used in formulas for design, under the assumption of simple loading upon which the derivation of the formula rests. They are properly, in many cases, only factors of design, the actual stresses differing greatly from them. The more simple the loading or thorough the analysis, the more nearly will the assumed stress approach the actual maximum stress.

Working Stresses and Factors of Safety.—It has never been considered safe to use a working stress just inside the stress causing rupture, so a factor of safety has always been employed. Formerly this was an arbitrary value and often took no account of the manner in which the load was applied. It is now customary to assume working stresses which have been proven by practice to be satisfactory. These are usually determined with reference to some property of the material, such as the elastic limit.

The statement often made that there is no fixed rule governing the selection of a working stress is as unsatisfactory as it is true; however, the results of Wöhler's tests on the fatigue of materials under various forms of loading furnish a means of checking the values of working stress in common use in a partially rational manner at least. These experiments were upon wrought iron and steel, and the laws deduced therefrom may be considered strictly applicable only to ductile materials having approximately the same properties.

Fatigue tests for the purpose of comparing different kinds of steel have more recently been made in which the stress used was considerably beyond the elastic limit, actually bending the test pieces to a point of

permanent set at each repetition of the load. These tests showed the excellence of a certain steel greatly to the disparagement of other high-grade steels; and there is no doubt that in case of miscalculation or accident, steel showing high fatigue values under these tests would resist failure under such abuse better than that with which it was compared; but while there is a feeling of safety in the use of material which will best withstand the most vigorous tests, the author doubts if the tests mentioned alter the applicability of the laws deduced from the older tests, to rational design.

From a discussion of many experiments on repeated stresses by Wöhler and others, Prof. Merriman (Mechanics of Materials) states the following laws:

1. The rupture of a bar may be caused by repeated applications of a unit stress less than the ultimate strength of the material.

2. The greater the range of stress, the less is the unit stress required to produce rupture after an enormous number of applications.

3. When the unit stress in a bar varies from zero up to the elastic limit, an enormous number of applications is required to cause rupture.

4. A range of stress from tension into compression and back again, produces rupture with a less number of applications than the same range of stress of one kind only.

5. When the range of stress in tension is equal to that in compression, the unit stress that produces rupture after an enormous number of repetitions is a little greater than one-half the elastic limit.

Prof. J. B. Johnson (Materials of Construction), discussing numerous stress diagrams for steel, reaches certain conclusions, some of which may be added to the laws just given, and are as follows:

6. The "apparent elastic limit" is found between 60 and 70 per cent. of the ultimate strength in tension.

7. The "apparent elastic limit" in compression is practically the same as that in tension.

8. The ultimate strength in compression is practically equal to the "apparent elastic limit."

Prof. Johnson refers to a column test on p. 360 of Materials of Construction which confirms this third statement (Law 8) in which:

$$\frac{\text{length}}{\text{least radius of gyration}} = \frac{l}{r} = 20.$$

All authorities do not agree with this statement, nor do all tests confirm it, especially for smaller values of l/r, but it is on the side of safety and will be assumed as true in the following discussion.

The term "apparent elastic limit" used by Prof. Johnson, also called by him the "commercial elastic limit," refers to the elastic limit in ordi-

nary use, in distinction from the "true elastic limit" or "limit of proportionality," which is somewhat less than the elastic limit.

Stress is produced in engine parts in three ways, as follows:

1. *Static stress*, produced by an unchanging load.

2. *Repeated stress*, produced by a constant repetition of all or a part of the load, producing stress of one kind, tension or compression. The limiting case is expressed by Law 3, the stress ranging from zero to maximum.

3. *Reversed stress*, which changes from tension to compression. The limiting case is given by Law 5, in which the tension equals the compression.

Formulas expressing Wöhler's results may be found in the works already referred to. The limiting cases given by Laws 3 and 5 are the most important, intermediate conditions being few, or seldom known with accuracy in engine design.

Let S_U = the ultimate strength of the material in lb. per sq. in.
S_E = the elastic limit in lb. per sq. in.
$n = S_E/S_U$.

Then from Laws 3, 5 and 8, the stress at failure for the three conditions are given in Table 67.

TABLE 67

Load	Static		Repeated	Reversed
	Tension	Compression		
Stress.............	S_U	S_E	S_E	$S_E/2$

Good practice dictates that under no condition should the members of a machine or structure be strained beyond the elastic limit; so making the maximum static stress in tension equal to the elastic limit, and reducing the other values in the same proportion, we have the values of Table 68.

TABLE 68

Load	Static		Repeated	Reversed
	Tension	Compression		
Stress............	$S_E = nS_U$	nS_E	nS_E	$\frac{n}{2}S_E$

This undoubtedly places the live load stresses well within the limit of proportionality. Taking the elastic limit as the basis of stress measurement for ductile materials, in which this property is as well defined as any other, we may consider the first factor of safety f_1, as the ratio of the elastic limit to the values given in Table 68. For tension, with different values of n, f_1 is given in Table 69.

TABLE 69

Load		Static	Repeated	Reversed
f_1	$n = n$	1	$1/n$	$2/n$
	$n = 0.5$	1	2.00	4.00
	$n = 0.6$	1	1.67	3.34
	$n = 2/3$	1	1.50	3.00
	$n = 0.7$	1	1.43	2.86

Selecting the value of f_1 when $n = \frac{2}{3}$ gives a dead-load factor which will bear a greater ratio to the live-load factor than if a smaller value of n were assumed; and since f_1 is based on the elastic limit, this assumption is more apt to place the static stress within the elastic limit when the live-load stresses are within the limit of proportionality. Values of f_1 for the different ways of producing stress are given in Table 70.

TABLE 70

Load	Static		Repeated	Reversed
	Tension	Compression		
f_1	1	1.5	1.5	3

On account of some uncertainty as to just what constitutes the elastic limit, this factor f_1 may be said to insure stresses within the point of possible failure when the commercial elastic limit is used as a basis. If the material is furnished by specification or the properties known from test, f_1 should provide against failure for the applications of load given. It is usually better to increase the factor somewhat, as the properties of any grade of material vary more or less, and in many cases the material is known only in a general way by the designer. The factor f_1 may then be multiplied by a second factor f_2. For general engine design a practical value for this may be taken as:

$$f_2 = 2.$$

In cases where lightness is imperative, or where strains due to high temperature might be increased by thicker metal, this factor may be de-

creased, approaching unity if the properties of the material are well known.

Uneven distribution of stress, initial stress due to tightening nuts or driving keys (and greater than the calculated stress caused by the working load), or other straining actions not easily calculated, may be provided for by a third factor f_3.

Suddenly Applied Loads.—If the full load were applied instantaneously but without shock, it is shown in treatises on applied mechanics that the stress produced is double that caused by the same load gradually applied. Prof. Unwin says that "practical cases rarely approximate to these conditions."

Shock.—This may be caused by lost motion being suddenly taken up. Even with badly worn bearings where there is a perceptible "knock," it is probable that but a small percentage of the maximum load on any part is applied as sudden load or shock.

Factor of Judgment.—Provision for sudden load and shock, and for other unknown straining actions may be included in the third factor f_3 just mentioned. It is obvious that this is more nearly a factor of judgment and experience than the others.

The total factor of safety is then a product of the three factors, or:

$$f = f_1 f_2 f_3 \tag{1}$$

The factor f_1 may be assumed as a fixed minimum limit; f_2 may vary according to the accuracy with which the elastic limit of the material is known, but will be taken as 2, as previously stated, for ordinary engine work. The product of f_1 and f_2 may be taken as a *standard factor* f_A, or:

$$f_A = f_1 f_2 \tag{2}$$

This suits all cases for simple applications of stress given in Table 71.

TABLE 71

Load	Static		Repeated	Reversed
	Tension	Compression		
f_A	2	3	3	6

For the cases of irregular loading just given, f_A must be multiplied by the factor of judgment f_3, or:

$$f = f_3 f_A \tag{3}$$

In the design of engine details which follows, practical working stresses for ordinary conditions, based largely upon experience, have been assumed; from these the total factor has been found, and by using f_A as given in

Table 71, the value of f_3 was obtained. If S is the working stress assumed and S_E the elastic limit:

$$f = \frac{S_E}{S} = f_A f_3$$

or:

$$f_3 = \frac{S_E}{f_A S} \tag{4}$$

This method seems to place the cart before the horse, but as both S and f_3 depend upon experience and judgment, one serves as a check upon the other, no matter which is first selected.

The less accurate the analysis, the greater must f_3 be. In the use of formulas which err largely on the side of safety, such as most of those for flat plates, the factor of judgment may be reduced; but when there are several theories giving results differing considerably, as in the case of combined bending and torsion, the selection of the least safe formula offset by a large factor of safety is ill advised.

The use of such formulas is sometimes based upon their apparent agreement with tests to destruction, but working loads never impose these conditions upon the material, and such tests are no measure of its fatigue-resisting properties It is better in such cases, when the limit of elastic strength cannot be determined by test, to adopt the safest rational formula, selecting the factor of safety as for more simple cases of straining action.

A thorough analysis of even the best designs would disclose deviations from the assumptions made in the derivation of the formulas and selection of working stress. Though a quantitative analysis is often impracticable, a study of the kind of strains possible through a lack of exact conformity to a design by the shop, will assist in making assumptions and in determining the nominal working stress; but no matter how thoroughly we study, in the end the factor of safety will always be an absolute necessity, although sometimes considered to reflect upon the ability of designers.

In applying Table 71 to beams, the factor for static load may be taken the same as for tension. When the beam is long and unsupported laterally, there are a number of formulas which have been devised to limit the load. This comes under structural design but is worthy of consideration.

Cast iron and other brittle materials, having no marked elastic limit, the factor of safety must be based upon the ultimate strength; but shrinkage strains, and the possibility of unsound castings necessitate the exercise of a good deal of judgment.

Under ordinary conditions a factor of 4 for static loads may be considered sufficient. This may be assumed to correspond to f_A just found for

ductile materials, with the exception that the ultimate strength will be used as a basis for cast iron. In the absence of experimental data for repeated and reversed stresses, the same ratio of factors may be assumed as for the ductile materials. Factors for cast iron are given in Table 72, suitable alike for tension and compression.

TABLE 72

Load	Static	Repeated	Reversed
f_A	4	6	12

The factor f_3 may also be applied as already given.

As the compressive strength of cast iron is at least five times as great as the tensile strength, it may perhaps be permissible to assume that the compressive stress may be five times the tensile stress when the reversal factor f_A is used, based on the tensile stress. Or for reversed stress of equal intensity, the compressive stress may be considered as one-fifth of its actual value. Then from the chart of Fig. 326, this would give a standard factor f_A of 7.2 instead of 12.

When cast iron is used where failure might be attended with disaster, as in steam cylinders, the factor f_3 should be increased accordingly; on the other hand, high temperature in gas engine cylinders calls for thinner walls with a consequent higher maximum allowable stress.

The *materials* most usually subjected to stress in engines are forged machinery steel, steel castings and iron castings. Wrought iron was formerly much used, but has been almost entirely superseded by steel, usually in the form of forgings, but sometimes, as in locomotive frames, by steel castings. The elastic limit is well defined in steel castings, and factors of safety may be applied as for forgings.

The *mechanical properties* of a given material may vary with the composition, treatment, form and size of specimen, and even with the method of testing. They are not always the same in tension and compression, but except in the case of cast iron, they may be assumed equal. The values given in Tables 73 and 74 are safe for ordinary engine design.

E = modulus of elasticity in tension or compression.
E_S = modulus of elasticity in shear.
S_E = elastic limit in tension or compression.
S_{ES} = elastic limit in shear.
S_U = ultimate strength in tension or compression.
S_{US} = ultimate strength in shear.
S_{UR} = modulus of rupture in bending.

All the above values are in pounds per square inch.

TABLE 73

Material	E	E_S	S_E	S_{ES}
Wrought iron	26,000,000	10,000,000	27,000	21,000
Machinery steel	29,000,000	11,200,000	38,000	29,000
Steel casting	24,000,000	9,200,000	30,000	23,000

TABLE 74

Material	E	E_S	S_U		S_{US}	S_{UR}
			Tension	Compression		
Cast iron	12,000,000	4,600,000	16,000	90,000	18,000	24,000

For static shearing stress, the same factor of safety may be used as for tension.

Higher speeds in engines of older type, the extremely high-speed engines used in automobiles, airplanes, etc., and the steam turbine have been in part responsible for the development of steels of higher grade, and the properties of some of the most reliable and widely used will be given here.

The notation is as follows:

C = carbon	P = phosphorus
Mn = manganese	Ni = nickel
Si = silicon	Cr = chromium
S = sulphur	V = vanadium

Table 75, from the Proceedings of the American Society for Testing Materials, 1905, gives specifications for carbon and nickel steels. C.A. denotes carbon steel annealed; C.O., carbon steel, oil tempered; N.A., nickel steel, annealed, etc. The elastic limit in Table 75 is taken as that point at which the proportionality changes.

For bending tests, a specimen 1 by ½ in. shall bend cold 180 degrees without fracture on outside of bent portion, as follows: (*a*) around a diameter of ½ in.; (*b*) around a diameter of 1 in.; (*c*) around a diameter of 1½ in.; (*d*) no bending test required.

Chemical composition: P and S not to exceed 0.04 per cent. in carbon or nickel steel, oil tempered, or 0.05 per cent. in locomotive forgings. Mn not to exceed 0.60 per cent. Ni 3 to 4 per cent. in nickel steel.

TABLE 75.—SPECIFICATIONS FOR STEEL

Steel forgings	Kind of steel	S_E	Elongation in 2 in., per cent.	Reduction in area, per cent.
Solid or hollow forgings, no diameter or thickness	C	37,500	18	30(*c*)
to exceed 10 in.	C.A.	40,000	22	35(*b*)
	N.A.	50,000	25	45(*a*)
Solid or hollow forgings, diameter not to exceed	C.A.	37,500	23	35(*b*)
20 in., or thickness of section 15 in.	N.A.	45,000	25	45(*a*)
Solid forgings, over 20 in.	C.A.	35,000	24	30(*c*)
Solid forgings.	N.A.	45,000	24	40(*a*)
Solid or hollow forgings, diameter or thickness	C.O.	55,000	20	45(*b*)
not over 3 in.	N.O.	65,000	21	50(*b*)
Solid rectangular section, thickness not over 6 in.,	C.O.	50,000·	22	45(*b*)
or hollow with walls not over 6 in. thick.	N.O.	60,000	22	50(*b*)
Solid rectangular section, thickness not over 10	C.O.	45,000	23	40(*b*)
in., or hollow with walls not over 10 in. thick.	N.O.	55,000	24	45(*b*)
Locomotive forgings.	.....	40,000	20	25(*d*)

Bulletin 100 of the Bureau of Mines, entitled: Manufacture and Uses of Alloy Steels, by Henry D. Hibbard, gives much valuable information. Tables 76 to 78 give some selected values from this source.

W850, A538 indicates that the sample was quenched in oil at 850° C. and the hardness drawn in air at 538° C. O926 denotes quenching in oil at a temperature of 926° C.

Nickel Steels.—As stated by the author of the bulletin mentioned, nickel steel containing 3.25 to 3.5 per cent. of nickel and known as ordinary nickel steel, has a high value for structural purposes such as bridges, gun forgings, machine parts, engine and automobile parts, and any similar line of service too severe for simple steels. It has been stated that alloy steels possess no advantage over simple steels if not heat treated, but that the alloys may even have a deleterious effect; but nickel steel, when used in bridge work, is used in the natural or annealed condition, when the additional strength and ductility is due only to the presence of the nickel.

Table 76 gives the composition and properties of ordinary nickel steel, and it may be seen that it may be made suitable for any structural purpose for which it is not too expensive.

Nickel-chromium Steels.—These steels are perhaps the most important structural alloy steels, and their field of application is continually being enlarged. They are seldom used in any but a heat-treated condition. By suitable treatment small pieces may have as high physical properties

TABLE 76.—PROPERTIES OF ORDINARY NICKEL STEEL

No.	Composition: C, per cent.	Mn, per cent.	Si, per cent.	S, per cent.	P, per cent.	Ni, per cent.	Condition	S_E	Elongation, per cent.	Contraction, per cent.
1	0.28	0.57	....	0.03	0.02	3.44	Natural state	56,670	21.2*	50
2	0.40	0.64	....	0.02	0.01	3.43	Annealed	51,400	12.4†	33
3	0.40	0.55	....	0.03	0.01	3.70	Annealed	56,060	15.8†	40
4	0.20	0.65	....	0.04	0.04	3.50	Annealed	43,000	27.0	62
5	0.20	0.65	....	0.04	0.04	3.50	W850, A538	95,000	20.0	72
6	0.20	0.65	....	0.04	0.04	3.50	W800, A316	140,000	14.0	61
7	0.30	0.65	....	0.04	0.04	3.50	Annealed	63,000	27.0	63
8	0.30	0.65	....	0.04	0.04	3.50	W800, A593	87,000	25.0	68
9	0.30	0.65	....	0.04	0.04	3.50	W800, A399	123,000	15.0	57
10	0.30	0.65	....	0.04	0.04	3.50	W800, A316	187,000	13.0	57
11	0.25	0.74	0.21	0.01	0.01	3.55	W843, A316	177,000	14.0	60
12	0.25	0.74	0.21	0.01	0.01	3.55	W843, A538	117,000	20.0	67

*In 8 in. †In 18 ft.

as any known steel, and with any elastic limit from 40,000 to 250,000, accompanied by ductility that is high as compared with strength, the ductility naturally lessening with increase of elastic limit.

Nickel-chromium steel may be made some cheaper than simple nickel steel of the same strength and ductility, as it contains a smaller amount of the alloying elements, which are also less expensive than nickel. Table 77 gives the properties of nickel-chromium steels.

TABLE 77.—PROPERTIES OF NICKEL-CHROMIUM STEEL

No.	Composition: C, per cent.	Mn, per cent.	Si, per cent.	S, per cent.	P, per cent.	Ni, per cent.	Cr, per cent.	Condition	S_E,	Elongation in 2 in., per cent.	Contraction, per cent.
1	0.55	0.41	0.22	0.03	0.02	1.53	1.14	Annealed	75,000	31	66
2	0.18	0.27	0.05	0.04	0.02	1.28	1.59	Annealed	51,000	37	71
3	0.15	0.34	0.13	0.02	0.01	1.28	0.37	Annealed	42,000	38	64
4	0.25	0.32	0.10	0.03	0.02	1.45	1.20	Test piece	81,500	35	68
5	0.25	0.32	0.10	0.03	0.02	1.45	1.20	Full size eye bar	80,900	7	49
6	0.40	0.74	0.24	0.03	0.02	3.45	1.20	W830, A371	175,000	10	43
7	0.36	0.53	0.11	0.04	0.01	1.53	0.70	W830, A566	125,000	20	65
8	0.21	0.41	0.22	0.03	0.02	3.52	1.11	W830, A682	75,000	24	66
9	0.98	0.44	0.16	0.01	0.01	2.02	0.98	W843, A427	186,000	10	46
10	0.48	0.44	0.16	0.01	0.01	2.02	0.98	W893, A649	120,000	18	61
11	0.38	0.28	0.27	0.02	0.01	3.01	0.65	W843, A649	90,000	25	69

Chromium-vanadium Steels.—These steels are the latest development and have gained an extensive market. They are much like chrome-nickel steels but have a greater contraction of area for a given elastic limit. They are much easier to machine; a chrome-vanadium steel with an elastic limit of 150,000 lb. may be machined rapidly, while a chrome-nickel steel of the same strength would quickly dull a tool if cut at the same speed.

Chrome-vanadium steel is more free from surface imperfections than other steels containing nickel, vanadium improving the quality, and though vanadium is much more costly than nickel, the smaller amount required enables chrome-vanadium steel to compete with nickel-steel regarding cost.

Chrome-vanadium steel is nearly always used in a heat-treated condition, although there are some exceptions. The properties of chrome-vanadium steel are given in Table 78.

Table 78.—Properties of Chrome-vanadium Steel

No.	Composition							Condition	S_E,	Elongation in in., per cent.	Contraction, per cent.
	C, per cent.	Mn, per cent.	Si, per cent	S, per cent.	P, per cent.	Cr, per cent.	V, per cent				
1	0.57	0.84	0.27	0.03	0.01	1.36	0.31	Natural state	75,750	28.1	68.5
2	0.46	0.48	0.20	0.02	0.01	1.17	0.14	Natural state	52,500	34.0	71.0
3	0.18	0.32	0.18	0.02	0.01	0.74	0.20	Natural state	42,900	43.0	75.0
4	0.30	0.65	0.10	0.04	0.04	0.90	0.18	Annealed	45,000	35.0	69.0
5	0.30	0.65	0.10	0.04	0.04	0.90	0.18	W899, A704	101,000	20.0	64.0
6	0.30	0.65	0.10	0.04	0.04	0.90	0.18	W899, A454	180,400	10.0	43.0
7	0.30	0.65	0.10	0.04	0.04	0.90	0.18	W899, A315	200,000	10.0	52.0
8	0.28	0.45	0.26	0.02	0.01	1.00	0.18	O 899, A676	79,000	34.0	75.0
9	0.40	0.75	0.26	0.01	0.01	1.00	0.17	O 926, A676	120,000	20.0	53.0
10	0.40	0.75	0.26	0.01	0.01	1.00	0.17	O 926, A426	200,000	11.0	48.0
11	0.57	0.37	0.20	0.02	0.01	0.69	0.22	...... A426	177,500	14.0	57.0
12	1.06	0.36	0.22	0.02	0.02	0.95	0.11	...... A648	126,750	21.0	49.0
13	0.41	0.49	0.12	0.03	0.03	1.09	0.11	...... A754	77,250	33.0	70.0
14	0.25	0.50	0.10	0.03	0.02	0.95	0.75	....	113,100	18.0	56.0

Samples 11, 12 and 13 were hardened before being drawn at the temperature given.

The foregoing tables place an excellent variety of steels at the disposal of the designer, enabling him to meet every demand.

Heat treatment has little effect upon the modulus of elasticity, which for all steels is between 28 and 30 million, the average value being given in Table 73.

Aluminum castings, alloyed with copper, used for automobile and

similar engine crank cases, may be taken as having a tensile strength equal to cast iron.

The chart of Fig. 326 is given to aid in the selection of factors of safety when a fraction of the maximum load is repeated or reversed; a straight-line variation is assumed, and as previously stated, the elastic limit is taken as a basis for wrought iron, forged steel and steel castings, and the ultimate strength for cast iron and other brittle materials.

In selecting working stresses, the smaller machines belonging to a heavy class are made proportionally heavier than the larger machines; the stresses are reduced to bring this about. But in a small class of

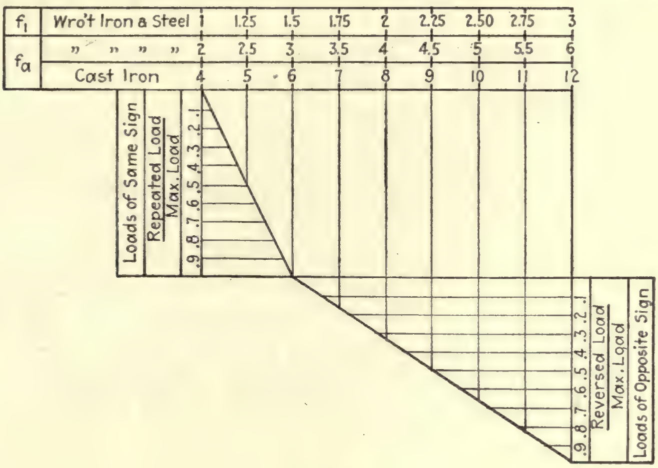


FIG. 326.

machines, such as automobile and airplane engines, the working stresses are often higher than in heavy rolling-mill engines. No one would think of using a ½-in. bolt in a place where strength is required in the latter, even though calculation showed it ample, while for the automobile engine a ½-in. bolt is a large bolt relatively.

Machines to be handled by unskilled labor should have a high factor of judgment; they should be as near "fool proof" as possible without removing all profit.

160. Selection of Formulas.—In the more simple applications of load there is little divergence of opinion regarding the formulas applying to calculations for stiffness and strength, but for loading involving complicated straining actions, different formulas are sometimes used for the same purpose, giving results often widely at variance. This is due to a lack of agreement with tests or to a lack of test data in the case

of the more theoretical formulas, and in some cases to the impossibility of a satisfactory rational treatment, leading to the use of empirical formulas. The formulas most commonly subjected to a variety of treatment are for:

1. Cylinder walls.
2. Combined bending and twisting.
3. Rectangular sections in torsion.
4. Struts.

The relative importance of maximum stress and maximum strain (deformation) is involved in the first two.

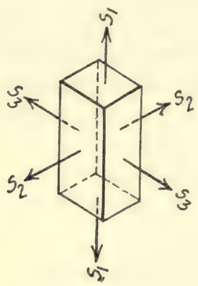


FIG. 327.

Let S = tensile or compressive unit stress.
S_S = shearing unit stress.
E = modulus of elasticity (Young's modulus).
E_S = modulus of transverse elasticity (in shear).
e = deformation (extension or compression).
m = the reciprocal of Poisson's ratio.

All stresses are in pounds per square inch.

Let the parallelopiped in Fig. 327 be acted upon by three forces, producing stresses, S_1, S_2 and S_3. It is shown in any good treatise on mechanics of materials or applied mechanics that the stress S_1 produces a deformation in the direction of action equal to:

$$e_1 = \frac{S_1}{E}$$

The stresses S_2 and S_3 each produce a deformation in a plane at right angles to their direction of action equal to:

$$e_1 = \frac{S_2}{E} \cdot \frac{1}{m} \qquad \text{and} \qquad e_1 = \frac{S_3}{E} \cdot \frac{1}{m}$$

The fraction $1/m$ is known as Poisson's ratio and is usually between $\frac{1}{3}$ and $\frac{1}{4}$.

If S_2 or S_3 is a compressive stress, elongation occurs; if tensile, the result is compression in the direction of stress S_1. The total deformation in this direction is the sum of these, or the resultant deformation is:

$$e_R = \frac{S_1}{E} + \frac{S_2}{mE} + \frac{S_3}{mE}.$$

Arranging so that compressive stress may be given the minus sign, and

letting S_R be the resultant simple stress which would produce the elongation e_R, gives:

$$S_R = Ee_R = S_1 - \frac{S_2 + S_3}{m} \tag{5}$$

The same treatment may be made in the direction of S_2 or S_3, but it is assumed that S_1 is the greatest.

If S_2 and S_3 are compressive and S_1 tensile, or vice versa, it is obvious that S_R is greater than S_1. The "maximum strain theory" limits S_R to the safe tensile or compressive stress. In some applications S_3 is zero, the equation becoming:

$$S_R = Ee_R = S_1 - \frac{S_2}{m} \tag{6}$$

According to (6), if S_1 and S_2 are both tension, S_R is less than S_1, indicating an increase in tensile strength due to the effect of S_2. Prof. Cotterill says: "An addition to the *tenacity* of a material, consequent on the application of a lateral tension, can, however, hardly be considered as intrinsically probable, and such direct experimental evidence as exists is against the supposition." However, other prominent authorities use the maximum strain theory in its entirety, the equivalent stresses being called "true stresses" by Merriman.

Wherever applicable, the maximum strain theory will be used in this book but only when S_1 is of different sign from S_2 and S_3 (tensile if the others are compression and *vice versa*); otherwise the effect of these secondary stresses will be neglected, insuring results always on the safe side.

If, in (6), S_2 is equal to S_1, but is of opposite sign, it may be shown that a shearing stress S_S of equal intensity is produced. Then letting $S_1 = S_S$ and $S_2 = -S_S$, (6) becomes:

$$S_R = S_S + \frac{S_S}{m}$$

or:

$$S_S = \frac{m}{m+1} \cdot S_R \tag{7}$$

In other words, a given shear stress is accompanied by an equivalent direct stress of greater intensity than the shear stress. This indicates that if the maximum strain theory is correct for stresses of opposite sign, the ratio of the resistance to shearing to that of tension or compression is equal to:

$$\frac{m}{m+1}.$$

If S_R and S_S are known, (7) gives:

$$m = \frac{1}{\frac{S_R}{S_S} - 1} = \frac{S_S}{S_R - S_S}$$

Some values of m are given in Table 79.

TABLE 79

Material	m		
	Goodman	Johnson	Average
Cast iron	3.0 to 4.7	....	3.8
Wrought iron	3.6	....	3.6
Steel	3.6 to 4.6	3.72	3.8
Brass	3.1 to 3.3	3.06	3.2
Copper	2.9 to 3.0	3.06	3.0

A safe value of m for most of the materials used in heat engine construction is $3\frac{1}{3}$, and this will be used in this book. This gives:

$$S_S = \frac{S_R}{1.3} = 0.77 S_R \tag{8}$$

a result agreeing well with values commonly given for ductile materials, although some authorities claim that the ratio S_S/S_R is much less.

Hereafter when S_R is the only stress appearing in the working formula, the subscript will be omitted, S denoting the maximum allowable, or the actual tensile or compressive stress.

If E_S is the modulus of transverse elasticity, or coefficient of rigidity, it may also be shown that:

$$E_S = \frac{m}{2(m + 1)} \cdot E \tag{9}$$

And for $m = 3\frac{1}{3}$:

$$E_S = \frac{E}{2.6} = 0.385\,E \tag{10}$$

161. Cylinder Walls.—There are several formulas for determining the thickness of a cylinder wall, some of which were compared by Alfred Petterson in the American Machinist of Feb. 15, 1900. The one most generally accepted, and probably the most accurate, is that of Lamé, which will be given here. Let:

p = internal pressure in pounds per square inch.
S_T = tangential, or hoop stress at inner surface, as given by the original formula of Lamé.
S_P = radial stress at the inner surface, due directly to the pressure and equal to it.
S = equivalent simple tensile stress due to S_T and S_P.
m = the reciprocal of Poisson's ratio.
r = internal radius of cylinder in inches.
r_1 = external radius of cylinder in inches.
D = internal diameter in inches.
t = thickness of cylinder wall in inches.

Let:

$$n = \frac{r_1}{r} = \frac{r+t}{r} = 1 + \frac{t}{r}.$$

The original Lamé formula gives:

$$S_T = p\frac{r_1^2 + r^2}{r_1^2 - r^2} = p\,\frac{n^2+1}{n^2-1} \qquad (11)$$

where S_T is obviously a tensile stress.
The radial, or normal stress is compressive, and is:

$$S_P = -p = -p\,\frac{n^2-1}{n^2-1} \qquad (12)$$

From Formula (6), Par. 2, the equivalent simple tensile stress is:

$$S = S_T - \frac{S_P}{m} = \frac{\frac{m+1}{m}\cdot n^2 + \frac{m-1}{m}}{n^2-1}\cdot p \qquad (13)$$

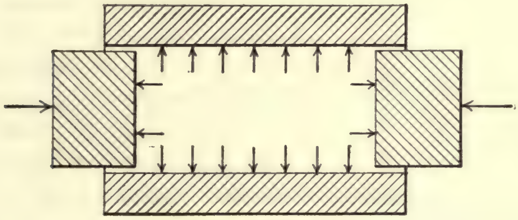


Fig. 328.

Formula (13), known as Birnie's formula, assumes no longitudinal force due to reaction of cylinder heads, the condition being shown by Fig. 328, in which the heads are supported independently of the cylinder. This condition is found only in engines fitted with separate cylinder liners, but as longitudinal stress, being tensile, supposes a reduction of S, or a greater allowable working pressure, it is neglected, as stated in Par.

160. If longitudinal stress is considered, the formula is known as Claverino's formula.*

Taking m as $3\frac{1}{3}$, (13) becomes:

$$S = \frac{1.3n^2 + 0.7}{n^2 - 1} \cdot p \qquad (14)$$

$$p = \frac{n^2 - 1}{1.3n^2 + 0.7} \cdot S \qquad (15)$$

and

$$n = \sqrt{\frac{S + 0.7p}{S - 1.3p}} \qquad (16)$$

$$t = r_1 - r = r(n - 1) = \frac{n - 1}{2} \cdot D \qquad (17)$$

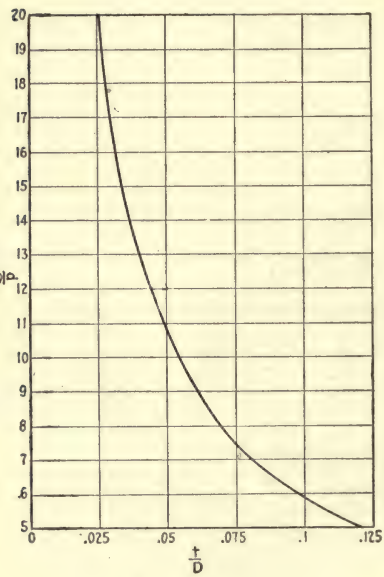


FIG. 329.

Formula (16) may be written:

$$n = \sqrt{\frac{\frac{S}{p} + 0.7}{\frac{S}{p} - 1.3}}$$

The chart in Fig. 329 will facilitate calculation. If S and p are known, $(n-1)/2$ may be found and used in (17); or if t and D are known:

$$\frac{n - 1}{2} = \frac{t}{D}$$

and from the chart S/p may be found.

When the pressure is small relative to the working stress, a simple formula, such as is used for steam boilers, will give results which are usually considered sufficiently accurate.

Assuming a pressure of 150 lb. per sq. in. gage, an allowable stress of 1500 lb., a common condition for stream engine cylinders, the thin cylinder formula is in error 9 per cent. as compard with (17). If radial stress is considered in the thin cylinder formula the error is still 6 per cent. Should the pressure be 500 lb. and the stress 3500, as for a Diesel oil engine, the error is nearly 12 per cent., and with radial stress, nearly 8 per cent.

* In hydraulic cylinders for high pressure, Claverino's formula gives results more nearly agreeing with practice than Birnie's formula. The formula for n is:

$$n = \sqrt{\frac{S + 0\,4\,p}{S - 1.3\,p}} \text{ if } m = 3\tfrac{1}{3}. \quad \text{Then } t \text{ may be found from (17).}$$

162. Combined Bending and Twisting.—Let:

S = simple tensile or compressive stress produced by bending, or more generally, by any simple application of load.

S_S = simple shearing stress produced by twisting or by a simple shearing load.

S_R = The equivalent simple tensile or compressive stress which would produce the same deformation as that resulting from the joint action of S and S_S. This is taken as the actual stress under a given load, or maximum allowable working stress.

M = the bending moment in inch-pounds.

M_S = the twisting moment in inch-pounds.

M_R = a bending moment which, with a given section modulus, would produce S_R.

z = modulus of section in bending.

z_S = modulus of section in torsion.

m = the reciprocal of Poisson's ratio.

The maximum direct stress, tensile or compressive, produced by the combination of a simple direct stress with a simple shearing stress is given by the equation:

$$S_1 = \frac{S}{2} + \frac{1}{2}\sqrt{S^2 + 4S^2{}_S} \tag{18}$$

This is called the major principle stress and is sometimes the only one considered; but for the maximum strain theory, the minor principle stress, of opposite sign and normal to S_1, must be considered, and is given by the equation:

$$S_2 = \frac{S}{2} - \frac{1}{2}\sqrt{S^2 + 4S_S^2}$$

Then from (6), Par. 160, the equivalent simple stress is:

$$S_R = S_1 - \frac{S_2}{m} = \frac{m-1}{2m}\cdot S + \frac{m+1}{2m}\sqrt{S^2 + 4S_S^2} \tag{19}$$

and when $m = 3\frac{1}{3}$:

$$S_R = 0.35S + 0.65\sqrt{S^2 + 4S_S^2} \tag{20}$$

Formulas (19) and (20) are general for any case where S and S_S may be found for the same spot, the location giving the maximum value of S_R being the weakest place in any machine part.

For circular sections only, the surface shearing stress due to torsion is uniform, and a maximum at all points for a given twisting moment, and is:

$$S_S = \frac{M_S}{z_S}$$

The maximum bending stress is also at the surface and is:

$$S = \frac{M}{z}$$

For all sections formed by concentric circles:

$$z_S = 2z$$

Then:

$$S_S = \frac{M_S}{2z}$$

Substituting these values of S and S_S in (19) gives:

$$S_R z = M_R = \frac{m-1}{2m} \cdot M + \frac{m+1}{2m}\sqrt{M^2 + M^2_S} \qquad (21)$$

and for $m = 3\frac{1}{3}$:

$$M_R = 0.35M + 0.65\sqrt{M^2 + M_S{}^2} \qquad (22)$$

To reduce the use of large numerical quantities, it is convenient to take:

$$\frac{M_S}{M} = k \quad \text{then:} \quad \frac{S_S}{S} = \frac{k}{2}$$

Then (20) becomes:

$$S_R = S[0.35 + 0.65\sqrt{1 + k^2}] \qquad (23)$$

And (22) becomes:

$$M_R = M[0.35 + 0.65\sqrt{1 + k^2}] \qquad (24)$$

The chart in Fig. 330 will aid in the use of (23) and (24).

The difference between the use of (18) and (20) may be illustrated by an example:

Find the resultant stress in a shaft 10 in. in diameter when the simple stress due to bending is 5000 lb. and the ratio of twisting to bending moment is unity. By (18), $S_1 = 6035$, and by (20), $S_R = 6350$, the former involving an error of nearly 5 per cent. if the elongation theory is correct.

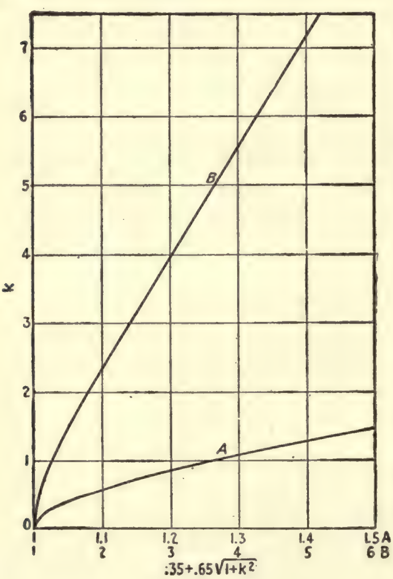


FIG. 330.

One other theory has received some consideration and has been incorporated in a number of text books. It assumes the resultant shearing stress as the limiting stress, the resultant bending stress being ignored. It therefore naturally reduces to a twisting moment formula

equated to the modulus of section for torsion. To avoid confusion, the formula will not be given, but the resultant shearing stress for the example just given is $S_{SR} = 3535$, which indicates that the shaft might safely be smaller. If the maximum strain theory is assumed, a corresponding direct stress given by (7) is: $S_R = 4600$; this, when compared with resultant stress 6350 given by (20), is in error 27 per cent. The error increases with small values of k, and unless increasing factors of safety are employed, undue tensile stress may be produced. This "maximum shear theory" is sometimes known as Guest's Law, although not resulting in Guest's formula, which is empirical.

Letting the bending stress S equal zero in (19) gives:

$$S_R = \frac{m + 1}{m} \cdot S_S$$

which is the same as given by (7), Par. 160. If the bending moment is zero, (21) may be written:

$$\frac{m}{m + 1} \cdot S_R \times 2z = S_S z_S = M_S \tag{25}$$

which is the equation for simple twisting; or if the twisting moment is zero, $M_R = M$. It therefore appears that general equations (19) and (21) give a satisfactory solution of combined bending and twisting for most problems occurring in heat engine design, and in their simplified form given by (23) and (24) are not difficult to apply.

From (7):

$$\frac{m + 1}{m} = \frac{S_R}{S_S}.$$

Then:

$$\frac{m - 1}{2m} = 1 - \frac{S_R}{2S_S}$$

and

$$\frac{m + 1}{2m} = \frac{S_R}{2S_S}.$$

Values of S_R/S_S are given by some experimenters, differing considerably from those given here, and no attempt is made to relate them to m. If desired, these values may be substituted in (21), giving safer results, especially for large values of M_S/M. If $m = 1$, (20) is the same as Guest's formula, although the latter is not derived in this way. This is the safest of all formulas for combined bending and twisting moment of round shafts, but perhaps is unnecessarily so. Recent values of S_E/S_{ES} for various grades of steel range from 1.45 to 2, a value for mild carbon steel being 1.546. This would change (24) to:

$$M_R = M[0.23 + 0.77\sqrt{1 + k^2}]$$

Assuming the theoretical relation given by (7), this gives:

$$m = 1.84.$$

That there is difference of opinion concerning combined bending and torsion is well known, but until the relation between direct and shearing stresses is better known, greater refinement seems unwarranted.

Since the above was written, an interesting article on combined stresses by Prof. A. Lewis Jenkins has appeared in the August number of the Journal of the A.S.M.E., p. 694; it is also in the Transactions, vol. 39, p. 929.

163. Rectangular Section in Torsion.—The torsional modulus of section which applies to circular sections does not express the stress relations in a rectangular section. The stress in the extreme fiber of the latter, instead of being a maximum, is zero, the maximum stress being at the surface of the center of the long side. It may probably be assumed that the stress at the center of the short side bears the same ratio to that of the long side as their distances from the center of the section, inversely. These stresses reduce to zero at the corners in a manner represented by a curved line, the exact form of which is probably unknown. The best expression for the modulus of section of a rectangular section is an empirical formula given by St. Venant as follows:

b = the length of the short side.
h = the length of the long side.
S_S = maximum shearing stress, at center of long side.
S_{S1} = shearing stress at center of short side.
M_S = the twisting moment.
z_S = modulus of section in torsion.

$$z_S = \frac{b^2h^2}{3h + 1.8b} \tag{26}$$

Let:

$$\frac{b}{h} = x$$

then:

$$M_S = z_S S_S = \frac{x^2}{3 + 1.8x} \cdot h^3 S_S = \frac{h^3}{C} \cdot S_S \tag{27}$$

where

$$C = \frac{3 + 1.8x}{x^2}$$

Then:

$$S_S = C\frac{M_S}{h^3} \tag{28}$$

where S_S is the maximum stress, which is at the center of the long side, as just stated:

Then the stress at the center of the surface of the short side may be taken as:

$$S_{S1} = \frac{b}{h} S_S = Cx \cdot \frac{M_S}{h^3}$$

Let:

$$C_1 = Cx = \frac{3 + 1.8x}{x}.$$

Then:

$$S_{S1} = C_1 \cdot \frac{M_S}{h^3} \tag{29}$$

The chart in Fig. 331 may be used to find C and C_1.

164. Struts.—Two of the most important engine parts—the piston rod and connecting rod—are struts. Therefore the selection of a suitable strut formula, which shall be safe while not demanding an excess of material, is essential to rational design. As already stated, this need not be a rational formula; this relieves an embarrassing situation, as the only formula which rests on any satisfactory rational basis is the long column formula of Euler, which, with few exceptions, is outside the range of ratios of length to radius of gyration found in practice.

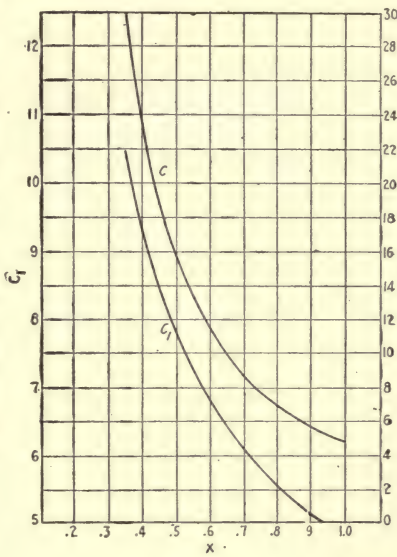


Fig. 331.

Although this formula, for practical conditions, is inferior to any one of several empirical formulas, it is surprising to note that it is still used by some authorities. However, for exceedingly long struts which fail by buckling, and in which the direct unit load is small, Euler's formula gives results agreeing well with experiment; and as it serves to fix a limit for an empirical formula to be discussed later, it is given here.

P = ultimate load on strut.

p = ultimate unit load (pounds per square inch).

A = area in square inches of section of maximum stress. This is at the center when both end conditions are the same.

l = actual length of strut in inches.

l_1 = the distance between two consecutive points of contrary flexure, sometimes called the *effective length.* It is this length which determines the strength of the strut.

r = the least radius of gyration, in inches, of section A.

E = The modulus of elasticity of the material (Young's modulus).

n = a factor depending upon the condition of the ends of the strut.

Then:

$$p = \frac{P}{A} = \frac{\pi^2 E}{\left(\frac{l_1}{r}\right)^2} = \frac{\pi^2 n E}{\left(\frac{l}{r}\right)^2} \tag{30}$$

From (30) it may be seen that n is used so that the formula may be given in terms of actual length; then:

$$n = \left(\frac{l}{l_1}\right)^2$$

Table 80 gives the theoretical conditions usually considered in strut discussions. They assume accurate dimensions, homogeneity of material, and the application of load at the exact gravity axis of the strut. The actual conditions are never accurately known in practice, and intermediate values of n are sometimes used which give values agreeing approximately with tests with such end conditions as are practically attainable, and will be mentioned later.

Of the several strut formulas in common use, the most satisfactory, in the author's opinion, is an empirical formula given by the late Prof. J. B. Johnson (Modern Framed Structures, and Materials of Construction), a brief discussion of which will now be given. The formula is:

Fig. 332.

$$p = K - q\left(\frac{l}{r}\right)^2 \tag{31}$$

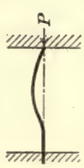
which is the equation of the parabola, K and q being constants. Prof. Johnson assumed that:

$$K = S_E$$

(see Par. 159, Law 8), and q is to be determined so as to bring a parabola tangent to Euler's curve. This of course occurs when the slope of the two curves is the same.

For convenience let $p = y$, and $1/r = x$ (see Fig. 332).

TABLE 80

Case		l_1	n	End conditions
1		$2l$	$\frac{1}{4}$	One end fixed; the other end free and unguided.
2		l	1	Both ends pivoted.
3		$0.7l$	2 (nearly)	One end fixed; the other pivoted, but guided in the direction of axis of fixed end.
4		$0.5l$		Both ends fixed, with common axis.

Then Euler's formula becomes:

$$y = \frac{\pi^2 nE}{x^2}$$

And Johnson's is:

$$y = S_E q x^2.$$

The slope of each curve is:

$$\text{Euler,} \quad \frac{dy}{dx} = -\frac{2\pi^2 nE}{x^3}$$

$$\text{Johnson,} \quad \frac{dy}{dx} = -2qx.$$

When the curves are tangent to each other, y and dy/dx are the same for both; then:

$$\frac{\pi^2 nE}{x^2} = S_E - qx^2 \tag{32}$$

and:

$$\frac{\pi^2 nE}{x^3} = qx \tag{33}$$

Dividing (32) by (33) gives:

$$x = \frac{S_E}{qx} - x$$

or:

$$x^2 = \frac{S_E}{q} - x^2$$

From which:

$$x^2 = \frac{S_E}{2q} \tag{34}$$

Substituting (34) in (32) gives:

$$S_E - \frac{S_E}{2} = \frac{2\pi^2 nEq}{S_E}$$

From which:

$$q = \frac{S_E^2}{4\pi^2 nE} \tag{35}$$

Johnson's formula then becomes:

$$p = S_E - \frac{S_E^2}{4\pi^2 nE}\left(\frac{l}{r}\right)^2 \tag{36}$$

The limit of application of this formula may be found from (34), which gives:

$$x^2 = \left(\frac{l}{r}\right)^2 = \frac{S_E}{2q}$$

or:

$$\frac{l}{r} = \sqrt{\frac{S_E}{2q}} \tag{37}$$

For any greater value of l/r, Euler's formula is to be used, which is:

$$p = \frac{\pi^2 nE}{\left(\frac{l}{r}\right)^2} \tag{38}$$

The value of q may be found which gives close agreement with tests under certain practical conditions and n may be found from (35). This value of n may be assumed to be the same for different materials tested with the same end conditions. The ordinary test conditions are for:

1. Round ends.

2. Pin ends. When the pins are of substantial diameter, friction resists buckling, making the strut stronger than for a round or pivot-ended strut.

3. Flat ends, which are stronger than pin ends, but not as strong as fixed ends.

For the purpose of this book the following values of n will be assumed, although they are some smaller than those used by Johnson:

1. Round ends, $n = 0.90$.
2. Pin ends, $n = 1.45$.
3. Flat ends, $n = 2.00$.

That they agree with tests in some cases at least may be seen from Fig. 333.

These actual end conditions do not occur in engine design, but are given here to assist the designer in determining a value of n which most nearly represents his practical problem. Special cases may arise in which the theoretical values of n in Table 75 may be safely used.

Figure 333 is an ultimate unit load curve for four formulas, with round ends, with the values of n just given. Curve A represents Euler's long column formula, and B, Johnson's parabolic formula. These curves are tangent to each other and together form the adopted curve. Curve C represents Ritter's formula as given by Goodman (Mechanics Applied to Engineering), in which the limiting stress at failure when l/r is zero is taken as the "crushing strength of a short specimen," or practically, the ultimate tensile strength. Curve D is also Ritter's formula as given by Kimball and Barr (Machine Design), in which the limiting stress at failure is taken as the elastic limit. With these curves is plotted, values from tables in the Pencoyd handbook (curve E) which are the average of many tests of struts composed of medium steel structural shapes. To obtain a fair comparison, the physical constants used in the formulas are the same as for the material used for the Pencoyd tests. From a study of the curves it may be observed:

1. That Euler's formula has little application for values of l/r ordinarily found in practice.

2. That Ritter's formula (Goodman) gives results considerably above the tests when S is the ultimate tensile strength.

3. That Ritter's formula (Kimball and Barr) gives values far below the test values when S equals the elastic limit. It is safe, but safety is not the only important consideration in good design.

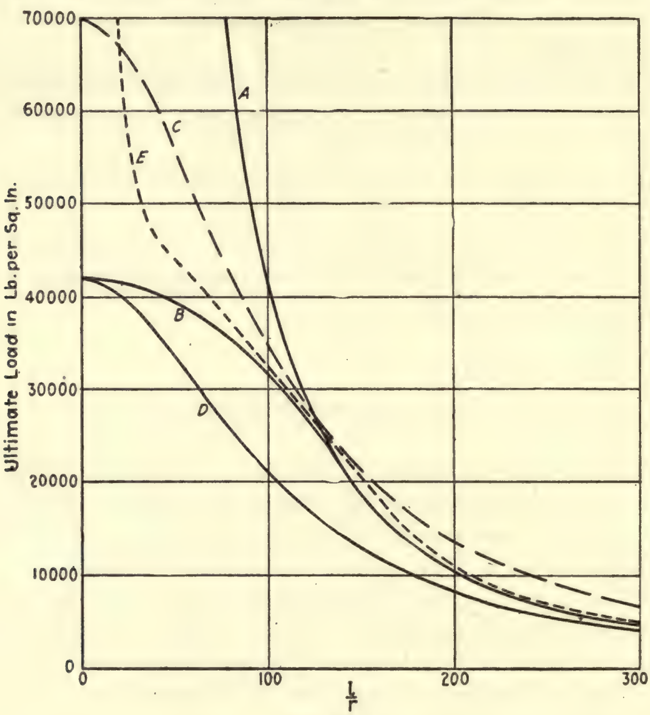


Fig. 333.

4. That Johnson's formula follows the tests very closely excepting for small values of l/r.

Ritter's formula is often given in text-books on machine design and it is no doubt a good safe formula for general design. It is generally mentioned as a rational formula, but as it gives a higher stress for a given unit load when the elastic limit is higher, this can not be true. In a certain gasoline engine connecting rod the unit load is 7100 lb. According to Ritter's formula the stress produced in the rod is 10,850 lb. when the elastic limit is 38,000; but when the elastic limit is 120,000,

the stress in the rod as given by the formula is 18,850, being 74 per cent. greater. This of course is inconsistent.

It is generally conceded that long struts fail when or before the elastic limit of the material is reached. If Johnson's parabola is made tangent to Euler's curve, it seems reasonable to assume that any failure occurring when the unit load lies on or below these combined curves must occur at or within the elastic limit. We may then employ factors of safety with the elastic limit as a basis for any value of l/r, which would not be feasible for formulas employing higher values for the crushing strength of short struts, even though correct.

It is probable, as stated in the Pencoyd handbook, that higher ultimate loads would obtain for round or square sections, due perhaps in part to absence of thin, unsupported flanges, and partly to the ability to obtain better end conditions. On the contrary, the elastic limit of the material decreases as the thickness increases, and this may more than offset the gain by symmetry of section.

Johnson's formula will be employed in this book, and using the physical constants given in Table 73, the following special formulas may be derived, which are conservative for materials commonly employed in engine construction. For cast iron there is no marked elastic limit. The value of K in Formula (31) is taken as 60,000, this giving results agreeing with tests.

Round ends

Steel. When $l/r \ngtr 117$: $p = 38{,}000 - 1.4\left(\frac{l}{r}\right)^2$ (39)

Wrought iron. When $l/r \ngtr 126$: $p = 27{,}000 - 0.79\left(\frac{l}{r}\right)^2$ (40)

Cast iron. When $l/r \ngtr 59$: $p = 60{,}000 - 8.5\left(\frac{l}{r}\right)^2$ (41)

Pin ends

Steel. When $l/r \ngtr 147$: $p = 38{,}000 - 0.87\left(\frac{l}{r}\right)^2$ (42)

Wrought iron. When $l/r \ngtr 166$: $p = 27{,}000 - 0.49\left(\frac{l}{r}\right)^2$ (43)

Flat ends

Steel. When $l/r \ngtr 173$: $p = 38{,}000 - 0.63\left(\frac{l}{r}\right)^2$ (44)

Wrought iron. When $l/r \ngtr 195$: $p = 27{,}000 - 0.355\left(\frac{l}{r}\right)^2$ (45)

Cast iron. When $l/r \ngtr 88$: $p = 60{,}000 - 0.38\left(\frac{l}{r}\right)^2$ (46)

When a strut is subjected to bending stress, from eccentric loading or otherwise, the sum of the bending stress at center of strut and direct unit load should not exceed the unit load allowed by the strut formula, using least radius of gyration. Should the end conditions not be the same in all planes, as in an engine connecting rod, the maximum value of q/r must be used, as it is obvious from (31) that this will give the minimum value of the ultimate load.

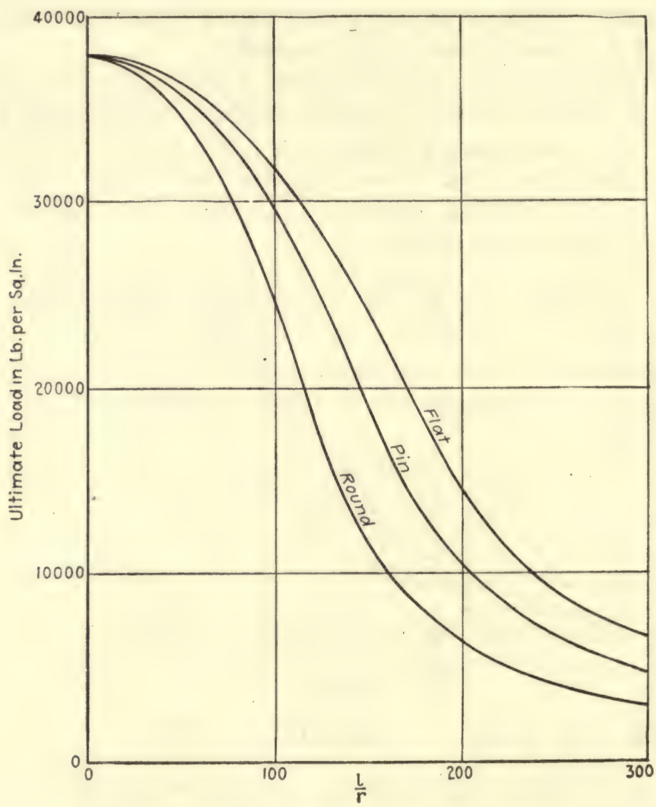


FIG. 334.

To facilitate calculations of steel struts, the chart in Fig. 334 for round, pin and flat ends is given, being plotted from special equations (39), (42) and (44).

165. Clearances and Tolerances.—In the various machinists' handbooks and books on general machine design there are many tables giving clearances and tolerances for different kinds of work and also much other

data relating specifically to the shop. While it is necessary for the designer to have such data, it seems to be outside of the scope of this book, so comparatively little is said on the subject. No absolute standard has been adopted, but many concerns have their own standards. These should be in the hands of all designers in a particular office so that there may be uniformity of practice, at least for that office. The standards should be adopted by the coöperation of engineering department and machine shops, and should not be changed unless experience shows that alterations are desirable.

166. Basis of Design.—The forces acting on engine parts are commonly computed for the maximum steam or gas pressure in the cylinder. If these alone may be considered, it is a simple matter to determine stresses in both magnitude and kind. For large, slow-speed engines, these

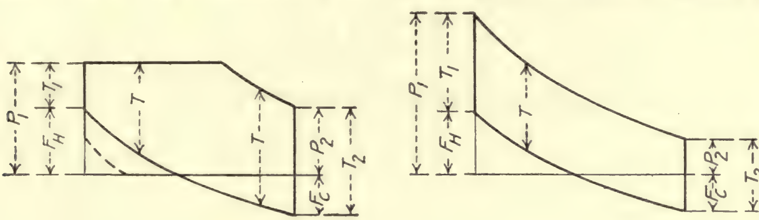


FIG. 335.

forces, with those produced by heavy weights such as the flywheel, are all that are practically necessary to consider; but for high-speed engines, the inertia forces may approach those due to steam or gas pressure, and may not safely be overlooked.

The forces acting normal to the line of stroke are comparatively small for most engine parts, so that by the use of combined indicator and inertia diagrams a comparatively simple method of selecting factors of safety may be devised which will cover ordinary cases.

Aside from attachment to foundation or other supports, the frame is not affected by inertia forces along the line of stroke; indicator diagrams only are required to determine the forces. This is also true of the cylinder. Practically all other parts are affected by inertia.

Single-acting Engines.—Fig. 335 shows stroke diagrams combined with inertia diagrams for single-acting steam and internal-combustion engines. Reference may also be made to Chap. XVI. Whether the resulting stresses are repeated or reversed will depend largely upon the amount of inertia. The inertia diagram may include only the reciprocating parts in determining the forces acting upon certain parts, or for some cases the connecting rod may be included.

The following formulas may assist in obtaining factors. From (33) and (34), Chap. XVI, if n is the ratio of connecting rod to crank length:

$$\frac{F_H}{F_C} = \frac{n+1}{n-1}.$$

Let:

$$\frac{P_2}{P_1} = q \quad \text{and} \quad \frac{F_H}{P_1} = k.$$

The notation is taken from Fig. 335. Then:

$$T_1 = P_1 - F_H = P_1(1 - k) \tag{47}$$

and:

$$T_2 = P_2 + F_C = P_1\left(q + \frac{n-1}{n+1} \cdot k\right) \tag{48}$$

Let the maximum value of T be T_M; then:

$$f_T = \frac{T_M}{P_1} \tag{49}$$

This may be called the pressure factor and may be a part of the factor of judgment.

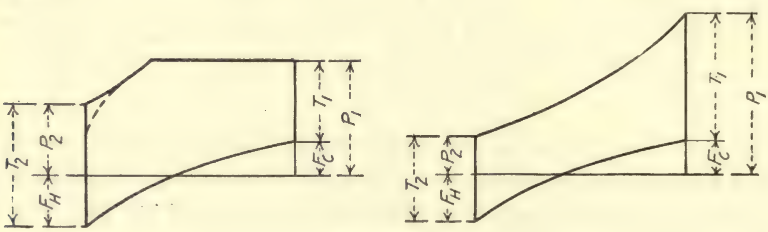


Fig. 336.

In the single-acting steam engine, compression may offset part or all of the inertia at the head end as shown in the dotted lines.

Double-acting Engines.—In these engines, the greater range of T is found in the crank-end diagrams, and these are shown for both steam- and internal-combustion engines in Fig. 336.

The equations for this case are:

$$T_1 = P_1 - F_C = P_1\left(1 - k\frac{n-1}{n+1}\right) \tag{50}$$

and:

$$T_2 = P_2 + F_H = P_1(q + k) \tag{51}$$

In the steam engine, compression may reduce P_2 and T_2, making them negative in some cases.

As an aid in determining factors Table 81 is given. As the single-acting steam engine is little used it is omitted.

TABLE 81

Part	Inertia effect of	4-cycle, single-acting internal-comb. engine		Double-acting			
				Steam		4-cycle Int. comb.	
		Load	Range	Load	Range	Load	Range
Cylinder	....	Repeated	O to P_1	Repeated	O to P_1	Repeated	O to P_1
Frame	....	Repeated	O to P_1	Reversed	O to P_1	Reversed	O to P_1
Piston	....	Repeated	O to P_1	Reversed	O to P_1	Reversed	O to P_1
Piston rod	Piston and ½ rod	Reversed	T_M to F_H	Reversed	T_{MH} to T_{MC}	Reversed	T_{MH} to T_{MC}
Crosshead pin	Piston, piston rod and crosshead	Reversed	T_M to F_H	Reversed	T_{MH} to T_{MC}	Reversed	T_{MH} to T_{MC}
Head end of connecting rod	Piston, piston rod and crosshead	Reversed	T_M to F_H	Reversed	T_{MH} to T_{MC}	Reversed	T_{MH} to T_{MC}
Crank end of connecting rod	Reciprocating parts and connecting rod	Reversed	T_M to F_H	Reversed	T_{MH} to T_{MC}	Reversed	T_{MH} to T_{MC}
Center of connecting rod	Reciprocating parts and ½ connecting rod	Reversed	T_M to F_H	Reversed	T_{MH} to T_{MC}	Reversed	T_{MH} to T_{MC}
Connecting rod for vibration	....	Reversed	Full	Reversed	Full	Reversed	Full
Crank, crank pin and shaft	Reciprocating parts and connecting rod	Reversed	T_1 to T_2 or T_1 to F_H	Reversed	T_1 to T_2	Reversed	T_1 to T_2

The symbols T_{MH} and T_{MC} are maximum effective pressures during the stroke from head to crank end, and from crank to head end respectively. T_{MH} may be found from (47) or (48). It is possible for T to have a maximum value at other than dead-center positions, especially in steam engines when the cut-off is long, but the difference will not be great. Where T_1 and T_2 are less than P_1, the latter has been used for steam engine reciprocating parts in Table 81. It is obvious that a reversal from one stress to another becomes a repeated stress if one of the stresses is zero.

The twisting load on the shaft is practically a repeated load. The load due to weight of wheel, etc. is always a reversed load, and as it is difficult to use both of these factors in the combined stress formula, it is probably better to assume a reversed load when combining with a wheel load, making the factor of judgment unity except for very severe service.

When cranks are in a position to receive a large turning effort—perhaps 40 degrees or more away from either dead center—the stresses due to both bending and twisting may be taken as repeated stresses with a factor of safety of 3 (for ductile materials, and 6 for brittle materials). In this case, actual pressures, including inertia, must be used, the factor not being based upon maximum steam or gas pressure, as no pressure factor is used. It is difficult to give pressure factors near mid-stroke, especially for the internal-combustion engine. A factor of judgment may be used if desired.

In designing engines, it is a simple matter to plot diagrams such as are given in Figs. 335 and 336, and this is recommended.

As an illustration, application will be made for five values of head-end inertia as follows: when $F_H = P_1$, or $k = 1$; when $F_H = P_1/2$, or $k = 0.5$; and when $k = 0.25$, 0.125 and zero. The condition when $k = 0$ would only apply at starting and would be used only for engines working with frequent stops. Values of n and q are also assumed.

It may be seen from Table 81 that aside from the cylinder, frame, and the connecting rod for vibration, the standard factors for which are obvious, there are but two groups, the reciprocating parts and the revolving parts. Factors for these two groups will be tabulated in Table 82; the several values of inertia will be assumed as the inertia of the parts involved; for parts acted upon by the inertia of fewer other parts, the factors may be reduced a small amount.

For obtaining the standard factor f_A, Fig. 326 may be used. For cast iron and other brittle materials the factor of safety is based upon the ultimate strength, and the values of Table 82 should be multiplied by 2,

or in case of reversal the compressive stress may be assumed to have one-fifth of its actual value as stated in Par. 1.

A factor of judgment may be applied, or f_T may be considered as a part of the factor of judgment and may in some cases be equal to it. It must be remembered that the factor finally determined assumes the part acted upon only by the maximum steam or gas pressure, even though in reality the maximum force is not a maximum when the pressure is maximum. This is provided for by f_T and f_A.

TABLE 82

Parts	k	Single-acting engines: 4-cycle internal-combustion engines $n=4$ $q=0.2$			Double-acting engines: Steam $n=5$ $q=0.5$			Double-acting engines: 4-cycle internal-combustion $n=4$ $q=0.2$		
		f_a	f_T	f_af_T	f_a	f_T	f_af_T	f_a	f_T	f_af_T
Reciprocating parts	1.000	5.4	1.00	5.4	5.34	1.50	8.00	5.75	1.20	6.90
	0.500	6.0	0.50	3.0*	5.50	1.00	5.50	5.15	0.70	3.60
	0.250	4.0	0.75	3.0	6.00	1.00	6.00*	5.65	0.85	4.80
	0.125	3.4	0.87	3.0	6.00	1.00	6.00	5.85	0.92	5.40*
	0.000	3.0	1.00	3.0	6.00	1.00	6.00	6.00	1.00	6.00
Revolving parts	1.000	3.0	1.00	3.0	3.70	1.50	5.55	4.00	1.20	4.80
	0.500	6.0	0.50	3.0	5.00	1.00	5.00*	6.00	0.70	4.20*
	0.250	4.4	0.75	3.3	5.70	0.83	4.75	4.60	0.85	3.90
	0.125	4.0	0.87	3.5*	5.05	0.92	4.65	4.00	0.92	3.70
	0.000	3.6	1.00	3.6	4.50	1.00	4.50	3.60	1.00	3.60

Table 82 may be used with judgment by calculating F_H and getting the value of k. For Corliss engines with a single eccentric, the cut-off would never be as long as assumed in fixing the value of q.

It is obvious that for large, slow-speed engines, k would not likely be as great as unity, but it may be greater in high-speed engines. It is best to lay out diagrams as previously suggested, or calculate the inertia at both ends of the stroke, but Table 82 shows some interesting results and may be used as a guide in determining factors. At starting, the maximum gas pressure is applied to the parts of internal-combustion engines, and for this the pressure factor is unity, and the standard factor should never be less than for static loading. The minimum value in Table 82 amply provides for this.

A comparison of the factors for the steam engine and the single-acting internal-combustion engine will show why the latter, although carrying higher pressure, is not proportionally heavier, but sometimes

lighter than the steam engine. Factors marked* may be taken as safe factors for ordinary conditions when a careful analysis is not to be made.

In all design it must be kept in mind that cost of production will always be a great factor in selection. As stated at the beginning of Par. 142, economy, effectiveness, safety and durability are the criteria of good design, and to strike a proper balance between these is the work of the designer. There is no limit to the training and experience which may be applied to this work.

Reference will be made to this paragraph in the treatment of the various details.

CHAPTER XXII

CYLINDERS

Notation.

t = thickness of wall in inches.
D = diameter of cylinder in inches.
D_S = diameter of steam inlet in inches.
D_E = diameter of exhaust outlet in inches.
d = diameter of piston rod in inches.
d_S = diameter of stud in inches.
d_1 = diameter of stud at root of thread.
A = piston area in sq. in. = $\pi D^2/4$.
a = area of steam passage in square inches.
c = clearance distance.
p = unbalanced pressure in pounds per square inch.
f_a = standard factor of safety.
f_3 = factor of judgment.
S = tensile stress in pounds per square inch.
S_P = piston speed in feet per minute.
V = nominal average velocity of steam or gas in feet per minute.

167. The Cylinder.—The material most used for cylinders is hard, close grained, gray cast iron known as cylinder iron. If properly made, cast iron cylinders have sufficient strength, and with proper lubrication and care, the cylinder bore and valve seat attain a smooth, glass-like surface which gives but little frictional resistance and wears almost indefinitely.

Strength calculations are simple and few, the principal dimension to be determined being the thickness of the wall. Cylinder formulas were discussed in Par. 161, Chap. XXI, in which Formula (17) gives the thickness of the wall. The chart in Fig. 328 of the same chapter greatly facilitates calculation. Allowing for other considerations than strength, (17) may be written

$$t = \frac{n-1}{2} \cdot D + k \qquad (1)$$

where k provides for possible reboring, unequal thickness of wall due to shifting of cores in casting, porous material and other defects. This constant may range from 0.25 to 0.5, depending upon the quality of

foundry work produced in a particular foundry. For very small cylinders such as for small gasoline engines, k may be very much less; and for cylinder linings, such as are used in Diesel engines, it may be zero, as when wear occurs the lining may be replaced.

The value of n in (1) is given by (16), Chap. XXI, and is:

$$n = \sqrt{\frac{S + 0.7p}{S - 1.3p}} \tag{2}$$

The stresses in cylinder walls are repeated stresses due to the steam or gas pressure only, inertia having no effect. Table 72 of Chap. XXI gives 6 as the standard factor of safety f_A. Using the tensile strength of cast iron from Table 74, Chap. XXI, which is 16,000, gives a maximum working stress of 2670 lb. per sq. in. A common stress for steam cylinders is 1500; this gives a factor of judgment f_3 of 1.78, which is reasonable, due to the disastrous effect of steam cylinder failure, caused by escaping steam.

Much higher stresses are used in gas engine cylinders—as high as 3500 lb. and even higher. While pressures in such cylinders are probably more erratic than in steam cylinders, there is no escaping steam in case of failure, and due to high temperature, failure is more apt to be due to unequal expansion, the stresses due to which are increased by thick cylinder walls. Neglecting the factor of judgment but retaining the standard factor 6, a working stress of 3500 lb. would necessitate an ultimate tensile strength of 21,000 lb.; this is not excessive for the grade of material which should be used for cylinders, the value 16,000 being very conservative and suitable for general use when the quality of the material is uncertain.

Internal stress due to shrinkage of the metal in cooling is often much greater than operative stresses. For this reason it is often stated that rational formulas do not apply to cast iron. While this is partly true, there seems to be no better method of determining dimensions if the formulas are used with judgment.

Good foundry practice with careful cooling of castings is as essential as good design. The latter consists in an even distribution of material, making the thickness as uniform as possible, and avoiding pockets and constricted passages, the cores for which are apt to be displaced in casting.

Walls other than those of the cylinder barrel may be made according to (1), omitting k if subject to the high-pressure steam; if containing exhaust steam they may be $\frac{3}{4}$ as thick. With small cylinders this may lead to too much difference in thickness, in which case steam and exhaust

passages may have walls of the same thickness. Formula (1) is only an empirical formula when used for other than circular passages.

Cylinder flanges may be about 1.2 times the thickness of the barrel wall.

Flat surfaces should be carefully supported with ribs, but ribs should not be too generally used, especially on the outside of the casting. It is better to have the cylinder barrel free from ribs, as a cracked rib may mean a ruptured cylinder later The strength of flat surfaces when round or rectangular, and not supported by ribs, may be checked by flat plate formulas. For rectangular plates with uniform load, Leutwiler's Machine Design gives:

$$t = b\sqrt{\frac{pK}{S\left[1 + \left(\frac{b}{a}\right)^2\right]}} \tag{3}$$

in which a is the length and b the breadth of the plate and K is Bach's coefficients. For plates supported at the edge, K is 0.565; when fixed at the edge K is 0.375 when the material is cast iron. For mild steel, K is 0.36 and 0.24 for free and fixed edges respectively.

For circular plates with uniform load the same authority gives:

$$t = a\sqrt{K \cdot \frac{p}{S}} \tag{4}$$

where a is the diameter. For cast iron, K is 0.3 and 0.2 for free and fixed edges respectively, and for mild steel, 0.19 and 0.13. For most practical cases the plates may be considered as neither free or fixed. It is well to calculate both ways when a compromise may be made with judgment.

It is well when possible to provide for free expansion of the cylinder barrel. It is true that the usual Corliss engine cylinder has the cylinder ends connected by the steam chest and exhaust passage, which, with the cylinder itself, carry three different mean temperatures; these cylinders operate indefinitely without failure, even with superheated steam in some cases, and when they do fail it may usually be traced to some other cause. However, for high superheat, the free cylinder is probably better.

In any steam cylinder, the exhaust steam should not come in contact with walls having live steam on the other side, as heat from the latter is quickly taken up by the exhaust steam, resulting in condensation and loss. An air space should always be left.

Several points in cylinder design may be shown by Figs. 337 and 338, which are for the cylinder drawings for the 20 by 48-in. Corliss engine of Chap. XII. The covers of the valve chambers are called bonnets, the

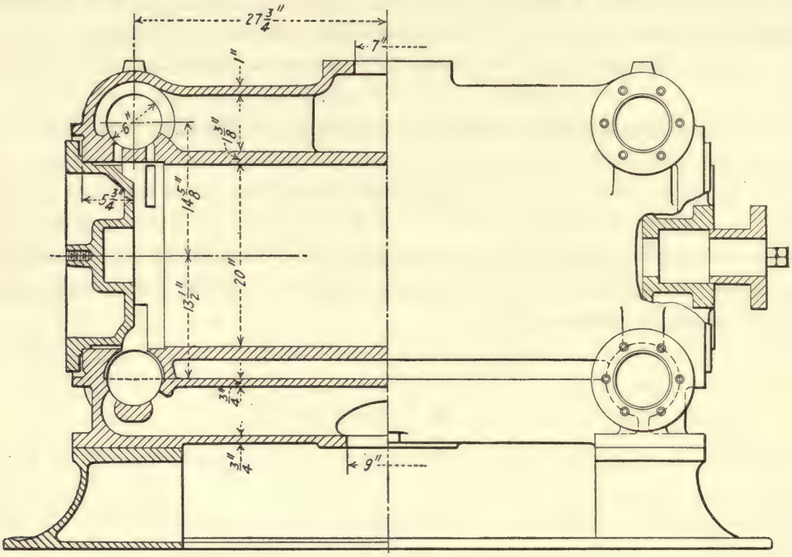


FIG. 337.

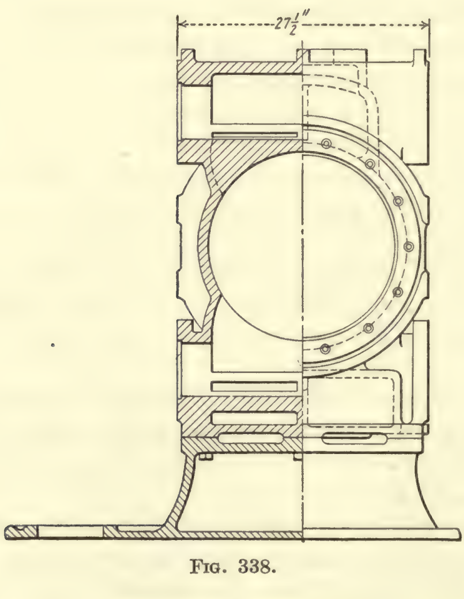


FIG. 338.

front bonnet forming a bracket for the valve gear. The back bonnets for the exhaust valve chambers are tapped for water relief valves. Parts of the cylinder depending upon the valve gear are given in Chap. XX.

Counterbore allows for reboring and for the piston ring to "wipe over" a small amount to prevent wearing a shoulder in the cylinder near the end of the stroke. There is no definite rule for the amount, but 1/16 in. is usually ample; too great an amount may result in slapping of the ring at the ends of the stroke.

The clearance c is as small as is considered safe and may be as small as 1/8 in.; most builders prefer to allow ample in the interest of safety and from 1/4 to 1/2 in. is more usual, depending upon the size of the cylinder. Table 83 gives values of counterbore and clearance for steam cylinders of different size, which may be used as a guide. In view of the discussion of clearance volume in Par. 46, Chap. IX, and Par. 62, Chap. XII, the larger and safer values of c seem to be justified.

TABLE 83

Cylinder diameter	10	12	14	16	18	20	22	24
Counterbore	10¼	12¼	14¼	16¼	18⅜	20⅜	22⅜	24⅜
Clearance	¼	¼	¼	¼	¼	¼	⅜	⅜

Cylinder diameter	26	28	30	32	34	36	38	40
Counterbore	26½	28½	30½	32½	34½	36½	38½	40½
Clearance	⅜	⅜	⅜	⅜	⅜	⅜	½	½

For larger sizes the clearance and allowance for counterbore may be taken the same as the maximum values in the table.

In internal-combustion engines and the uniflow steam engine, clearance is determined by the required compression pressure.

168. Inlet and exhaust passages in steam cylinders are dependent upon the allowable nominal steam velocities; the area of a passage is given by Formula (2), Chap. XX, and is:

$$a = \frac{S_P A}{V} \tag{5}$$

in which V will be taken in ft. per min. in this chapter. The values of V are given by Formulas (3) and (4), Chap. XX.

In steam engines the steam and exhaust connections are for standard round piping; then we may write:

$$a_S = \frac{\pi D_S^2}{4} \qquad a_E = \frac{\pi D_E^2}{4} \qquad A = \frac{\pi D^2}{4}$$

where subscripts S and E denote steam and exhaust respectively. Then (5) becomes:

$$D_S = D\sqrt{\frac{S_P}{V_S}} \tag{6}$$

and:

$$D_E = D\sqrt{\frac{S_P}{V_E}} \tag{7}$$

The next larger diameters of standard wrought pipes may be used if D_S and D_E are intermediate values. Values of V_S and V_E may be found from the curves of Fig. 339, which were plotted from (3) and (4), Chap. XX.

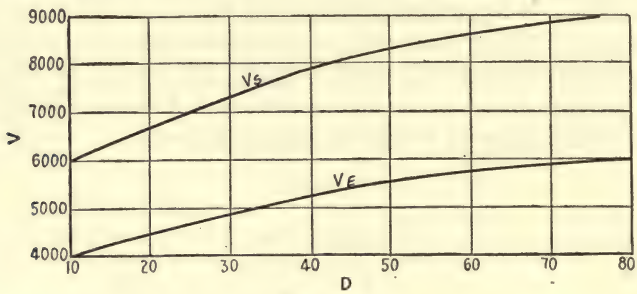


FIG. 339.

Formula (5) may also be used for internal-combustion engines by using the proper value of V. This also applies to (6) and (7) when the passages are of circular section. Lucke says that V should not exceed 3000. Measurements of the cylinder of a well-known automobile engine gives for the entrance to the cylinder a velocity of 8400 ft. per min. and for the exit 6800 ft., for a piston speed of 1000 ft. per min. A modern vertical gas engine gives a gas velocity through the inlet opening of the cylinder head of 5380 ft. per min., and through the exhaust opening, 7850 ft., with a piston speed of 640 ft. These velocities are all nominal velocities as explained in Par. 142, Chap. XX. Velocities through ports and valves of both steam and internal-combustion engines are discussed in Chap. XX.

169. Cylinder heads are largely of empirical design as there are no satisfactory rational formulas for determining the thickness of metal. In shallow heads which set into the cylinder but a small distance, a flat circular plate formula may be used as a check. For deep heads, such as used on Corliss cylinders, the strength depends largely upon the ribs, which form sort of a truss. Such a head is shown in Fig. 340.

As a guide the following dimensions are given:

$$t_1 = 1.1t \qquad t_2 = 0.6t_1$$

It will be noticed that the head has a snug fit in the cylinder for but a short distance, from ¾ to 1 in., depending on the cylinder diameter, after which it is cut away $\frac{1}{32}$ in., or sometimes less. It was once common to have the entire flange surface fit tight against the cylinder, with paper or some other packing between; but now an annular space only comes in contact with the cylinder; this surface, about ¾ in. wide, permits a greatly increased pressure per sq. in. and is more easily kept steam-tight. This surface is sometimes ground in place, but this is not necessary for a steam-tight joint.

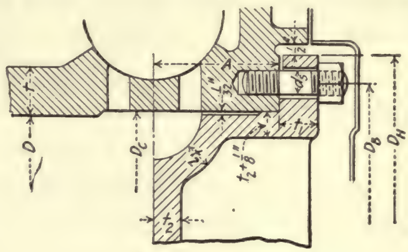


FIG. 340.

It is well to keep the stud circle as near the inner edge as possible without danger of breaking out; good results are obtained if this distance is about 1.2 times the stud diameter, and this same distance may be taken to the outer edge of the head. Table 84 gives the dimensions of high-pressure and low-pressure heads,

TABLE 84

High-pressure heads							Low-pressure heads						
D, in.	D_C, in.	D_H, in.	D_B, in.	d_S, in.	No. studs	S, for $p = 100$	D, in.	D_C, in.	D_H, in.	D_B, in.	d_S, in.	No. studs	S, for $p = 100$
10	10¼	14¼	12¼	¾	8	3260	34	34½	40½	37½	1¼	20	5090
12	12¼	16¾	14½	⅞	8	3370	36	36½	42½	39½	1¼	22	5180
14	14¼	18¾	16½	⅞	10	3670	38	38½	44½	41½	1¼	22	5780
16	16¼	21¼	18¾	1	10	3660	40	40½	46½	43½	1½	24	5890
18	18⅜	23½	21	1	12	3870	42	42½	48½	45½	1¼	24	6470
20	20⅜	25½	23	1	14	4080	44	44½	50½	47½	1¼	26	6560
22	22⅜	27½	25	1	16	4320	46	46½	52½	49½	1¼	28	6680
24	24⅜	30	27¼	1⅛	16	4080	48	48½	54½	51½	1¼	30	6760
26	26½	32	29¼	1⅛	18	4250	50	50½	57	53¾	1⅜	30	6240
28	28½	34	31¼	1⅛	20	4470	52	52½	59	55¾	1⅜	30	6740
30	30½	36½	33½	1¼	20	3980	54	54½	61	57¾	1⅜	32	6800
32	32½	38½	35½	1¼	22	4110	56	56½	63	59¾	1⅜	32	7330
34	34½	40½	37½	1¼	24	4240	58	58½	65	61¾	1⅜	34	7380
36	36½	42½	39½	1¼	26	4390	60	60½	67½	64	1½	34	6440
38	38½	45	41¾	1⅜	26	4100	62	62⅝	69½	66	1½	34	6870
40	40½	47½	44	1½	24	4060	64	64⅝	71½	68	1½	36	6920
42	42½	49½	46	1½	24	4470	66	66⅝	74½	70½	1⅝	36	6280
44	44½	51½	48	1½	26	4530	68	68⅝	76½	72½	1⅝	36	6670
46	46½	53½	50	1½	28	4610	70	70⅝	78½	74½	1⅝	36	7070
48	48½	56½	52½	1⅝	28	4260	72	72⅝	80⅝	76⅝	1⅝	38	7080
50	50½	58½	54½	1⅝	28	4650	74	74⅝	82⅝	78⅝	1⅝	38	7490
							76	76⅝	85¼	81	1¾	38	6860
							78	78⅝	87¼	83	1¾	38	7220
							80	80⅝	89¼	85	1¾	38	7500

with the number of studs, and the stress at the root of the thread for a pressure of 100 lb. per sq. in. The stress for any other pressure may be easily found as it is proportional to the pressure. The notation is as on Fig. 340.

For Corliss high-pressure cylinders, a depth of head equal to the diameter of the valve is satisfactory; the writer has used, for high-pressure heads:

$$\text{Depth} = 0.2D + 1.75 \tag{8}$$

and for low-pressure heads:

$$\text{Depth} = 0.17D + 1.8 \tag{9}$$

There is no such similarity between the designs of heads for internal-combustion engines, but the same general principles apply. The load on cylinder-head bolts or studs is practically a dead load, and the standard factor of safety could be taken as 2 if the stress due to screwing up were known. It is better, however, to take the repeated load factor 3, based upon the total steam pressure and divided by the total bolt area at root of threads, which is the assumption in Table 84; this may be called the nominal stress.

Small bolts are much more liable to be damaged by a wrench, so the factor of judgment may be:

$$f_3 = 1 + \frac{1}{d_S} \tag{10}$$

The factor of safety is then:

$$f = f_A f_3 = 3\left(1 + \frac{1}{d_S}\right) \tag{11}$$

in which d_S is the stud diameter. The safe stress for machinery steel, from Table 73, Chap. XXI is:

$$\frac{38{,}000}{f}.$$

High-grade steel is sometimes used, especially for internal-combustion engines.

With cylinder-head castings for internal-combustion engines, great care must be exercised in design and in the foundry work. There must be an even distribution of metal with no abrupt changes from thin to thick metal. Ribs must not be placed so that they make the casting too rigid and prevent the uneven expansion due to difference in temperature. Cored passages should be as easy as possible.

Stuffing Box.—If any special form of metallic packing is to be used, the stuffing box should be designed accordingly, but for the numerous soft

packings on the market, Fig. 341 gives good results if proportioned according to Tables 85 and 86, and the following formulas:

$$A = 0.15D_B \quad B = 2d_S \quad C = 1.5d_S \quad E = D_B + 2.4d_S$$
$$F = E + 2.4d_S \quad K = D_B - d$$

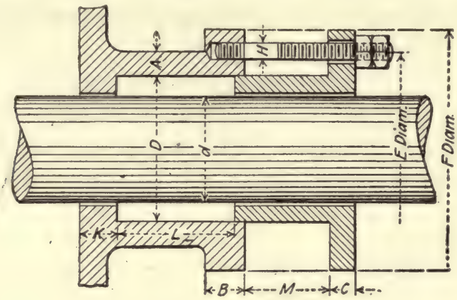


Fig. 341.

Table 85

d, in.	D, in.	L, in.	M, in.
$\leqq$ 3½	d + 1½	4½	3½
>3½ or <5	d + 1¾	5¼	4
$\geqq$ 5	d + 2	6	4½

Table 86

d, in.	d_S, in.	No. studs
3	⅞	2
4	1	2
5	1⅛	2
6	1	3
7	1⅛	3
8	1¼	3
9	1½	3

The stuffing-box flange is made thick to offset the weakening effect of the box, which cuts away the ribs at the center.

170. Cylinder Lagging.—Steam cylinders are covered with about 1½ in. of some heat-resisting material in the form of plaster, such as asbestos or magnesia. Outside of this is the lagging, usually of sheet steel, and provision should be made for fastening this to the cylinder. This may be seen in some of the designs which follow.

A water jacket is required for internal-combustion engines, except for very small cylinders, which may be air-cooled. No lagging is required.

Besides the illustrations of the following paragraphs, some features of design may be found in Chaps. III and V.

The foundry and machine shop should be kept in mind with a view to reducing the cost of production, and no design should be used which has been known to give trouble in operation.

DESIGNS FROM PRACTICE

171. Steam Engine Cylinders.—The Corliss engine cylinder of Figs. 337 and 338, while not designed for an actual engine, is substantially the

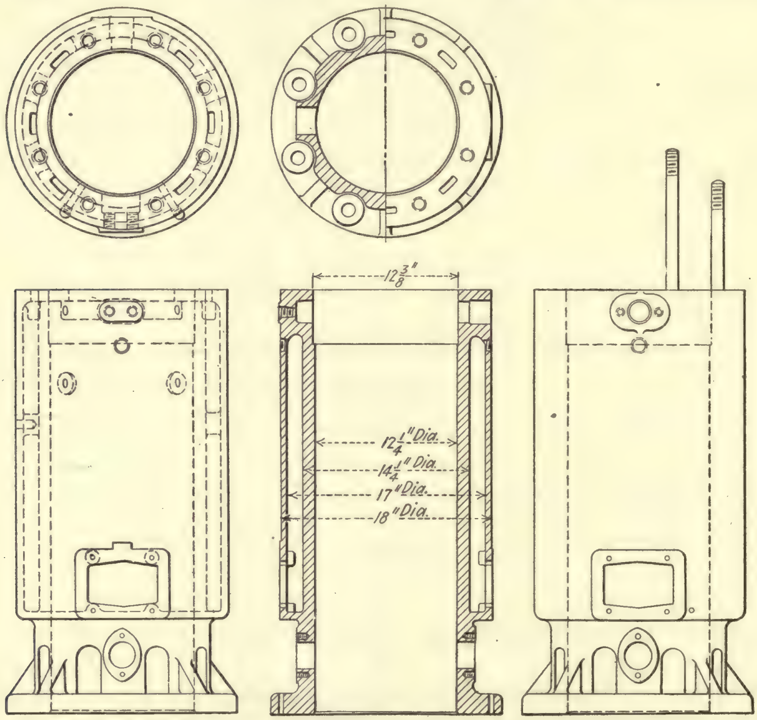


Fig. 342.—Bruce-Macbeth gas engine cylinder.

design used by the author for several years and shows the characteristic features of Corliss cylinders. The cylinder foot is included; openings are provided for the vacuum dashpots, but in some cases these are secured to the foundation independently. The cylinder foot is usually bolted to the foundation.

172. Internal-combustion Cylinders.—A vertical gas engine cylinder is shown in Fig. 342. This is the design of the Bruce-Macbeth Engine Co., Cleveland, Ohio. With a stress of 3500 lb. and a gage pressure of 400 lb., Fig. 328 and Formula (17) of Chap. XXI give a wall thickness of practically $\frac{3}{4}$ in. The actual thickness being 1 in. makes k in (1) equal to $\frac{1}{4}$ in. The jacket wall is made equal to one-half the cylinder-wall thickness.

With a gage pressure of 400 lb., the nominal stress in the cylinder-head bolts at root of thread is 8850 lb. If the elastic limit of the bolt

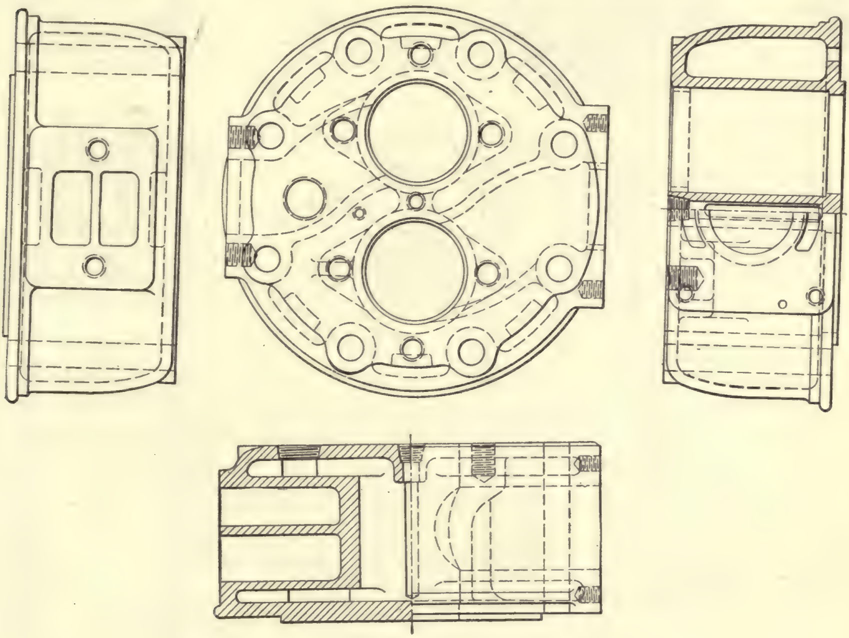


FIG. 343.—Bruce-Macbeth cylinder heads.

material is 38,000, the factor of safety is 4.3. Taking the repeated-load standard factor as 3, gives a factor of judgment of 1.43 instead of 2, as given by (10); the former is no doubt ample.

The head for this cylinder is shown in Fig. 343. The valves and valve cages are shown in Chap. XX.

Fig. 344 shows the air-cooled cylinder of the Franklin Automobile Engine Steel ribs are cast in the cylinder as shown. These are surrounded by a jacket and air is drawn through the passages between the ribs.

The cylinder wall, measured from the bottom of the ribs is $\frac{3}{16}$ in. thick. With a gage pressure of 350 lb. and no allowance for wear, etc., Formula (1) of this chapter gives a stress of 3325 lb., while a pressure of 400 lb. gives 3800 lb. With the standard factor of 6 for repeated load for cast iron, neglecting the factor of judgment, these stresses give nec-

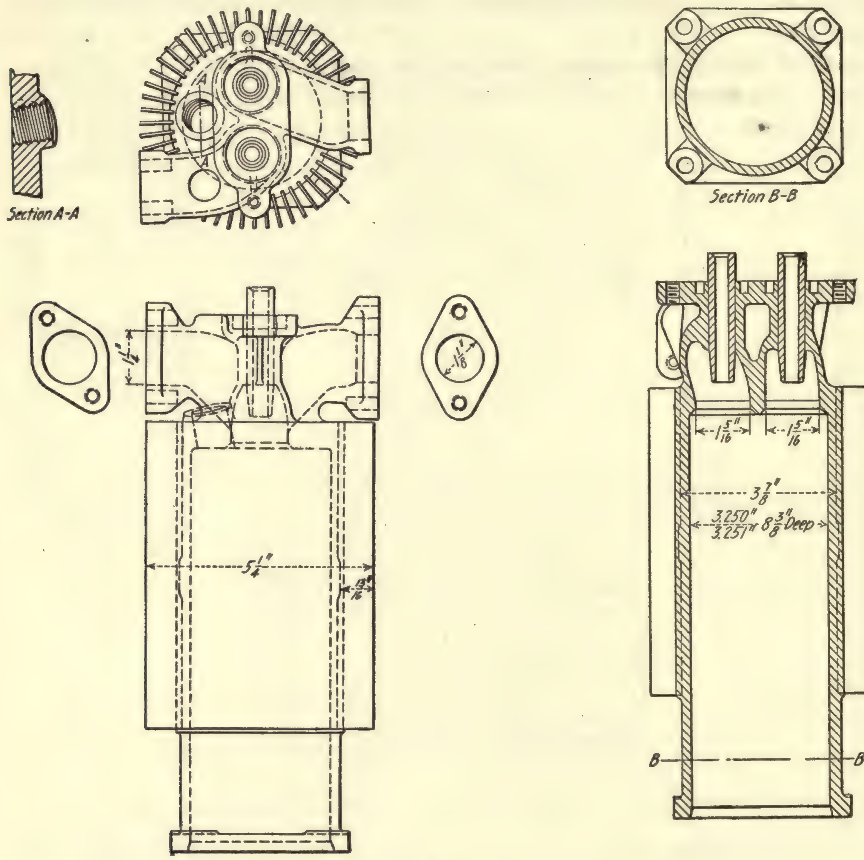


FIG. 344.—Cylinder of Franklin automobile engine.

essary ultimate strengths of 20,000 and 22,800 lb. respectively. The latter is no doubt conservative, as the better grades of gray iron have a tensile strength as high as 30,000 lb.

There are four $\frac{3}{8}$-in. bolts holding the cylinder to the frame. By the S.A.E. standard, they have 24 threads per inch. The stress at root of thread for direct thrust only is 9000 lb. for a maximum explosion pressure of 350-lb. gage, and 10,300 for 400 lb. pressure. This gives a factor

of 4.2 and 3.7 respectively for the conservative value of the elastic limit given in Table 73, Chap. XXI. Both of these are greater than the standard factor 3 for repeated load, giving factors of judgment of 1.4 and 1.23. This would, no doubt, provide for the side thrust due to the connecting rod. The maximum pull on the bolts occurs when the product $P_N a$ (Fig. 345) is a maximum; as before, neglecting the stress due to screwing up, the stress is due to a repeated load. If n is the ratio of the length of connecting rod to length of crank, (12), Chap. XVI, gives:

$$P_N = \frac{P \sin \theta}{n\sqrt{1 - \left(\frac{\sin \theta}{n}\right)^2}}.$$

The pull on the bolts for the arrangement of bolts in Fig. 344 is:

$$P_B = \frac{P_N a}{b}$$

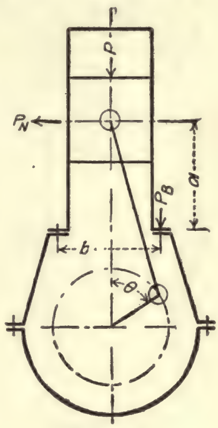


FIG. 345.

Assuming the diagram of Fig. 274, Chap. XX applies to this engine—which is not strictly true—the values of P_B were calculated for all positions covering that giving the maximum value; corresponding pressures were taken from the indicator diagram in the same figure, and the maximum combined stress from these is 11,300 lb. per sq. in., and occurs at position 1, or very near head-end dead center. This does not include the inertia of the rod, which might have modified the result somewhat. From the same diagram, the stress produced by maximum gas pressure is 8620 lb.

The stress due to P_N is transmitted through the rod, so inertia forces must be included in the value of P used (this should properly be the p_A of Fig. 204, Chap. XVI, instead of P_N); but in obtaining the direct load on the bolts in line of stroke, inertia has no effect and gas pressure only must be used.

For the value of mild steel in Table 73, Chap. XXI, the maximum stress just found gives a factor of safety of 3.36, but S.A.E. standard bolts have an elastic limit of 60,000 lb., giving a factor of 5.3, or a factor of judgment of 1.77.

In a steam engine, the maximum steam pressure continues to a point beyond that at which P_B is a maximum, so that the stress in the bolts (neglecting initial stress) is greater than that produced by direct steam pressure in the line of stroke.

Fig. 347 shows a side elevation and plan of a pair of cylinders of the

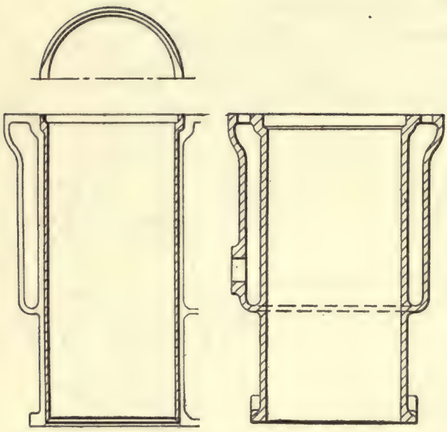


FIG. 346.—Sections of Sturtevant cylinder.

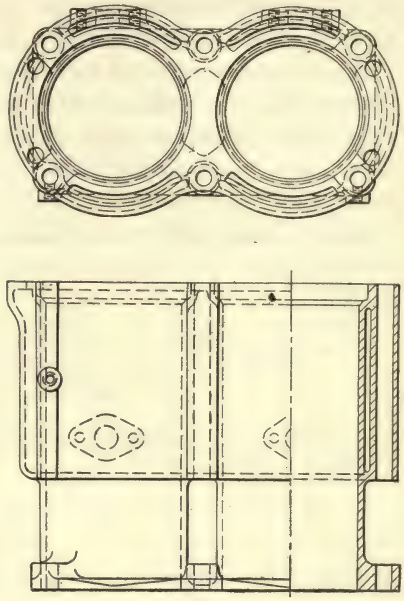


FIG. 347.—Sturtevant airplane engine cylinder.

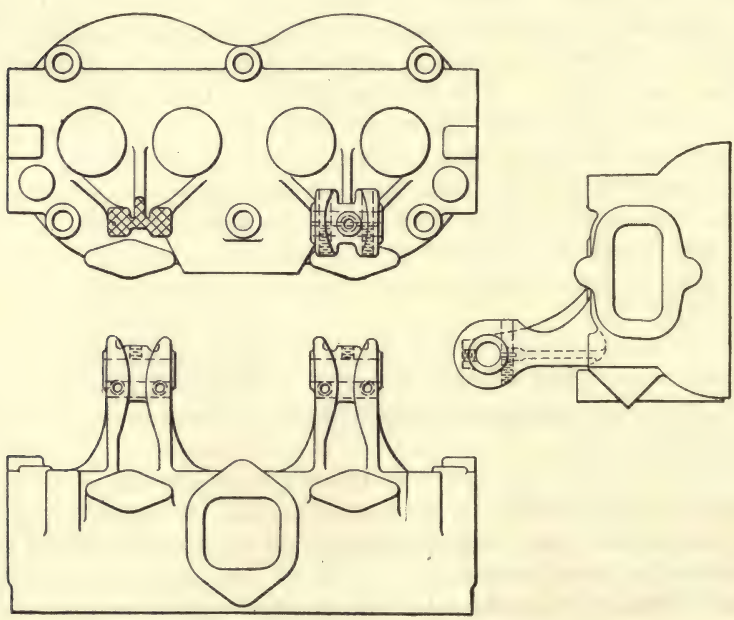


FIG 348.—Sturtevant cylinder heads.

Sturtevant airplane engine, without liners. Fig. 346 shows a section without sleeve, and a section with the sleeve in place. The cylinders are attached to the crank case by bolts extending through lugs as shown at section, and through the cylinder head.

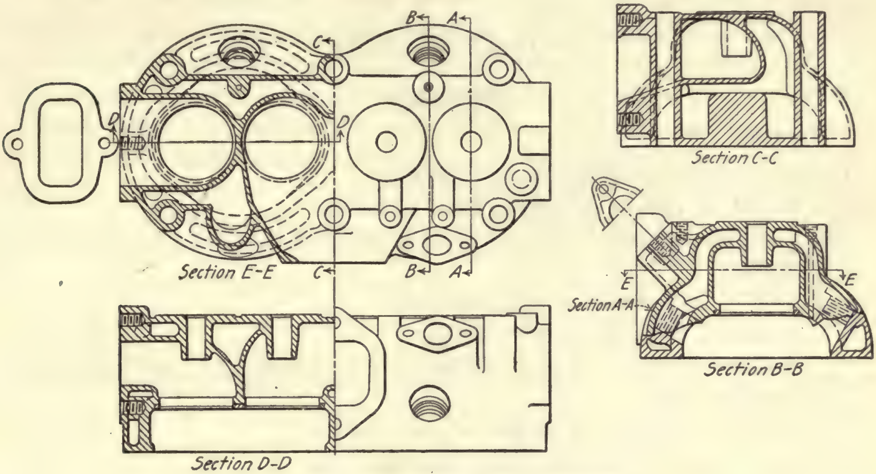


FIG. 349.—Sturtevant cylinder heads.

Fig. 348 gives outside views of two cylinder heads cast enbloc, with rocker arm fulcrums; the fulcrums were formerly cast separately and bolted to the head. Fig. 349 shows such a head, which also better gives the detail design.

CHAPTER XXIII

PISTONS

Notation.

D = diameter of cylinder bore in inches.
F = length of piston, or piston face, in inches.
L = distance between the centers of gravity of the two semi-circles, in inches.
a = area of half circle in square inches.
r = radius of cylinder bore in inches.
r_1 = radius of outside of piston ring when free, assumed constant for an eccentric ring.
t = thickness of ring in inches, at any section.
t_M = maximum thickness of ring—opposite cut.
w = width of ring in inches.
I = moment of inertia of ring section.
z = modulus of section of piston section.
M = bending moment at any section of ring.
E = modulus of elasticity.
S = bending stress in piston; also at any section of ring when in cylinder, exerting pressure p_N.
ks = stress developed in stretching ring over piston to place it in groove.
s_M = maximum bending stress when ring is in cylinder (at section of thickness t_M).
p = maximum unbalanced pressure per square inch in cylinder.
p_N = normal pressure in pound per square inch of ring upon cylinder wall.
W = total maximum pressure in pounds, upon engine piston.

173. Pistons are made in several different styles, the most common for steam engine use being the box piston, cast in one piece as in Figs. 361 and 363, and the built-up, or bull-ring piston shown in Fig. 360. In the former, ribs are usually provided for strengthening the piston; the holes in one side of the piston necessitated by the core, are tapped and plugged. The cores between the ribs are sometimes connected, leaving holes in

the ribs. This type of piston has given some trouble, due to the formation of cracks where the walls join hub and rim; this may be due to poorly distributed metal or improper cooling of the casting. This design is not much used on very large engines.

The bull-ring piston is composed of the spider, or body, the bull ring (sometimes called junk ring), the follower, and, in common with the solid piston, one or more packing rings, which prevent the leakage of steam past the piston. The stiffening ribs contain bosses into which are tapped the follower bolts. A circular rib connects the radial ribs and is provided with set screws and lock nuts, by means of which the bull ring may be adjusted to take up wear. If wear is excessive the cylinder may be rebored and a new bull ring provided. Trouble is sometimes experienced by the breaking of follower bolts, and as this has been attributed to expansion, long studs, tapped into the farther side of the piston have been used by some builders, as shown in Fig. 360; the stud is thus allowed to stretch and bend slightly without undue stress.

The wearing surface of the piston is sometimes provided with rings or strips of Babbitt metal, or some other anti-friction metal; this is more common in large pistons. Cast iron upon cast iron usually gives the best wear, but an occasional exception arises, when a change to a babbitted bull ring gives better results, due, no doubt, to some defect in the metal of cylinder or ring.

When a single packing ring is used and made in sections, or where two "snap rings" are used, the bull ring is made in one piece, but for the usual single ring it is in two parts, as shown in Fig. 360.

When lightness is required in a large piston, perhaps largely for the purpose of balancing, as in marine engines, conical pistons are sometimes used, as in Fig. 362. Sometimes the box piston is made conical.

Single-acting steam engines and internal-combustion engines have trunk pistons, which also serve as crossheads, examples of which are shown in Figs. 364 and 365. Trunk pistons should be designed so that the thrust due to the angularity of the connecting rod may be transmitted from cylinder wall to wrist pin without distorting the piston; the proper placing of ribs provides for this in very light pistons, as seen in Fig. 365.

The *material* used in pistons is usually cast iron, but there is no reason, aside from expense, why the spider and follower of a bull-ring piston may not be steel castings. Pistons for automobile engines are sometimes made of an aluminum alloy, but it has probably not been tried sufficiently long to insure success. Morley, in his Strength of Materials, says that under repeated stress, pure aluminum tends to "creep," or gradually fail. Aluminum also has a high coefficient of expansion—more than twice as

great as for iron. If enough aluminum is used to greatly lighten the piston, some trouble may possibly ensue. According to the American Machinists' Handbook, aluminum becomes granular and easily broken at about 1000° F.

The working stress may be taken as 1200 lb. for cast iron pistons, and 5000 lb. for steel castings. From Tables 73 and 74, Chap. XXI, this gives a factor of judgment over the standard reversed stress factor of 1.11 for cast iron, and zero for steel.

There is no strictly rational method of determining the stress. The flat plate formulas would apply only to the simple plate piston, which is

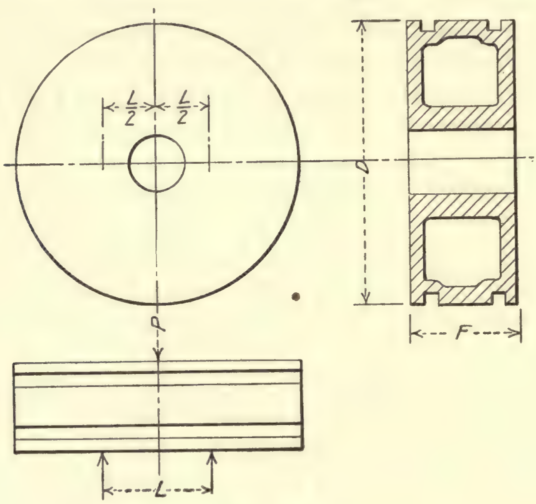


Fig. 350.

rarely used. The piston has been considered as a beam supported at the centers of gravity of the two parts on either side of a diameter, by one of the leading manufacturers, with the maximum piston load applied at the center. Possible rupture would occur along a diameter, which would be resisted by the weakest section. This would be through the cored holes of the box piston, the plugs not adding to the strength. This method may be used as a check and will be given. Fig. 350 is self-explanatory.

From mensuration:

$$\frac{L}{2} = \frac{D^3}{12a}$$

and as:

$$a = \frac{\pi D^2}{8}, \quad L = \frac{4D}{3\pi}.$$

From the beam equation:

$$Sz = \frac{WL}{4}; \text{ and as } W = \frac{\pi p D^2}{4}$$

$$Sz = \frac{pD^3}{12} \tag{1}$$

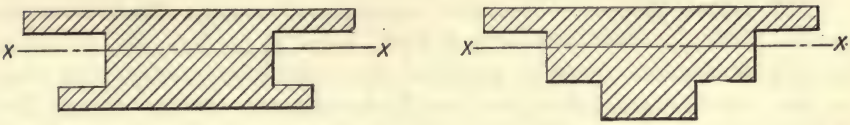


Fig. 351.

S may be found for a given design, or the necessary value of z for a given stress.

In most cases the section may be put into a simple form as in Fig. 351, omitting the follower and bull ring from the built-up piston.

The center of gravity may be found by calculation, locating the neutral axis xx; the moment of inertia may then be found and divided by the distance of the extreme fiber from the neutral axis, giving the minimum value of z, and this must be used in (1).

For the conical piston, Fig. 352, an empirical rule, based in part upon a flat plate formula, is:

$$T = 1.825\sqrt{\frac{pD}{S}} \cdot \sin\theta \tag{2}$$

For cast iron, if $S = 1200$ and $\theta \overline{\gtrless} 60$:

$$T = 0.046\sqrt{pD}$$

For steel casting, if $S = 5000$:

$$T = 0.023\sqrt{pD}$$

T_1 may equal 0.5 T.

Using the factor 7.2 for cast iron, as mentioned in connection with Table 72, Chap. XXI, the working stress may be 2220 lb. when the ultimate strength is 16,000 lb.

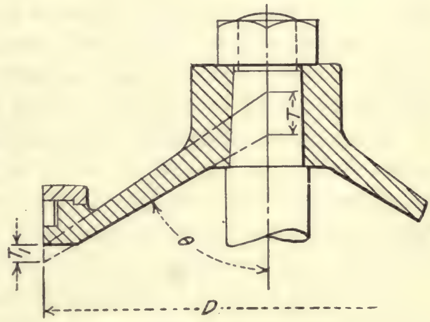


Fig. 352.

The hub into which the piston rod is fitted will be considered under the subject of piston rods. Trunk pistons will be discussed under cross-heads.

The length of piston F must be such as to allow of sufficient strength

and to give ample wearing surface. For constructional reasons, F is proportionally larger in small than in large engines. For steam engines, an empirical formula giving satisfactory results is:

$$F = \frac{D}{4} + 2'' \tag{3}$$

174. Piston Rings.—Numerous varieties of piston rings are on the market, but the most common is some form of spring ring, so called because it is made larger in diameter than the cylinder bore, and when in place, exerts a pressure upon the cylinder surface. In bull-ring pistons this pressure is augmented by springs placed between the packing ring and bull ring at intervals, as shown in Fig. 360. When flat springs are used, lugs are cast on the ring to keep them in place. Such rings are usually cut at an angle of 45 degrees and a "keeper" provided to prevent leakage, as shown in Fig. 353.

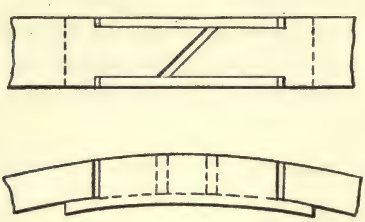


FIG. 353.

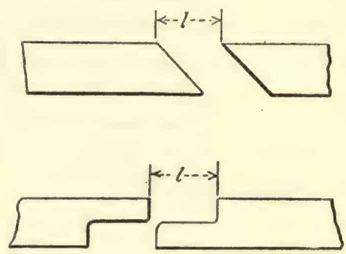


FIG. 354.

Rings providing all of their own spring are called "snap rings;" these are either made of uniform thickness or eccentric, the latter approximating a form which will remain circular under uniform pressure at any diameter which does not strain the ring beyond the elastic limit. Two methods of cutting these rings are shown in Fig. 354.

Some authorities claim that the ring of uniform thickness is superior to the eccentric ring; they are more common, possibly because cheaper to construct. The design of the snap ring is often empirical, but a fairly satisfactory rational method is not difficult and will be given. The eccentric ring best lends itself to such a discussion.

The ring must be designed so that it will not be stressed above its elastic limit when in place in the cylinder, or while sprung over the edge of the piston while placing in the groove. The latter process is done but seldom and the stress may be higher than the working stress, which, however, is constant and may be high.

Figure 355 shows the ring in place, Fig. 356 shows it free, while

Fig. 357 shows it being placed in the groove. A somewhat smaller opening would suffice for the last, but the assumption made is safe and simplifies calculation.

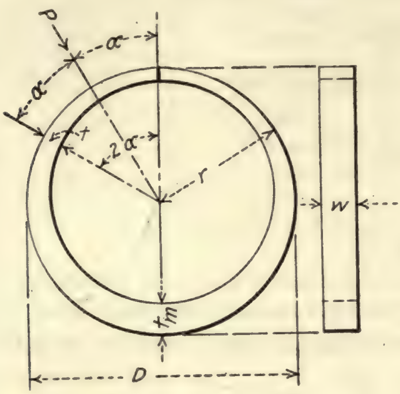


FIG. 355.

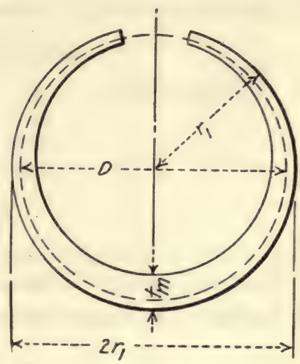


FIG. 356.

In Fig. 355, the force P (which is the normal pressure per sq. in. multiplied by the projected area in angle 2α upon which it acts) produces a bending moment M at x. Angle 2α may be any angle, but it is convenient to take it as 20 degrees, then increase it by increments of

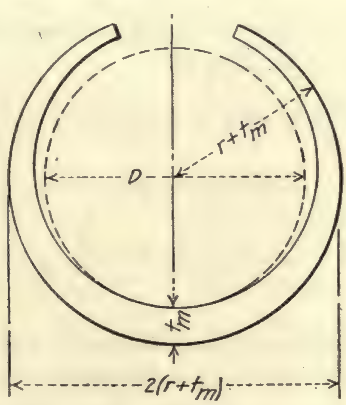


FIG. 357.

20 degrees until the maximum thickness at 180 degrees is reached. In each case P acts midway between the cut and the section x. In this discussion the radius will be taken to the outside of the ring instead of

at the neutral axis of the sections, involving a small error; the bent beam theory is also ignored.

In Fig. 355:

$$P = p_N w \cdot 2r \sin\alpha = 2p_N wr \sin \alpha \tag{4}$$

Taking moments about x (any section):

$$M = Pr \sin \alpha = 2p_N wr^2 \sin^2\alpha \tag{5}$$

Also at any section:

$$M = \frac{wt^2 S}{6} \tag{6}$$

and:

$$I = \frac{wt^3}{12} \tag{7}$$

From the general equation for change of curvature from free radius r_1 to radius of cylinder bore r, substituting M and I from (3) and (4):

$$\frac{1}{r} - \frac{1}{r_1} = \frac{M}{EI} = \frac{2S}{Et} = \text{constant} \tag{8}$$

That is, the same relation must hold for every section.

Assuming the stress developed by placing the ring in the groove (Fig. 357) as kS;

$$\frac{1}{r_1} - \frac{1}{r + t_M} = \frac{2kS}{Et} = \text{constant} \tag{9}$$

Taking S and t at the maximum section (opposite the cut) and equating values of $1/r_1$, from (8) and (9) gives:

$$t_M = r\left[(1 + k)\frac{S_M}{E} + \sqrt{(1 + k)\frac{S_M}{E}\left[(1 + k)\frac{S_M}{E} + 2\right]}\right] = Kr \tag{10}$$

Equating (5) and (6) when $2\alpha = 180$, $t = t_M$ and $S = S_M$, and solving for p_N, which is constant, gives:

$$p_N = \frac{S_M}{12}\left(\frac{t_M}{r}\right)^2 = \frac{S_M}{12}\cdot K^2 \tag{11}$$

Value of t at any Section.—Substituting (5) and (7) in (8) when $2\alpha =$ 180, and again when it equals any angle, gives:

$$\frac{1}{r} - \frac{1}{r_1} = \frac{24p_N r^2}{Et_M^3} = \frac{24p_N r^2 \sin^2 \alpha}{Et^3} \tag{12}$$

from which:

$$t = t_M\sqrt[3]{\sin^2 \alpha} \tag{13}$$

Table 87 is given to facilitate calculation.

TABLE 87

2α	$\sin \alpha$	$\sin^2\alpha$	$\sqrt[3]{\sin^2\alpha}$
20	0.17365	0.0302	0.3112
40	0.34202	0.1172	0.4892
60	0.50000	0.2500	0.6300
80	0.64279	0.4130	0.7450
100	0.76604	0.5870	0.8375
120	0.86603	0.7500	0.9085
140	0.93969	0.8830	0.9595
160	0.98481	0.9700	0.9900
180	1.00000	1.0000	1.0000

It is usually accurate enough to draw a circle touching as many of the points found by (13) as possible.

From (8), conveniently taking $S = S_M$ and $t = t_M$, the radius of the ring when free is:

$$r_1 = \frac{1}{\dfrac{1}{r} - \dfrac{2S_M}{Et_M}} = \frac{r}{1 - \dfrac{2S_M}{KE}} = mr \tag{14}$$

Length of Cut.—Referring to Fig. 354, the length cut out of the ring after it is turned to radius r_1 is:

$$2\pi(r_1 - r)$$

This allows it to fit in the cylinder when finished, after which, a small amount c may be cut out; the total cut is then:

$$l = 2\pi(r_1 - r) + c = 2\pi r(m - 1) + c \tag{15}$$

The amount c cut out after the ring is turned or ground to exactly fit the cylinder may be taken as:

$$c = 0.006D + 0.02'' \tag{16}$$

which is an empirical formula.

Maximum Stress.—For a ring of given dimensions the maximum stress in place is, from (8):

$$S_M = \frac{Et_M}{2}\left(\frac{1}{r} - \frac{1}{r_1}\right) \tag{17}$$

And when being placed in the groove:

$$kS_M = \frac{Et_M}{2}\left(\frac{1}{r_1} - \frac{1}{r + t_M}\right) \tag{18}$$

To assist in calculation, or to determine different relations approximately, Table 88 has been calculated for eccentric rings. The stresses are high, but for the grade of cast iron used for rings, are not excessive.

TABLE 88

$(1+\kappa)\frac{S_M}{E}$	K	If $E = 12{,}000{,}000$ and $\kappa = 1.2$			
		S_M	κS_M	p_N	m
0.000915	0.0436	5,000	6,000	0.79	1.0194
0.001095	0.0480	6,000	7,200	1.15	1.0212
0.001280	0.0519	7,000	8,400	1.57	1.0228
0.001465	0.0555	8,000	9,600	2.05	1.0246
0.001650	0.0592	9,000	10,800	2.63	1.0258
0.00183	0.0623	10,000	12,000	3.23	1.0270
0.00201	0.0654	11,000	13,200	3.92	1.0290
0.00219	0.0684	12,000	14,400	4.68	1.0300
0.00238	0.0714	13,000	15,600	5.52	1.0310
0.00256	0.0741	14,000	16,800	6.41	1.0325
0.00274	0.0767	15,000	18,000	7.35	1.0335
0.00293	0.0795	16,000	19,200	8.43	1.0347
0.00311	0.0819	17,000	20,400	9.51	1.0361

Rings of Uniform Section, although cut, sprung together, and turned to fit the cylinder, do not exert a uniform pressure. To do so, r, p, E and t must be constant, and (12) gives:

$$r_1 = \frac{1}{\frac{1}{r} - \frac{24p_N r^2}{Et^3}\sin^2\alpha} = \frac{1}{\frac{1}{r} - q\sin^2\alpha}$$

$$= \frac{r}{1 - \frac{24p_N \sin^2\alpha}{K^3E}} = \frac{r}{1 - \frac{2S_M\ \sin^2\alpha}{KE}} \tag{19}$$

The value of r_1 varies at every point, but Unwin gives an approximate method for finding the form of the free ring by assuming r_1 the same over an angle of 30 degrees. Fig. 358 shows the method, in which $ab' = aa'$ $b'b'' = a'a''$, etc. The distance $ca, = r_1$ for the arc ab', $c'b'$ for the arc $b'b''$, $c''b''$ for the arc $b''b'''$, etc. The method is exact if these arcs are indefinitely small, so that greater theoretical accuracy would be obtained by taking a larger number of divisions; but the errors of draftsmanship increase with too small divisions however, offsetting the theoretical advantage.

Large rings are sometimes hammered to this form, but more commonly it is neglected. All rings, however, should be turned or ground to the diameter of the cylinder when sprung together. A common method of making small rings, whether eccentric or of uniform thickness, is to cast a pipe and cut the rings from it. A more refined way, and one which will not cut away the best metal, which is always nearest the skin, is to

cast the rings separately, allowing but slight finish, then grind to size.

Number of Rings.—An empirical formula for width or ring is:

$$w = 0.02D + 0.2'' \tag{20}$$

which may be used as a guide. As the width increases but slowly with the

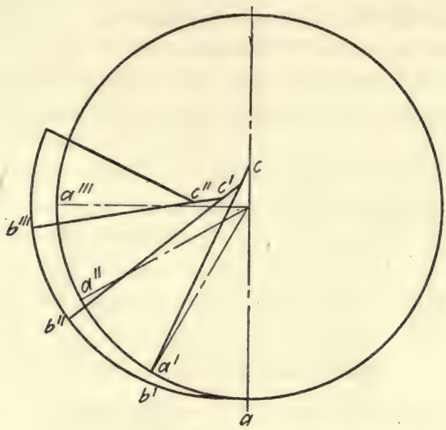


FIG. 358.

cylinder diameter, it is customary to retain the same width for several diameters, advancing by sixteenths or even eighths of an inch for the larger sizes. The chart of Fig. 359 is plotted from (20) with the changes mentioned.

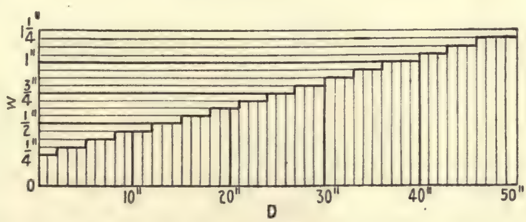


FIG. 359.

If p is the maximum unbalanced pressure per square inch in the cylinder, the following relation may be of assistance in fixing conditions:

$$p_N\, wn = \frac{p}{40}$$

from which:

$$p_N n = \frac{p}{40w} \tag{21}$$

This is an empirical formula in which n is the number of rings. There is

no definite rule for determining the number of rings. If p_N is assumed, n may be found; if n is assumed, p_N may be found, and from Table 88, K and m may be taken for determining the thickness and free diameter. It is obvious that great refinement in these selections is not necessary.

It may be considered that n is 2 for steam engines, and should a single ring be used the width may be twice as great; otherwise the thickness will be excessive. As already stated, single rings are provided with springs, so that a thinner ring may be used, giving greater flexibility and a more uniform pressure against the cylinder wall.

Internal-combustion pistons have from 3 to 10 rings, 3 being a common number for small engines. Many Diesel engines have 6 rings.

Rings must be fitted into the grooves so that they are free to adjust themselves. F. A. Halsey, in the Handbook for Machine Designers, says that a little side play in piston rings is desirable, and that the steam pressure will force the ring against the side of the groove, preventing leakage.

DESIGNS FROM PRACTICE

175. Steam Engine Pistons.—Fig. 360 shows a built-up piston used on the Bass-Corliss engine. A modification is made in the original design

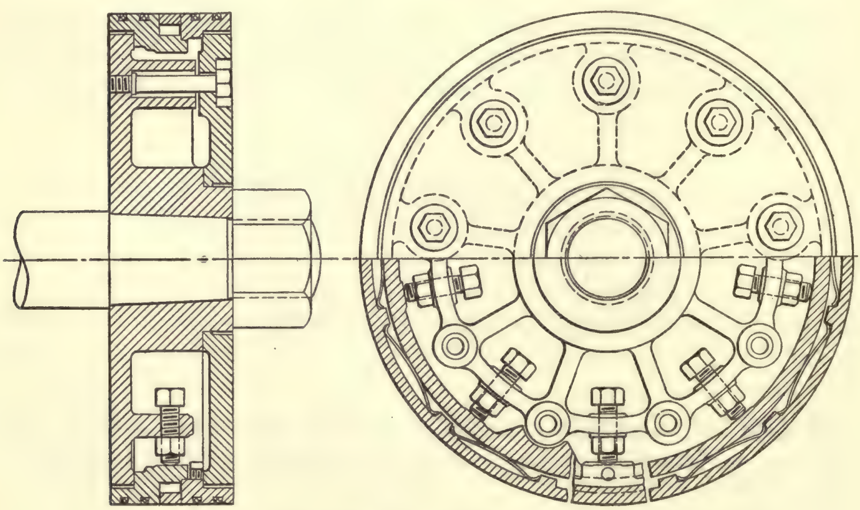


Fig. 360.—Bass-Corliss piston.

by making the follower bolts as described in Par. 173. While Babbitt metal is shown in the bull ring, this is not standard.

Using the method of stress determination described in connection

with Fig. 351, the stress from (1) is 2400 lb. with 100 lb. steam pressure. With 125 lb. the stress is 2980 lb. It is probable that the actual stress is not so high; at any rate this piston has been successfully used with a pressure of 125 lb. But it is more satisfactory to have (1) give a lower stress, and for this reason Formula (3) was devised by the author and gives a greater depth. The change makes but little difference, however, in this particular design, still giving 2875 lb. stress for 125 lb. pressure. With the ultimate strength of cast iron 16,000 lb., this gives a factor of safety of but 5.57, less than that suggested for repeated stress. The

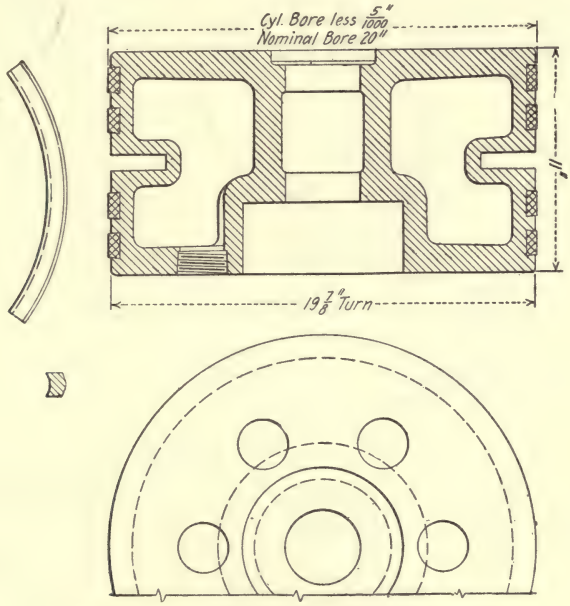


FIG. 361.—McIntosh and Seymour piston.

possible factor for reversed stress in cast iron suggested in connection with Table 72, Chap. XXI, is 7.2. With this as a basis the factor of judgment is 0.77, or less than unity, which, in view of the fact that the follower plate and bull ring were ignored in the calculation, may be reasonable. If the piston spider were a steel casting the factor of judgment is 1.74; it would probably equal at least unity for semi-steel or some of the better grades of cast iron often used. Unwin gives 3000 lb. as the working stress in pistons.

Fig. 361 shows a piston designed for Type *F* gridiron-valve engines built by the McIntosh and Seymour Corporation, Auburn, N. Y. It is a

box piston containing no ribs, but having a wide face and rather thick walls. The holes through which the cores were held are drilled out and

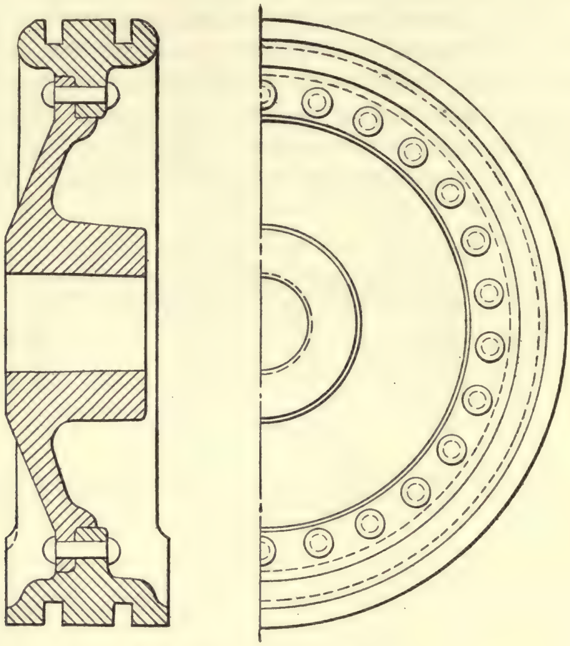


FIG. 362.—Locomotive piston.

tapped with a 2-in. pipe tap, plugged, and then these plugs are faced flush with the piston as it is finished in the lathe.

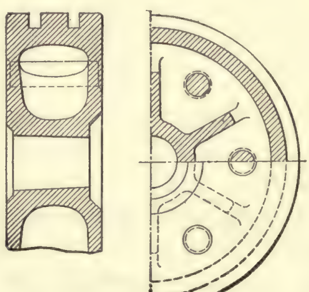


FIG. 363.—Locomotive piston.

For strength calculations, this piston is composed of two flat plates; one of these plates has a central load, and this is resisted at the edge by

a portion of the pressure on the other plate transmitted through the cylindrical portion of the piston. The other plate, exposed to a uniform steam pressure has a central load, and at the edge a load caused by the resistance of the first plate to bending. Morley's Strength of Materials contains the elements of an analysis of this condition, and it may be

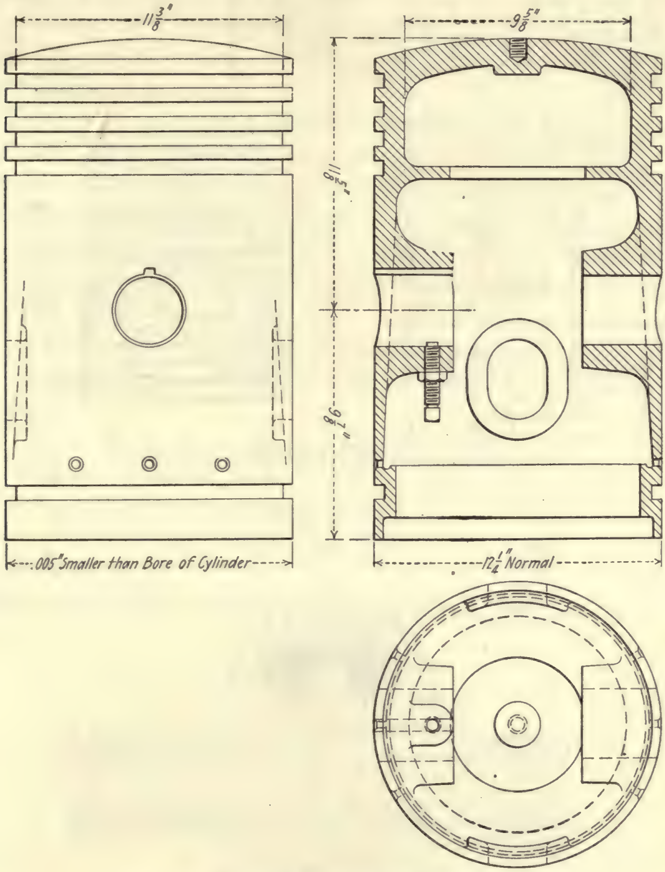


FIG. 364.—Bruce-Macbeth gas engine piston.

possible to combine them; it is probably not practical, however, and the design must be mostly empirical.

Figure 362 shows a locomotive piston designed by the American Locomotive Co. It is a conical piston with cast steel body riveted to a cast iron bull ring. With a pressure of 200 lb., a working stress of 5000 lb.

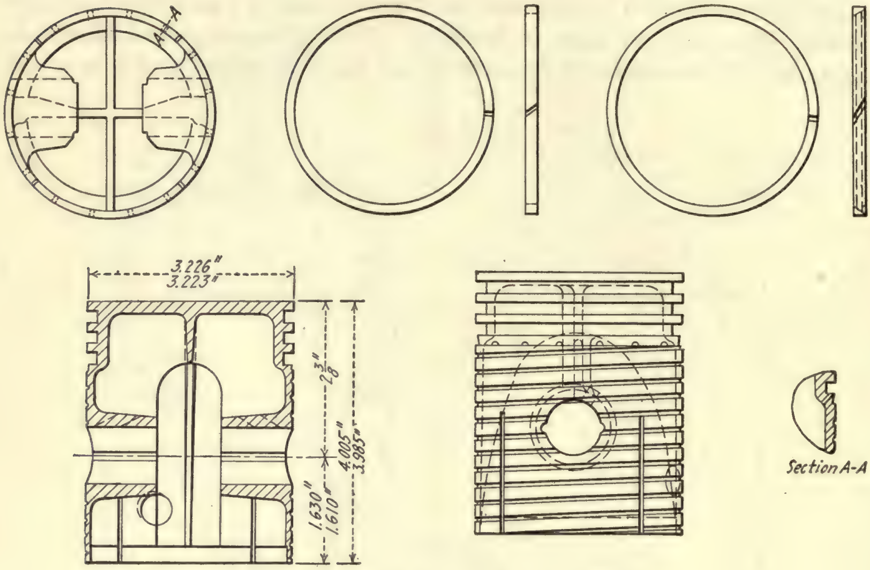


FIG. 365.—Franklin automobile engine piston.

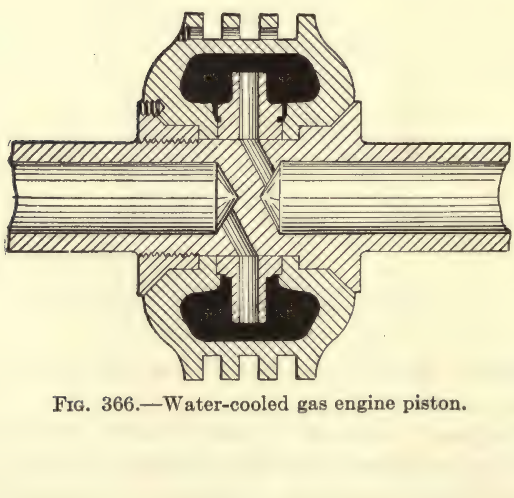


FIG. 366.—Water-cooled gas engine piston.

and using the mean angle ($90° - 20.5°$), T equals $1\frac{3}{4}$ in. This is but $\frac{1}{8}$ in. more than the dimension given.

Another style of locomotive piston by the same builders is shown in Fig. 363. This is a box piston with ribs, the outer ends of which are cut away, allowing the core to extend around the piston. The core holes are filled by driving in rivets which are countersunk, and faced in finishing the casting. This piston is cast iron. From (1), the stress with 200 lb. pressure is 1340 lb. Using the standard factor 7.2, the factor of judgment is 1.66. If the standard factor 12 is used (see Chap. XXI), the factor of judgment is unity.

176. Internal-combustion Engine Pistons.—A gas engine piston used by the Bruce-Macbeth Engine Co., Cleveland, Ohio, is shown in Fig. 364. There is little opportunity for strength calculation in such pistons, and the determination of their dimensions is largely a matter of judgment based on experience. The bearing surface must be sufficient to keep the bearing pressure within certain limits and this is considered in Chap. XXVI.

In the larger engines running at slow and medium speeds, the inertia effect is not great, and sufficient thickness may be used so that ribs are not necessary.

Figure 365 is the piston of the automobile engine built by the Franklin Automobile Co., Syracuse, N. Y. It is very light, and ribbed for strength and stiffness. A helical groove is turned upon the surface for oil distribution. There are practically no strength calculations which may be made for this piston.

A water-cooled piston for a double-acting gas engine is shown in Fig. 366.

CHAPTER XXIV

PISTON RODS

Notation.

D = diameter of cylinder bore in inches.

d = diameter of body of piston rod in inches.

r = radius of gyration of rod section.

t_1 = thickness of wall of crosshead neck, considered as a thick cylinder.

t_o = ditto for piston hub.

p = maximum unbalanced pressure per square inch in cylinder.

p_1 = normal pressure per square inch in rod fit, due to taper or forcing.

P = total maximum unbalanced pressure on piston. Also used for total ram pressure in making pressed fit.

P_A = portion of total piston load taken by taper when rod is in compression.

P_F = portion taken by friction due to fit.

P_B = portion taken by shoulder.

S_T = tensile stress through key eye at diameter d_2.

S_R = tensile stress at root of thread.

S_C = compressive stress in portion of key in rod, and in rod, due to key.

S_K = compressive stress at shoulder.

S_1 = hoop stress in neck of crosshead due to taper fit.

S_o = hoop stress in hub of piston due to taper, or forced straight fit.

l = length of piston rod in inches, taken from center of piston face to center of crosshead pin.

n = taper from axis in inches per foot.

μ = coefficient of friction.

f = factor of safety.

m = number of threads per inch.

T = tons per inch of diameter, per inch of length, required to make a pressed fit—the maximum, as fit is complete.

T_1 = tons per inch of diameter required to make a pressed fit.

177. Piston Rod Formulas.—There are various formulas for determining the diameter of piston rods, most of which take no account of the method of attachment, which is usually the weakest part of the rod. Some formulas consider the length, and while it is well to check the rod as a strut, in few instances will it be found weak in this respect when end fastenings have been properly provided for. Too elaborate an analysis may not be necessary, but from an examination of a number of piston rod failures, it is evident that certain important points are sometimes overlooked.

There are many methods of fastening the rod to crosshead and piston. The best crosshead connection in the author's opinion is the taper fit and key when properly designed and constructed, with not too steep a taper, but one which will usually allow the removal of the rod without a hydraulic press. This also applies to the piston end when piston design permits of its convenient application, but as this is seldom so, the taper fit and nut may be used. The nut may be circular, partially or wholly concealed within the piston, and tightened by a special wrench, but the plain nut is simplest, and while it may increase the clearance volume slightly, the effect upon economy is negligible. Standard thread is too coarse for large diameters, so to facilitate tightening, a finer thread is usually employed; five or six threads per inch gives good results.

Steep taper with no shoulder is sometimes employed, but the shoulder is preferable, as it definitely fixes the length of the rod. In some designs, as in Fig. 371, the end of the rod "bottoms" in the crosshead, an excellent design but more difficult to fit.

A straight fit with shoulder and nut is sometimes used for the piston end, and is good practice; but a pressed fit with rod end cold-riveted is to be condemned as unreliable.

As the formulas derived will cover most forms of attachment, the analysis first employed will be for a rod having a taper fit and key at the crosshead end, and a taper fit and nut at the piston end. It is assumed that when the rod is in tension the entire load is carried by the smallest rod section cut by the key at the crosshead end, and by the section at the root of the thread at the piston end. In compression, at both ends, the load is in part taken by the wedging action of the taper which is resisted by the hub considered as thick cylinder; by the friction of the wedging action; and the remainder by the shoulder.

The *crosshead end* is shown by Fig. 367, all dimensions being in inches. The area wd_2, subjected to crushing, is not more that one-half of the area $2wb$, in shear; therefore, crushing will be considered and shear ignored in the present discussion. With low compressive stress, shearing is not likely to begin.

For equal strength in tension and compression, for the most economical dimensions:

$$S_C w d_2 = S_T\left(\frac{\pi d_2^2}{4} - w d_2\right)$$

or:

$$w = \frac{\pi d_2}{4} \cdot \frac{S_T}{S_T + S_C}.$$

Also:

$$P = \frac{\pi D^2 p}{4} = w d_2 S_C = \frac{\pi d_2^2}{4} \cdot \frac{S_T S_C}{S_T + S_C}$$

from which:

$$d_2 = D\sqrt{\frac{p(S_T + S_C)}{S_T S_C}}.$$

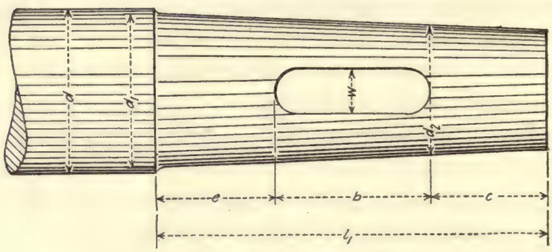


FIG. 367.

Substituting in the expression for w gives:

$$w = \frac{\pi D}{4}\sqrt{\frac{S_T p}{S_C(S_T + S_C)}} \qquad (1)$$

From Fig. 367:

$$d_1 = d_2 + \frac{n}{6}(b + e).$$

If $b + e = x d_1$:

$$d_1 = \frac{d_2}{1 - \frac{nx}{6}}.$$

Substituting the value of d_2 just found, gives:

$$d_1 = \frac{D}{1 - \frac{nx}{6}}\sqrt{\frac{p(S_T + S_C)}{S_T S_C}} \qquad (2)$$

If a thrust P_A will produce an allowable normal unit pressure p_1:

$$P_A = \frac{n}{12} \cdot \pi d_1 l_1 p_1.$$

The additional thrust which may be resisted by friction under pressure p_1 is:

$$P_F = \pi d_1 l_1 p_1 \mu.$$

Then:
$$P_A + P_F = \pi d_1 l_1 p_1 \left(\frac{n}{12} + \mu\right).$$

If this is equal to P, no shoulder is required, but if less, the remainder, P_B, must be carried by the shoulder. Letting $n/12 + \mu = k$:

$$P_B = P - (P_A + P_F) = \frac{\pi}{4}(D^2 p - 4 d_1 l_1 k p_1).$$

Also:

$$P_B = \frac{\pi}{4}(d^2 - d_1^2) S_K.$$

Equating these two values of P_B and solving for d gives:

$$d = \sqrt{\frac{D^2 p - 4 d_1 l_1 k p_1}{S_K} + d_1^2} \tag{3}$$

It will be found convenient to determine P_1 for a given value of S_1, the allowable stress in the crosshead neck. From (16) and (17), Chap. XXI, the thick cylinder formula for free ends is:

$$1 - \frac{2t_1}{d_1} = \sqrt{\frac{S_1 + 0.7p_1}{S_1 - 1.3p_1}} \tag{4}$$

or:

$$p_1 = \frac{\left(\frac{2t_1}{d_1} + 1\right)^2 - 1}{1.3\left(\frac{2t_1}{d_1} + 1\right) + 0.7} \cdot S_1 = qS_1 \tag{5}$$

Substituting (5) in (3) gives:

$$d = \sqrt{\frac{D^2 p - 4 d_1 l_1 k q S_1}{S_K} + d_1^2} \tag{6}$$

If t_1 is required for known rod dimensions, solving for q from (6) gives:

$$q = \frac{D^2 p - S_K(d^2 - d_1^2)}{4 d_1 l_1 k S_1} \tag{7}$$

And from (5):

$$q = \frac{\left(\frac{2t_1}{d_1} + 1\right)^2 - 1}{1.3\left(\frac{2t_1}{d_1} + 1\right)^2 + 0.7} \tag{8}$$

Solving for t_1 in (8) gives:

$$t_1 = \frac{d_1}{2}\left[\sqrt{\frac{1 + 0.7q}{1 - 1.3q}} - 1\right] \tag{9}$$

the value of q being found from (7).

If all dimensions are known, (7) gives:

$$S_1 = \frac{D^2p - S_K(d^2 - d_1^2)}{4d_1l_1kq} \tag{10}$$

Formula (10) may be used for investigation. If the rod has no shoulder and is not bottomed, $S_K = O$; then (3) and (6) fail, and q or S_1 may be obtained from (7) or (10), and t_1 determined from (9). A steeper taper is then required to prevent excessive hoop stress.

The *piston end* of the rod is shown in Fig. 368. The same analysis may be used as for the crosshead end when the rod is in compression, replacing d, d_1, l_1, t and S_1 by d_C, d_O, l_O, t_O and S_O. Then (6) becomes:

$$d_C = \sqrt{\frac{D^2p - 4d_Ol_OkqS_O}{S_0} + d_O^2} \tag{11}$$

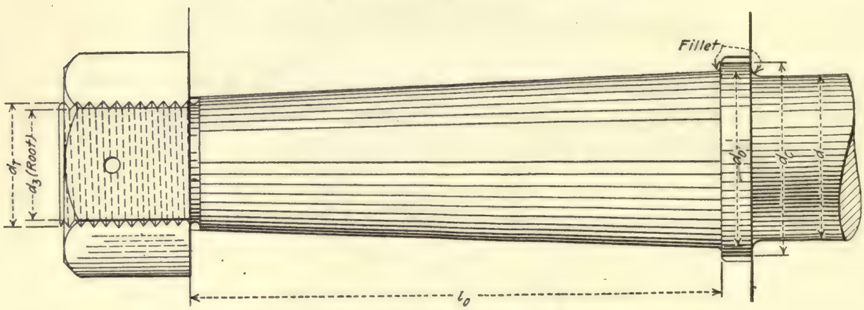


FIG. 368.

Referring to (6), (11) and (5), it may be seen that, if:

$$d_O = d_1$$

and:

$$\frac{t_O}{d_O} = \frac{t_1}{d_1}$$

and if the taper is the same at both ends of the rod, d_C will equal d, and the collar may be omitted if $l_OS_O = l_1S_1$; or if:

$$\frac{S_O}{S_1} \geqq \frac{l_1}{l_O} \tag{12}$$

As the piston hub is strengthened by the walls and ribs, this condition may easily be assumed, if in doing so the resulting value of d_T is not too small.

To find d_T, the diameter of the screwed end:

$$P = \frac{\pi D^2p}{4} = \frac{\pi d_3^2S_R}{4}.$$

Then:

$$d_3 = D\sqrt{\frac{p}{S_R}} \tag{12a}$$

and:

$$d_T = d_3 + \frac{1.3}{m} = D\sqrt{\frac{p}{S_R}} + \frac{1.3}{m} \tag{13}$$

Then:

$$d_O \overline{>} d_T + \frac{l_O n}{6} + 0.125 \tag{14}$$

If d_O is greater than d_1, a collar may be forged on the rod as shown in Fig. 368, of diameter d_C, which may be found by (11); or safely, and more simply:

$$d_C = d_O + d - d_1 \tag{15}$$

With a straight forced fit in the piston, n equals zero and $k = \mu$, which may be used in (11). It is safe, however, to take $k = O$ in (11). This will be further discussed in Par. 178. If a key is used, the fit diameter may be found from (2), making n zero.

Numerical Values.—The piston rod is commonly made of forged, open hearth steel, known as machinery steel, the elastic limit of which is given as 38,000 in Table 73, Chap. XXI. In the key eye and in the thread at the piston end, the initial stress is much greater than that due to the working load in good design and construction, so the load may be considered static with a standard factor of safety in tension of 2. Due to unknown stresses from the driving of the key, the factor of judgment may be taken from 2 to 2.5. The standard factor in compression is 3, but as the stress is apt to be more evenly distributed, the factor of judgment for the key may be from 1.25 to 1.5.

It is safer to assume cast iron as the material for the crosshead, and as the load is practically static, the standard factor is 4. As the driving load is reversed—although of small stress value—the factor of judgment may be from 1.5 to 1.75.

The factor of judgment for the piston hub may safely be unity, due to the reinforcing of walls and ribs. These stresses are all practically static, and are not affected by changing loads due to steam and gas pressure and inertia.

As lubrication is more effective with a taper fit than with a straight fit, the coefficient of friction may be 0.1. A good taper is 1 in. per ft., total.

From the foregoing, and from other considerations of good design, the following data may be taken:

$S_T = S_R = 8000$, $S_C = S_K = 10{,}000$, $S_1 = 2500$, $S_O = 4000$, $n = 0.5$, $\mu = 0.1$, $b = d_1$, $e = c = 0.8d_1$, $l_1 = 2.6d_1$, $t_1 = d_1/2$, $t_O = d_O/2$. From the above: $k = 0.1417$, $x = 1.8$ and $q = 0.509$.

The crushing stress S_C is sometimes taken more than twice as great, in which case it is necessary to check for shear; but the lower value reduces unit bending load on the key considered as a beam, lessening deflection, and the large bearing surface minimizes the chance of damage due to driving the key.

Substituting the above values in the various equations gives:

$$d = 0.0189D\sqrt{p} \tag{16}$$

$$d_1 = 0.0177D\sqrt{p} \tag{17}$$

$$w = 0.00524D\sqrt{p} \tag{18}$$

$$l_1 = 0.046D\sqrt{p} \tag{19}$$

The minimum value of e may be $0.6d_1$, but l_1 must not be decreased without checking by (6). The distance $e + b$ must not be greater than the value used to determine d_1 and d_2. It is well to make $b \overline{\gtrless} d_1$.

From (12):

$$\frac{l_O}{l_1} = \frac{S_1}{S_O} = \frac{2500}{4000} = 0.625.$$

If $l_O \overline{\gtrless} 0.625l_1$, no collar will be required, and d_O may equal d_1 as in Fig. 369, if this value is not less than that given by (14). Then from (14):

$$d_T \overline{\gtrless} d_O - \frac{l_O n}{6} - 0.125 = d_O - \left(\frac{l_O}{12} + 0.125\right) \tag{20}$$

Also from (13):

$$d_T \overline{\gtrless} 0.011D\sqrt{p} + 0.216 \tag{21}$$

The greater value of d_T must be used. It is usually preferable to have:

$$d_O \overline{\gtrless} d_1$$

The piston rod as a strut may be checked by Formula (36), Chap. XXI. The rod is not fixed at the ends as this would involve the binding of crosshead and piston between guides and cylinder walls, but the large bearing surfaces tend to keep the ends of the rod in line, and for safety it may be considered as a pin-ended strut; it probably is between a pin end and a flat end in strength. Formula (42), Chap. XXI may then be used as the special formula, taking l from the center of the piston face to the center of the crosshead pin. Solving for d gives (where $d = 4r$):

$$d \overline{\gtrless} \sqrt{\frac{fD^2p + 14l^2}{38{,}000}} \tag{22}$$

The factor of safety f is made up of the standard factor for reversed load, and a factor of judgment of from 1.4 to 1.6, due to the possibility of

some eccentricity from adjustment of the bull ring. Taking f as 9 for double-acting steam engines, (22) becomes:

$$d \geqq \frac{\sqrt{D^2 p + 1.56\, l^2}}{65} \tag{23}$$

If this value is greater than that given by (6) or (16), it must be used; but usually it will give a smaller value and may be used when lightness is especially desirable, or for short-stroke engines, in which case the rod ends must be proportioned as already explained, resulting in a collar at each end.

The factor $f_A f_T$ for double-acting steam engines, for parts loaded as the piston rod body, is also given as 6 in Par. 166, Chap. XXI, making the factor of judgment 1.5 for the value just assumed.

Example.—As an example of design, a rod will be proportioned for the 20 by 48 in. Corliss engine of Chap. XII, the unbalanced steam pressure being 125 lb. per sq. in.

From (16), $d = 4.225$, say $4\frac{1}{4}''$.
From (17), $d_1 = 3.96$, say $4''$.
From (18), $w = 1.162$, say $1\frac{3}{16}''$.
From (19), $l_1 = 10.29$, say $10\frac{1}{4}''$.
Also $e = 3.175$, say $3''$, and $b = 4''$.

In rounding up fractions it is safe never to make d_1 less than the value given by the formula, while $e + b$ must never be greater. If it is desired to save space, e may be made $\frac{3}{4}$ of the calculated value, especially with a steel crosshead.

From Chap. XXIII, the piston face is 7 in. ($= l_o$), which is greater than $0.625 l_1$, and no collar will be required. Then $d_o = d_1$. From (20), $d_T = 3.29$, say $3\frac{1}{4}$ in. From (13), $d_T = 2.716$ in., so that $3\frac{1}{4}$ in. from (20) is safe.

Following tentatively through the design of the details involved, the length of the rod l is about 86 in.; then from (23), $d = 3.82$ in., showing that the rod is safe as a strut.

The above calculations are for open hearth hammered steel with an elastic limit of 38,000 lb.; for other steels the general formulas must be used, and working stress substituted according to the judgment of the designer.

178. Pressed Fits.—Straight fits are sometimes used on one or both ends of the rod. These vary from a sliding fit to a forced fit capable of withstanding the entire load, and in some cases, with the addition of riveting over the end, the load is so carried. The riveting, however, of cold steel adds but little to the safety, and in general the method is

untrustworthy. If supplemented by nut or key, straight fits of various degrees of tightness are satisfactory when once assembled, but are difficult to take apart if forced.

A simple method of computing the stresses due to forced fits, and perhaps as reliable as any, is to assume a coefficient of friction, and determine the relation between ram pressure, thickness of hub wall, normal pressure, and stress resulting therefrom. The method was in part covered by the taper fit analysis already given, and the notation already applied to the crosshead end will be used, d_1 being the uniform diameter of the fit. P will also be used for total ram pressure and T for tons per inch of diameter per inch of length of the fit, due to P. Then:

$$P = 2000Td_1l_1 = \pi d_1 l_1 p_1 \mu \tag{24}$$

From which:

$$T = \frac{P}{2000d_1l_1} = \frac{\pi p_1 \mu}{2000} \tag{25}$$

From (5) and (24):

$$p_1 = \frac{P}{\pi d_1 l_1 \mu} = \frac{2000T}{\pi\mu} = qS_1 \tag{26}$$

in which, from (8):

$$q = \frac{\left(\frac{2t_1}{d_1} + 1\right)^2 - 1}{1.3\left(\frac{2t_1}{d_1} + 1\right)^2 + 0.7} \tag{27}$$

From (26):

$$q = \frac{p_1}{S_1} = \frac{2000T}{\pi\mu S_1} = \frac{P}{\pi\mu S_1 d_1 l_1} \tag{28}$$

Then from (9):

$$t_1 = \frac{d_1}{2}\left[\sqrt{\frac{1 + 0.7q}{1 - 1.3q}} - 1\right] \tag{29}$$

These equations give the relations mentioned and are general; they will be referred to in other chapters.

Taking P as the piston load and writing $d_1 = KD$, (25) may be written:

$$T = \frac{Dp}{2550Kl_1} \tag{30}$$

Or, in tons per inch of fit diameter:

$$T_1 = Tl_1 = \frac{Dp}{2550K} \tag{31}$$

From (31) and (26):

$$S_1 = \frac{Dp}{4K\mu q l_1} \tag{32}$$

It is obvious that T_1 is not the measure of stress in the hub, but it is nearly always the quantity given in limiting pressed fits. If l_1 is constant, T_1 varies directly as the cylinder diameter, but this is usually not the case, l_1 usually changing with the cylinder diameter, although sometimes not at the same rate.

While it is proper to use a pressed fit for piston rods, it is safer not to count on their holding power, but to provide a key or nut, with a collar to take the entire load. An example will illustrate. Assume a 20 in. engine with 125 lb. steam pressure, and for the piston end, changing subscripts as in the previous paragraph, let $d_O = 4$ in. and $l_O = 7$ in. Assume a stress of 4000 lb. in the piston hub and a coefficient of friction of 0.25. Let the outside diameter of the piston hub be 8 in.

From (27), q is 0.509; from (26), $p_1 = 2036$. From (25), $T = 0.8$ and from (24), P is 44,800 lb., the ram pressure required. The maximum piston load is 39,250 lb. The ram thrust is some greater, but as this value is allowed some fluctuation, it may be assumed as about the same in this particular case. If the fit is to carry the entire load there must be a factor of safety and the increase of stress will be proportional thereto. While the piston walls and ribs add to the strength of the hub, the uneven distribution of metal and the possibility of shrinkage strains may result in cracks if the stress is assumed much greater than would be permissible in the plain hub if there were no ribs. With a factor of safety of only 2, the stress in the hub would be 8000 lb., which is excessive for cast iron. To retain the stress at 4000 lb., T being 1.6, and from (26), q being 1.018, the thick cylinder formula fails, showing that a solid piston would not permit as low a stress.

The situation is not much relieved by assuming as high a coefficient of friction as 0.4, which is unwarranted, as the surface is always lubricated. It looks as if the margin of safety must be provided by undue stress and cold riveting, when no other fastening is used. It seems more rational to allow a reasonably snug fit and not to depend upon it for axial load; then d_C (Fig. 368) may be determined from (11), in which k is zero for a straight fit. The diameter of the threaded portion may be determined from (14), taking n as zero and checking by (12a) and (13).

The rod is sometimes fastened to the piston with a straight fit and nut, and screwed into the crosshead, where it is held by a lock nut or clamped by splitting the crosshead neck. As the ends are usually safe for this design if the rod is designed as a strut, Formula (23) may be used, and if it is desired that the diameter shall be the same for all strokes, the maximum stroke may be assumed.

As an example, let a rod be designed for the 20 by 48 in. engine of the

preceding paragraph. Using the same data as far as it applies, $l = 86$ and $d = 3.82$, say $3\frac{7}{8}$ in. Calling this d_C in (11), letting k be zero and solving for d_O, the diameter of fit, gives: $d_O = 3.162$, say $3\frac{1}{8}$ in. The threaded end may be 3 in., which is greater than the value given by (13). As all the load is carried by the shoulder and nut, a driving fit, or even a snug hand fit will be ample. It may be considered advisable to make the fit larger and use a collar. If the fit is the same diameter as the rod, the diameter of the collar may be found from (11), or, $d_C = 4.47$, say $4\frac{1}{2}$ in. The thread may be $3\frac{3}{4}$ in., and may be the same at both ends of the rod.

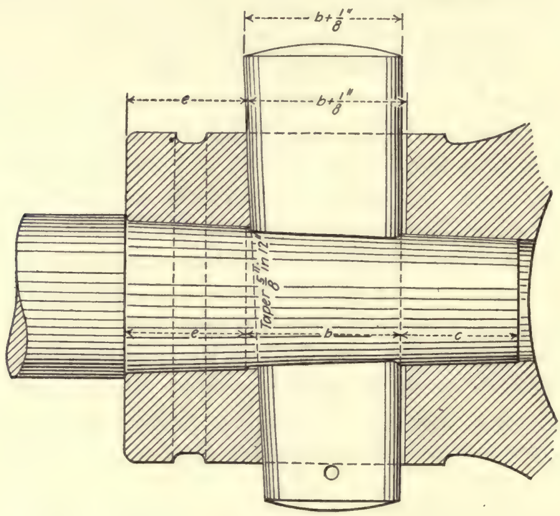


FIG. 369.

A small amount may usually be removed from the rods just designed, for the purpose of truing up when worn out of round or scored, so it may be advisable to add from $\frac{1}{16}$ to $\frac{1}{4}$ in.—depending upon the rod diameter—to the diameter of the body of the rod for this purpose, especially if the calculated diameter is not greater than given by (23). In any case, the shoulders must not be reduced by turning, to values less than those given by (6) and (11).

179. Designs from Practice.—Fig. 369 shows the crosshead end covered by Formulas (16) to (19), while Fig. 370 shows the screwed end. These have been used by the author on Corliss engines for a number of years and have been entirely satisfactory. In determining the diameter of the screwed-end rod, Formula (16) was used for the sake of uniformity; this usually gives a larger rod than is necessary for the

strength of the crosshead threaded end, and if the screwed end is to be standard construction, (22) may be used, and if it is desired to use the same diameter of rod for all strokes, it may be computed for the maximum stroke as previously stated.

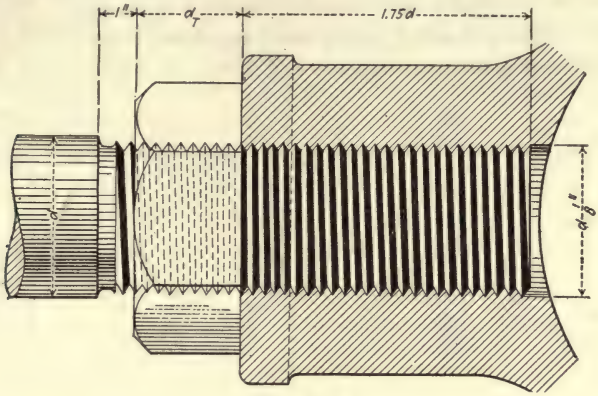


Fig. 370.

With the smaller rod it may be necessary in some cases to use the collar at the piston end, shown in Fig. 368, in order that d_3 may not be too small. This may readily be checked by (11), and the diameter of the threaded end by (21).

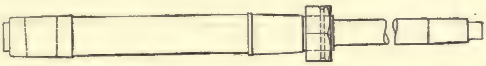


Fig. 371.—Locomotive piston rod.

Fig. 371 shows a locomotive rod used by the American Locomotive Co. A stress of 9500 lb. in tension is allowed at the least area through the key way. Considered as a static stress, this gives a factor of judgment of 2 if the elastic limit is 38,000 lb. Instead of a shoulder, this rod

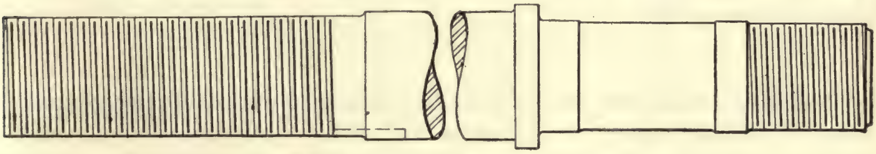


Fig. 372.—McIntosh and Seymour piston rod.

"bottoms" in the crosshead. A tail rod is provided which connects to a small crosshead whose guide is a projection from the cylinder head. The purpose of the tail rod is to relieve the pressure of the piston due to its weight, and prevent wear in the cylinder.

A rod used by the McIntosh and Seymour Corporation on their Type *F*, Gridiron-valve engine is shown in Fig. 372. The rod is screwed into the crosshead and held by a lock nut. The crosshead for the same engine is shown in Chap. XXVI, and the piston in Chap. XXIII. The fit in the latter is a straight hand fit, a clearance of 0.002 in. being allowed. This fit only extends a short distance from shoulder and nut, the intermediate portion being turned slightly smaller. A special nut is used, and

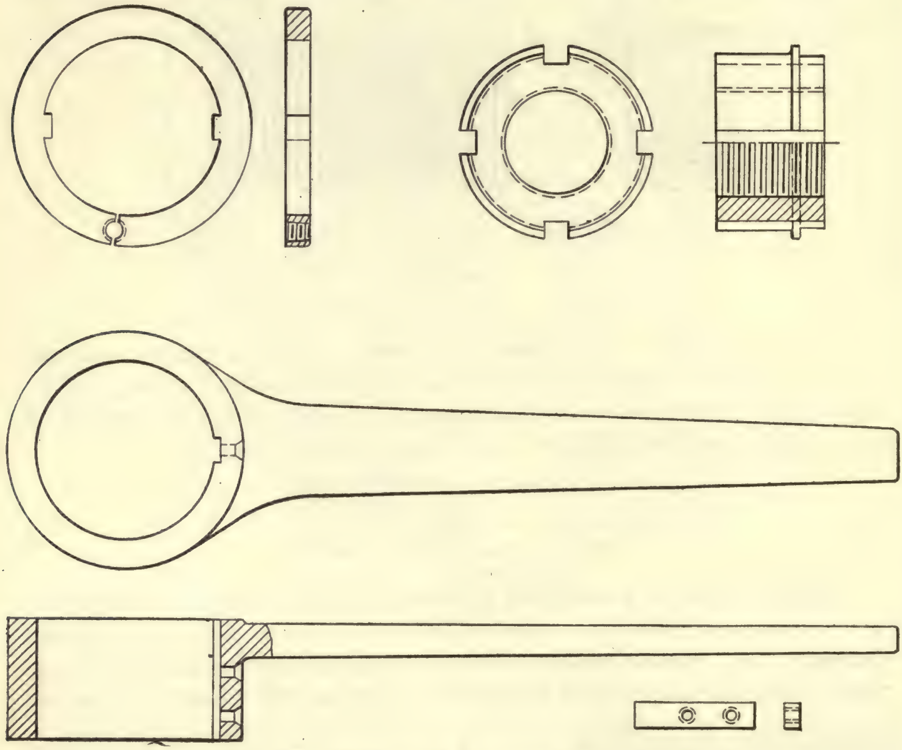


Fig. 373.—McIntosh and Seymour piston rod details.

is sunk into the piston and tightened by a special wrench. The wrench, nut and locking collar are shown in Fig. 373. The collar is split, and expanded into the counterbore of the piston by a taper set screw. The rod shown is for a 20 in. engine; its dimensions may be compared with those of the examples given, the length being practically the same. Rod and nut are steel forgings, the wrench a wrought iron forging and the locking plate cast iron.

A tandem compound rod is very similar to the rod shown in Fig. 371.

It may be designed by the foregoing methods, the main rod taking the maximum thrust of both pistons, and the tandem portion the thrust of the piston farthest from the frame. The low-pressure cylinder is commonly next the frame, but not always; by this arrangement, the main rod affects the effective piston area less. The strut length may be taken as before for the main rod, and from center to center of piston faces for the extension. If an intermediate crosshead is used, measurements will be to the center of its pin. The rod may also be screwed into the crosshead, and straight piston fits may be used.

With some large tandem steam engines, and with most large gas engines, an intermediate crosshead is used. For gas engines the rod is made hollow for water cooling, but the same general principles of design apply as already given.

CHAPTER XXV

CONNECTING RODS

Notation.

D = diameter of engine cylinder in inches.

d = diameter of round rod or depth of section in inches, in plane of vibration.

b = width of section in inches.

l = length of rod from center to center of pins in inches.

R = radius of crank circle in feet.

L = stroke of piston in inches.

A = area of rod section in square inches.

I = moment of inertia of rod section about axis perpendicular to plane of vibration.

r = radius of gyration of rod section in inches. This is the radius of gyration used in the strut formula, and does not necessarily correspond to the value of I just given (see Formulas (6) and (9)).

c = distance from neutral axis to extreme fiber ($= d/2$).

W = weight of rod in pounds.

w = weight per cubic inch of rod material.

w_1 = weight per inch of length of rod.

M = bending moment in pound-inches.

F = force in pounds, due to angular vibration, tending to bend rod.

P = total force exerted by piston in pounds.

p = unbalanced pressure per square inch in cylinder.

p_S = ultimate strength of rod as a strut.

S_E = elastic limit of rod material.

S = stress in rod due to bending.

S_R = stress resulting from bending and direct load.

a = acceleration at x normal to center line of engine.

V = tangential velocity of crank pin in feet per second.

N = revolutions per minute.

n = ratio of length of connecting rod to radius of crank circle ($= l/12R$).

q = a constant in strut formula which varies with end conditions (see Chap. XXI, Par. 164).

f_S = factor of safety for angular vibration.
f_P = factor of safety for strut load.
e = ratio of width to depth of rectangular section ($= b/d$).

180. Body of Rod.—Empirical formulas are often used to determine the dimensions of the body of the connecting rod. These formulas are suitable within a certain range of conditions under which they have proven satisfactory in practice, or have been checked by a more complete analysis. While too elaborate an analysis is neither necessary or desirable, a safe rational formula must take account of the combined effects of the strut load and the vibration, or whipping action. While methods of checking for stresses so produced are given in certain texts, the direct solution is omitted. By using Johnson's strut formula discussed in Chap. XXI, the diameter or depth of the rod may be solved for, and by numerical substitutions a reasonably convenient working formula derived.

In the derivation it is assumed that the maximum sum of the unit load at or near the center of the rod due to the piston thrust, and the compressive bending stress due to whipping, must not be greater than the safe unit load allowed by the strut formula. This may not be strictly scientific, but it seems reasonable, and checks well with more exact analyses of combined compression and bending. It is further assumed that the section is uniform throughout the length of the rod. This is no doubt always on the safe side and it greatly simplifies matters.

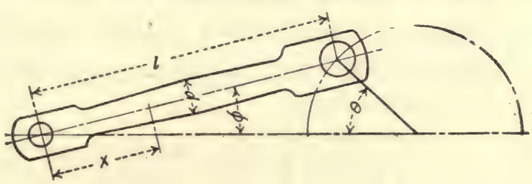


FIG. 374.

Fig. 374 is a sketch of a simple slider-crank mechanism, showing the outline of the rod and certain notation used in the discussion.

From Formula (31), Chap. XVI, the acceleration at right angles to center line of engine, of any point, is given by:

$$a = \frac{V^2}{R} \cdot \frac{x}{l} \sin \theta \tag{1}$$

The forces resulting from this acceleration will be assumed to produce a bending moment in the rod, but will be applied to the actual rod length and not to its projection upon the center line. No error of practical

significance is involved by this assumption. Then the force acting per inch of length is:

$$\frac{w_1}{g} \cdot \frac{V^2}{R} \cdot \frac{x}{l} \cdot \sin \theta.$$

As w_1, the weight per unit length is constant, the total force for a given value of θ is:

$$F = \frac{w_1}{g} \cdot \sin \theta \int_o^l a \cdot dx = \frac{w_1 l V^2}{2gR} \cdot \sin \theta = \frac{WV^2}{2gR} \cdot \sin \theta \qquad (2)$$

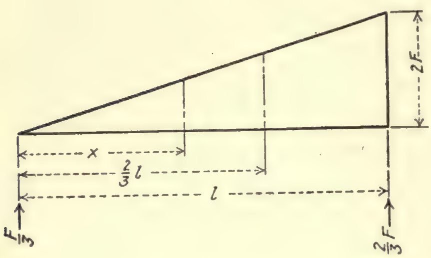


FIG. 375.

From (1) it is obvious that the load F due to inertia is distributed along the rod directly proportional to the distance from the center of the crosshead pin, as shown by the load diagram in Fig. 375. The distance of the center of gravity from the center of the crosshead pin is $2/3l$; this is the center of oscillation. The reaction on the crosshead pin is $F/3$, and on the crank pin $2F/3$. More exact values may be obtained by the method of Chap. XVI.

The general formula for bending moment is:

$$M = \text{moment of reaction} - \Sigma \text{ moment of loads.}$$

Then for the crosshead pin reaction, taking moments about any section distance x from the crosshead pin:

$$M = \frac{F}{3} \cdot x - \frac{w_1}{g} \cdot \frac{x}{3} \cdot \sin \theta \int_o^x a \cdot dx = \frac{w_1 V}{6gRl} (l^2 x - x^3) \sin \theta.$$

$$= \frac{WV}{6gRl^2} (l^2 x - x^3) \sin \theta \qquad (3)$$

When M is a maximum, $l^2 x - x^3$ is a maximum. This occurs when:

$$\frac{dM}{dx} = 0 = l^2 - 3x^2.$$

Or $x = l/\sqrt{3} = 0.577l$. Then (3) becomes:

$$M = \frac{l}{15.5} \cdot \frac{WV^2}{gR} \cdot \sin \theta \qquad (4)$$

Maximum stress due to combined strut and beam effect will occur between $0.577l$ and $0.5l$. When the rod is made largest at the center,

and tapers toward both ends, the center of oscillation and section of maximum bending moment are both moved nearer the center, so that the section of maximum combined stress is but a small distance from the center of the rod. If the rod tapers from the crosshead end to the crank-pin end, these points are moved toward the crank, but the rod section is increased in that direction. In either case, if the diameter or depth of rod section determined be considered as at the center of the rod, it is safe to assume that other sections will be ample, especially as the actual weight will be less than for a rod of uniform section in the first case, and the increasing depth of section provides for any discrepancy in the second.

From (11), Chap. XVI, the maximum thrust along the connecting rod is:

$$\frac{P}{\sqrt{1 - \left(\frac{\sin \theta}{n}\right)^2}} = mP.$$

To simplify the use of the formula, m may be taken as $1/\sqrt{1 - 1/n^2}$, its maximum value; or a somewhat larger value may be used to cover all cases.

The unit load on the rod as a strut is:

$$\frac{mP}{A} = \frac{\pi mpD^2}{4A}.$$

The unit stress due to bending is:

$$S = \frac{Mc}{I}.$$

Johnson's strut formula for ultimate unit load from (31), Chap. XXI is:

$$p_S = S_E - q\left(\frac{l}{r}\right)^2.$$

The whipping action always produces reversed stress with a factor of safety f_S; while for direct load, the stress is partly reversed with single- and double-acting engines. With internal-combustion engines the pressure factor affects the factor of safety and this is explained in Par. 166, Chap. XXI, which it is well to consult. It is then well that a separate factor of safety f_P be used for piston thrust, making it more convenient to use ultimate loads and stresses in the general equation, which, according to our assumption is:

$$p_S \overline{\gtrless} f_S S + f_P m \cdot \frac{P}{A} \tag{5}$$

From Table 82, Chap. XXI, it may be assumed that for ordinary cases the factor of safety for double-acting engines, both steam and gas, may

be 6, and for single-acting engines 3. This factor 3, however, is the product of the standard factor f_A and pressure factor f_T of Chap. XXI, and it is based upon the maximum steam or gas pressure being used to determine the load. Should determinations be made for rod dimensions at different crank angles for internal-combustion engines, using actual pressures (including inertia of reciprocating parts), the proper standard factor f_A (Chap. XXI) should be used instead of f_P, and this will depend largely upon the inertia as may be seen in Table 16, Chap. XXI. For ordinary work f_P may be taken as 3, and P as the maximum total pressure, determining the dimensions when the crank is 90 degrees from line of stroke, as with the steam engine, in which maximum thrust and maximum whipping action occur when in this position if cut-off is as late as one-half stroke, and engines are usually so designed for over-load purposes.

Substituting in (5) the values already given, we have:

$$S_E - q\left(\frac{l}{r}\right)^2 = f_S \cdot \frac{l}{15.5} \cdot \frac{WV^2c}{gRI} \cdot \sin\theta + f_P \cdot \frac{\pi mpD^2}{4A} \qquad (6)$$

For any form of section, where d is the diameter or depth of section, we may write:

$$r = \alpha d, \qquad A = \delta d^2, \qquad I = \sigma d^4, \qquad c = \frac{d}{2}.$$

Also:

$$R = \frac{L}{24}, \; V^2 = \frac{\pi^2 R^2 N^2}{900} = \frac{L^2N^2}{52{,}500}, \; l = 12nR = \frac{nL}{2}, \; W = wlA = \frac{w\delta nLd^2}{2}.$$

Substituting in (6) and solving for d gives:

$$d = K + \sqrt{K^2 + \frac{\dfrac{\pi m f_P pD}{\delta} + q\dfrac{nL}{\alpha}}{4S_E}} \qquad (7)$$

Where:

$$K = \frac{wf_S\delta n^2N^2L^3}{17{,}440{,}000 S_E\sigma} \cdot \sin\theta \qquad (8)$$

The moment of inertia I should be taken with the axis at right angles to the plane of vibration. From the strut formula, it is obvious that the resistance to failure is least when:

$$\frac{q}{r^2} = \text{maximum} \qquad (9)$$

If r is about an axis in the plane of vibration, the end conditions may be for a flat-ended strut in high-grade work, but more safely for pin ends. If r is about an axis at right angles to the plane of vibration, a round-ended strut should be taken, as the pins are in rotation and of no assist-

ance in guiding the strut; in fact, large pins, poorly lubricated, may produce a bending stress.

The length of connecting rods ranges from 4 to 6 times the crank length, the smaller values being used for gasoline engines, and the larger for horizontal stationary steam engines.

Numerical Values.—Usual practice is to make the connecting rod of an open hearth steel forging; then $S_E = 38{,}000$ and $w = 0.284$. Since the whipping action may be estimated with reasonable accuracy, the factor of judgment may be unity; then $f_S = 6$. For double-acting engines, $f_P = 6$ as already stated, and for single acting engines $f_P = 3$ for usual work. To provide for a slight eccentric load due to movement of pins in bearings, we may make $m = 1.1$, which is larger than any value of $1/\cos\phi$ found in practice. Then by substituting values of α, δ and σ, special formulas may be written for different sections.

For a circular section:

$$\alpha = \frac{1}{4}, \qquad \delta = \frac{\pi}{4}, \qquad \sigma = \frac{\pi}{64}.$$

As rupture would occur about an axis at right angles to the plane of vibration, $q = 1.4$. Then:

$$K = \frac{n^2N^2L^3}{24{,}300{,}000{,}000} \cdot \sin\theta \tag{10}$$

And:

$$d = K + \sqrt{K^2 + \frac{f_P pD^2 + 5.1n^2L^2}{34{,}500}} \tag{11}$$

Formulas (10) and (11) are also applicable to turned rods with sides planed to the width of the stub ends, when the maximum diameter is either at the center or at the crank end of the rod.

For a rectangular section let d be the depth in the plane of vibration, and b the width, and let $b = ed$. Then:

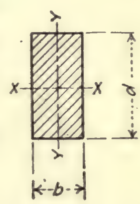


Fig. 376.

$$\delta = e \qquad \text{and} \qquad \sigma = \frac{e}{12}.$$

Assume round ends for flexure about axis xx, Fig. 376, and flat ends for axis yy. From Formulas (39) and (42), Chap. XXI, equal strength about both axes is obtained when:

$$\frac{1.4}{r_X^2} = \frac{0.63}{r_Y^2}$$

But:

$$r_X^2 = \frac{d^2}{12} \qquad \text{and} \qquad r_Y^2 = \frac{b^2}{12}$$

Then:
$$\frac{1.4}{d^2} = \frac{0.63}{b^2}$$

or:
$$e = \frac{b}{d} = 0.67.$$

Then if $e < 0.67$, the rod would fail about axis yy, and as this would usually be true:

$$\alpha = \frac{e}{\sqrt{12}} \quad \text{and} \quad q = 0.63.$$

As before, substituting in (7) and (8), special formulas for rectangular sections may be derived. Then:

$$K = \frac{n^2N^2L^3}{32{,}400{,}000{,}000} \cdot \sin \theta \tag{12}$$

And:

$$d = K + \sqrt{K^2 + \frac{f_P pD^2 + 2.18\,\dfrac{n^2L^2}{e}}{43{,}900e}} \tag{13}$$

For an I section, definite proportions must be assumed in order to derive a special formula. By trial, a section may be selected which will give a theoretical maximum of strength per lb. weight, but too light a section may have a tendency to buckle or twist, even though the usual beam and strut formulas may indicate sufficient strength. Thin flanges reduce the value of r about the y axis, which must be the axis used for this section in selecting q and r. A rather rugged section is given in Fig. 377, for which a special formula will be derived. For this section:

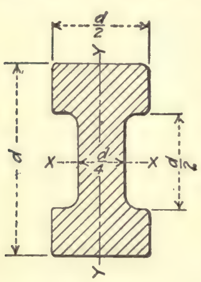


Fig. 377.

$$\delta = 0.375, \quad \sigma = 0.039, \quad \alpha = 0.125, \quad q = 0.63.$$

Then:

$$K = \frac{n^2N^2L^3}{40{,}400{,}000{,}000} \cdot \sin \theta \tag{14}$$

And:

$$d = K + \sqrt{K^2 + \frac{f_P pD^2 + 4.37n^2L^2}{16{,}500}} \tag{15}$$

Fig. 378 gives a number of connecting rod I sections used on locomotives.

Applications of Formulas (11), (13) and (15) will be made in the following examples:

(1) Find the diameter at center of a round rod for a 20 in. by 48 in. Corliss engine, running 100 r.p.m., the rod being 6 cranks long. The

maximum unbalanced steam pressure is 125 lb. per sq. in. From (10):

$$K = \frac{6^2 \times 100^2 \times 48^3}{24,300,000,000} = 1.63\cdot$$

From (11):

$$d = 1.63 + \sqrt{1.63^2 + \frac{(6 \times 125 \times 400) + (5.1 \times 36 \times 48^2)}{34,500}} = 6.5 \text{ in.}$$

Trooien's Formula, Bulletin of University of Wisconsin, No. 252, taking no account of speed gives 5 in. For 75 r.p.m., a more usual speed for the older Corliss engines of the same size, (11) gives 5⅝ in. diameter. The above shows that formulas based upon former practice, taking no account of speed, are not generally applicable. It is also apparent that for higher speeds a rod of round section is a little bulky.

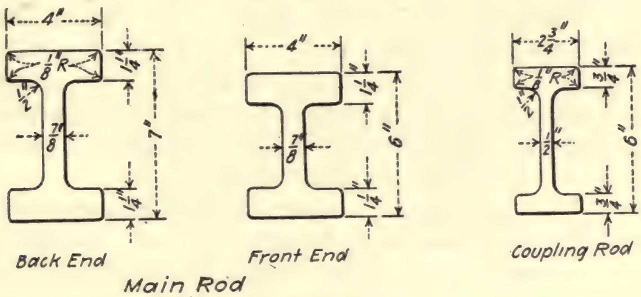


FIG. 378.

(2) Find a rectangular section for the same data, taking $e = 0.45$. From (12):

$$K = \frac{6^2 \times 100^2 \times 48^3}{32,400,000,000} = 1.23\cdot$$

From (13):

$$d = 1.23 + \sqrt{1.23^2 + \frac{(6 \times 125 \times 400) + \dfrac{2.18 \times 36 \times 48^2}{0.45}}{43,900 \times 0.45}} = 7.33 \text{ in.}$$

This gives a section 73 percent of the area of the round one, but still rather heavy.

(3) Find the depth of an I section for the same data, of the form of Fig. 377. From (14):

$$K = \frac{6^2 \times 100^2 \times 48^3}{40,400,000,000} = 0.985\cdot$$

From (15):

$$d = 0.985 + \sqrt{0.985^2 + \frac{(6 \times 125 \times 400) + (4.37 \times 36 \times 48^2)}{16,500}} = 7 \text{ in.}$$

This gives a section about 56 per cent. of the area of the round section and 77 per cent. of the rectangular section.

(4) Find the section at center of a gasoline engine rod. The engine is 3¼ by 4 in., running 1500 r.p.m. and the value of n is 4. The section is similar to Fig. 377, but with the following values:

$$\alpha = 0.224, \qquad \delta = 0.379, \qquad \sigma = 0.0295, \qquad q = 1.4.$$

The combined diagram of Fig. 187, Chap. XVI is used to determine p, and d will be calculated for several positions from (14) and (15). As actual pressures are used, the pressure factor of Par. 166, Chap. XXI is not required. According to the formulas of this paragraph, the standard factor for this case is 3.5, and this is taken as f_P in the formulas. The ratio q/r^2 of the strut formula is greater about the x axis, so is used.

Maximum pressure occurs at position 1 of the crank, which is 15 degrees from the head-end dead center. The depth of section for this position is 0.828 in.; at position 2, d is 0.739; at 3, it is 0.669. It is therefore a maximum in this case where the pressure is a maximum, the whipping action having less effect than the decrease in pressure as the piston moves away from the end of the stroke.

The actual depth of a rod for an engine of the same size and speed is 0.794 in., the calculated value being a trifle over 4 per cent. greater.

Taking the approximate method previously mentioned, with the method of Par. 166, Chap. XXI, and using the factor of safety given in Table 82 of the same chapter; then assuming the maximum whipping action with the crank 90 degrees from line of stroke, (14) and (15) give $d = 0.883$ in. This is on the side of safety and a little over 11 per cent. greater than the depth of the actual rod. In this method it was assumed that $p = 335$, the maximum gage pressure, and $f_P = 3$.

While being fairly rational and providing for the principle straining actions, Formula (7) gives results agreeing with practice.

181. Connecting Rod Ends.—Where the body of the rod joins the "stub end" there is direct repeated or reversed stress as at the center of the rod. There is also bending stress due to vibration and this may be more than is commonly supposed, although often neglected.

Formula (2) may be written:

$$F = \frac{WLN^2}{140{,}700} \cdot \sin \theta \tag{16}$$

Were the rod of uniform section, one-third of this would react approximately normal to the rod at the crosshead end, and two-thirds at the crank end. The reaction, especially at the crank end, may be greater

than this, although in part due to a heavy stub end which is partially balanced and may exert but little bending moment.

It is presumably safe to assume a uniform section unless the desire for extreme lightness necessitates greater refinement, in which case the principles of Chap. XVI must be resorted to. The weight is then:

$$W = 0.142nLA \tag{17}$$

where A is the area of section at the center of the rod. Taking Fig. 375 as a load diagram and using the notation of Fig. 379, the bending moment at dimension d_1 for the crosshead end is given by:

$$M_1 = \frac{Fl_1}{3}\left[1 - \left(\frac{l_1}{l}\right)^2\right] \tag{18}$$

At the crank end it is:

$$M_1 = \frac{2Fl_1}{3}\left[1 - \frac{l_1}{2l}\left(3 - \frac{l_1}{l}\right)\right] \tag{19}$$

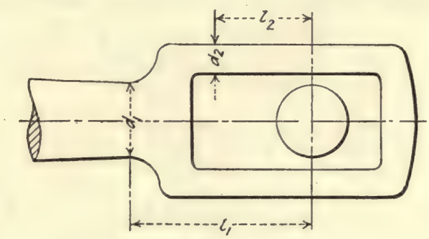


Fig. 379.

Formulas (18) and (19) assume a rod length from center to center of pins, of uniform section. While the moment may be greater than given by these formulas for a rod with the greatest depth at the crank end, the moment of the overhanging end of the stub—which is neglected—tends to offset this. The reaction on the pin is not necessarily the quantity with which to compute the bending moment on the rod.

At dimension d_2 the formulas apply by substituting l_2 for l_1. In this case the resistance is of two beams of depth d_2. Due to the possibility of more than one-half of the moment acting on one strap, the factor of safety may be increased somewhat. The stress at d_2 due to the steam or gas pressure is always a repeated stress, there being no compression beside that due to bending.

As assumed in deriving the formula for the body of the rod, the stress at failure may be taken as the sum of the product of each stress by its factor of safety, which may be different as previously explained. Then:

$$S_R = f_S S + f_P m \cdot \frac{P}{A_1} = \frac{f_S M_1 d_1}{2I_1} + \frac{f_P m P}{A_1} \tag{20}$$

where A_1 is the section area at dimension d_1. This formula also applies, at dimension d_2, and at both ends of the rod, by taking the proper subscript and values of M. For the strap (at d_2) kM must be substituted for M, and kP for P, where the minimum value of k is 0.5. As already stated, it is better to assume that more than one-half of both direct and bending load is applied to one side of the strap, and k may well be from 0.6 to 0.7.

If S_R is within the ultimate value used as a basis for applying the factor of safety—which is the elastic limit in this book for all ductile materials—the design may be considered safe.

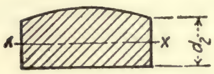


FIG. 380.

At section d_2 the form for a turned rod is shown on Fig. 380. The modulus of section about axis xx may be found graphically by the method given in Appendix 1, or the depth may safely be assumed as shown by the dotted line in the figure.

For rods of circular section at d_1, equation (20) may be written:

$$d_1^3 \times Xd_1 \times Y = 0$$

in which:

$$X = -\frac{4f_P mP}{\pi S_R}$$

and

$$Y = -\frac{32 f_S M}{\pi S_R}$$

If:

$$\frac{Y^2}{4} + \frac{X^3}{27} > 0$$

the solution of the cubic equation gives:

$$d_1 = \sqrt[3]{-\frac{Y}{2} + \sqrt{\frac{Y^2}{4} + \frac{X^3}{27}}} + \sqrt[3]{-\frac{Y}{2} - \sqrt{\frac{Y^2}{4} + \frac{X^3}{27}}} \qquad (21)$$

This may be applied to both ends of the rod, taking proper values of M, f_S and f_P, and taking $S_R = S_E$.

It is apparent from (21) that d_1 will vary with the distance l_1 in a very complicated manner. If there were no direct stress, the form of the rod might resemble, to some extent, a cubic parabola, which, if l_1 were zero, the value of d_1 measured at pin center would be zero. If there were no bending and the direct stress were tension, d_1 would be constant throughout the length of the rod. The theoretical line is some curve, and the required diameter becomes smaller more rapidly than the decrease of l_1. It is therefore safer to assume a value of l_1 large enough so that for points between this and the center of the rod, the diameter calculated from (21) will lie but little outside the conical surface between d_1 and d. It will then probably be safe to assume that approximately:

$$l_1 = \frac{l}{10}.$$

For rods with the maximum section at the crank end, calculations for d_1 are usually unnecessary at this end. With the value of l_1 just given, (18) may be written for the crosshead end:

$$M_1 = 0.033Fl = 0.016nFL \tag{22}$$

Also for the crank end (19) becomes:

$$M_1 = 0.057Fl = 0.028nFL \tag{23}$$

In determining d_2, the moment must be taken about the point giving the maximum value of S; when the construction is as in Figs. 381 and 382, this will usually be at the center of the wedge bolt, the area and modulus of section both being reduced here. For the design of Fig. 383, the maximum moment is where the strap joins the rod. The wedge bolt does not weaken the section at this point, but it is well to check for direct stress alone where the strap is cut away by the wedge bolt.

For a rectangular section, which may be assumed at d_2, (20) may be written:

$$d_2^2 - \frac{f_P mkP}{b_2 S_R} \cdot d_2 - \frac{6f_S kM_2}{b_2 S_R} = 0.$$

From which:

$$d_2 = \frac{f_P mkP}{2b_2 S_R} + \sqrt{\left(\frac{f_P mkP}{2b_2 S_R}\right)^2 + \frac{6f_S kM}{b_2 S_R}} \tag{24}$$

where k has the value mentioned in connection with (20).

Formula (24) may also be used to determine rod neck dimension d_1 for rectangular sections by using subscript 2 and making $k = 1$. It is more usual with rods of rectangular section to have straight sides tapered from crosshead to crank end. Then (24) may be used to check the ends, but (7) or (13) must be used for the depth of section at the center.

The value of d_1 may not be conveniently solved from (20) for an I section, but may be checked by it.

The wedge bolts were assumed rather arbitrarily in the formulas of Figs. 381 to 383, but as they may be unduly tightened, they were made more rugged than is sometimes done. If t is the taper in 12 in., the tension on the bolt, neglecting friction and initial tension is:

$$P_B = \frac{Pt}{12} \tag{25}$$

A single threaded bolt with nut is sometimes used, but if the brasses do not butt together so that the wedge is held firmly against the brass, the two tap bolts shown in Figs. 381 to 383 are preferable, as they lock the wedge more firmly. A more general expression for wedge bolt tension is given by (10), Chap. XXIX.

The formulas of this paragraph may be applied to the rod for the 20

by 48 in. Corliss. Figs. 381 to 383 may be considered as designs of the rod ends. From (17), as $d = 6.5$ in.:

$$W = 0.142 \times 6 \times 48 \times 33.2 = 1360 \text{ lb.}$$

From (16) and (22) the moment at d_1 for the crosshead end is:

$$M = 21{,}300.$$

From (23), at the crank end:

$$M = 37{,}500.$$

The total thrust on the piston P is 39,250 lb. Then taking the same values of f_P, f_S, m and S used for the center of the rod, $X = -8.69$ at both ends; $Y = -34.4$ at the crosshead end and -60.3 at the crank end. From (21), the neck diameter at the crosshead end is 4.12 in., and may be made 4⅛ in. At the crank end the diameter is 4.85 in. and may be made 4⅞ in.

For the dimension d_2 taken at the wedge bolt at the crosshead end, (20) gives $S_R = 28{,}150$ lb. for the dimensions given under Fig. 381, assuming the stress equally divided between the upper and lower side of the strap. At the crank end, with wedge between pin and rod as in Fig. 382, $S_R = 40{,}750$ lb. If the wedge is at the outer end, $S_R =$ 24,650 lb. These values are all well within 38,000 except the crank-end stub with wedge between pin and rod. It was stated that in (22), k must not be less than 0.5, and may be made 0.6 to 0.7. For the three values of stress just given, k is 0.675, 0.467 and 0.77 respectively. From the last two it is obvious that Fig. 383 is a better design than Fig. 382 for a round rod of the stroke and speed assumed in the example.

The wedge bolts are 1⅜ in. in diameter, the area at root of thread being 1.057 sq. in. The wedge taper is taken as 1½ in. per ft. Then from (25). the total maximum pull on the bolt, neglecting initial stress is:

$$P_B = \frac{39{,}250 \times 1.5}{12} = 4900 \text{ lb.}$$

And the stress is:

$$S = \frac{4900}{1.057} = 4630 \text{ lb.}$$

which is low, giving a factor of safety of 8.4 with an elastic limit of 38,000 lb.

182. Designs from Practice.—A number of designs which have been successful in practice will be illustrated. By assuming a standard engine as explained in Par. 63, Chap. XII, and Par. 72, Chap. XIII, the dimensions of stubs may be given in terms of pin diameter. This has been done by the author in Figs. 381 to 383, the formulas given under each figure. These designs were used for several years. The notation applies only

to the figures. The strap thickness was not determined by (22), but that it checks well with it may be seen by the example of the preceding paragraph.

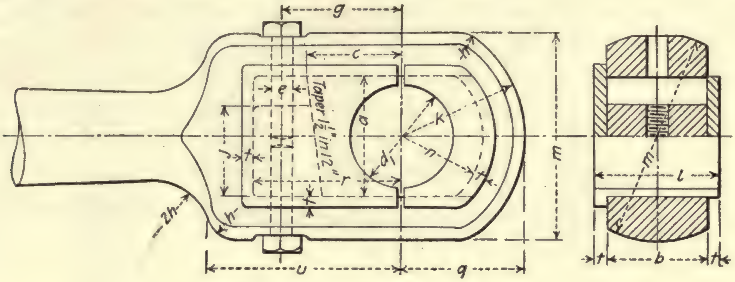


FIG. 381.—Crosshead stub.

$$a = 1.15d_1 \qquad b = 0.8l \qquad c = 0.9d_1 \qquad e = 0.15d + 0.35 \qquad g = c + e$$

$$h = 0.3d_1 \qquad j = 0.86d_1 \quad k = 1.15d_1 \quad m = d_1 + 0.8d \qquad n = 0.75d_1$$

$$q = 1.15d_1 \quad r = g + 1.1e \quad t = \frac{l - b}{2} \quad u = r + 0.4d_1$$

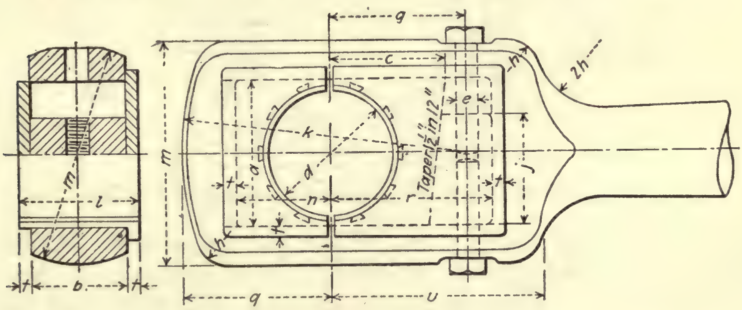


FIG. 382.—Crank pin stub.

$$a = 1.15d \qquad b = 0.8l \qquad c = 0.9d \qquad e = 0.15d + 0.35 \qquad g = c + e$$

$$h = 0.3d \qquad j = 0.86d \quad k = 2.25d \quad m = 1.8d \quad n = 0.75d \quad q = 1.15d$$

$$r = g + 1.1e \qquad t = \frac{l - b}{2} \qquad u = r + 0.4d$$

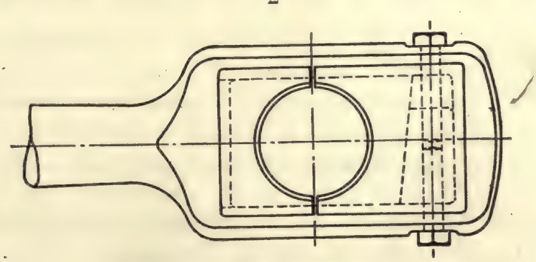


FIG. 383.—Crank pin stub.

The formulas for Fig. 382 apply in general to Fig. 383, any changes being obvious.

The stubs are for solid-end rods, much used for Corliss engines, and

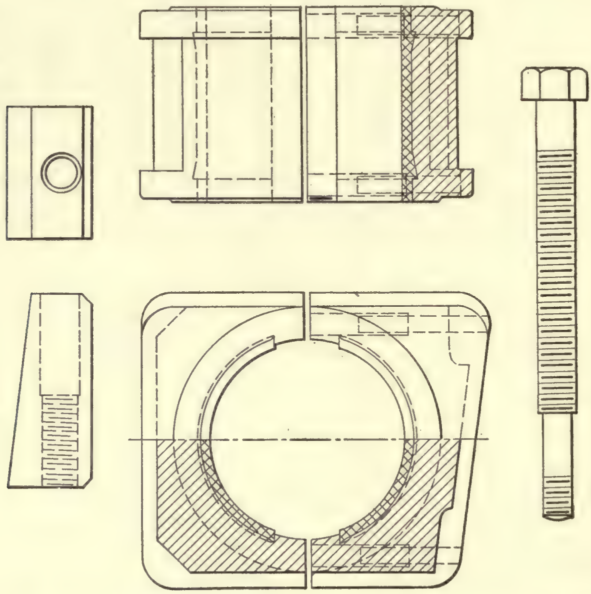


FIG. 384.—McIntosh and Seymour connecting rod details.

others of about the same class. While turned rods are shown, rectangular or I sections may be employed, in which case m is the depth and not the diameter, and may be made some less. The stubs were designed

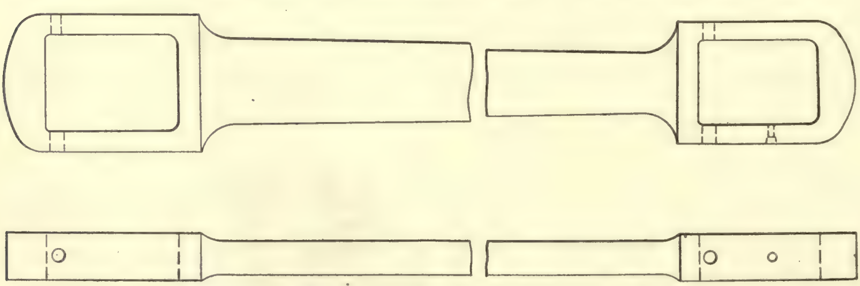


FIG. 385.—McIntosh and Seymour connecting rod.

for a maximum unbalanced steam pressure of 125 lb., and a pressure per sq. in. of projected area of 1200 lb. for the crosshead pin and 1000 lb. for the crank pin, and can therefore not be taken as generally applicable.

While Fig. 382 has been much used, Fig. 383 gives a stronger stub, especially for high speeds, as the bending arm l_2, Fig. 379, is less.

The boxes are made of brass or some of the bronzes, and it is usual to line only the crank-pin box with babbitt metal. The box flanges are sometimes omitted, the brass being flush with the rod end. The wedges and adjusting bolts are of steel and it will be noticed that they are of the same diameter in both stubs, e being in terms of the crank-pin diameter in both cases.

Oil grooves are shown in Chap. XI. The space between the two halves of the boxes is usually left open in stationary engine practice, but some engineers believe they should butt together, and when adjustment is

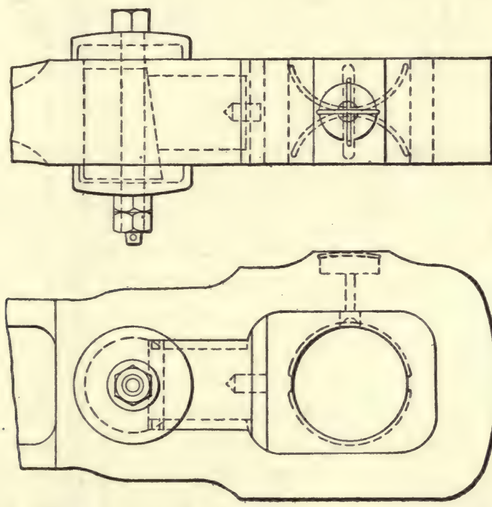


FIG. 386.—Crosshead stub for main locomotive rod.

made, they may be filed or machined; or they may be provided with "shims." In either case, the wedge is drawn tight against the box.

It is claimed that the design shown in Fig. 383 is preferable to that of Fig. 382, as adjustment is in the same direction, tending to keep the rod length from center to center constant. Both forms are standard.

Figure 384 shows details of the crank-end stub used on the McIntosh and Seymour Type *F* steam engine. The crosshead end is of the same design but smaller. The pin fits are the same length and both crosshead and crank ends are babbitted. It will be noticed that the wedge half of the box is drilled and tapped for "spreading bolts." These may be adjusted so that the wedge may be drawn up snugly against the box, making the filing of the brass or the use of shims unnecessary. The rod,

which is rectangular in section is shown in Fig. 385. The arrangement of wedges is that of the combination of Figs. 381 and 383, adjustment tending to keep the rod of constant length. There is a single through wedge bolt for each stub, of the type previously referred to.

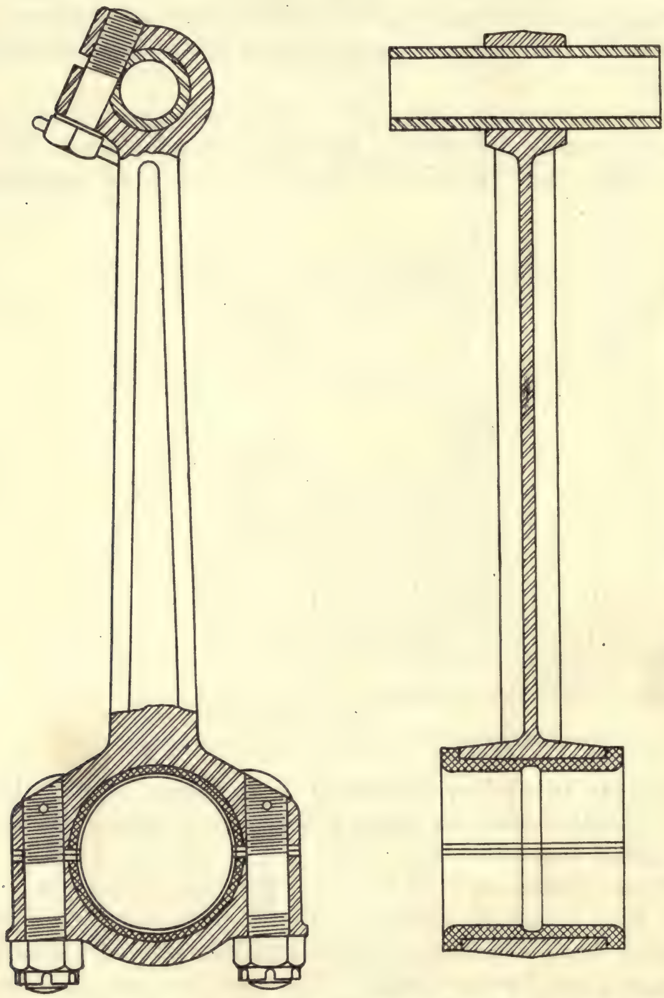


FIG. 387.—Franklin automobile engine rod.

Figure 386 shows the crosshead-end stub of the main rod of a locomotive, built by the American Locomotive Co. The brasses butt together which is usual locomotive practice. There are no flanges on the brasses of this stub. The adjusting wedge draws crosswise, pressing against a steel block, which in turn forces the brasses together.

Figure 387 shows the connecting rod used on the Franklin automobile engine. At the crosshead end the rod clamps to the pin, the wearing surfaces being in the piston hubs. This rod is a drop forging of channel section, suitable for very high speeds.

A marine rod end is shown in Fig. 387. A design of this type adapted to large engines is shown in Fig. 388. This may be modified in detail.

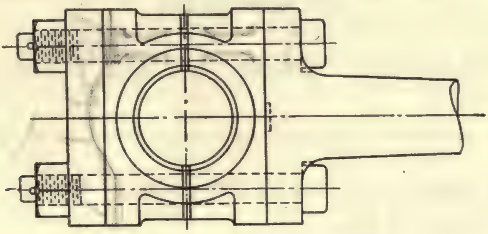


FIG. 388.—Marine-end stub.

When used for double-acting engines, one bolt must be strong enough to carry the fraction k (see Par. 181) of the maximum piston load, or the maximum value of T, Chap. XXI, Par. 166. The projection from the rod which holds the bolt must also be strong enough, considered as a cantilever, to take the same load. The load coming upon these parts may be determined by the equations of the paragraph just referred to.

In some marine steam engines having a four-bar guide, the same type of stub may be used for the crosshead-end of the rod, but usually a solid-end rod, similar to Fig. 381 or Fig. 386 is used A design used for a Diesel engine is shown in Fig. 389. The force on the rod, except that due to inertia, is all in one direction, so the adjustment does not require the strength necessary for a double-acting engine. Most Diesel engine rods are of circular section with but small increase in diameter from crosshead to crank end. The whipping action is small compared to the thrust due to gas pressure, and the circular section is a good form for strut loads.

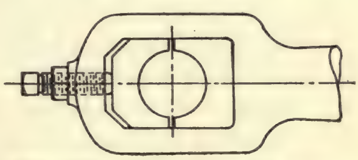


FIG. 389.—Diesel wrist-pin stub.

There have been many designs of rod ends, most of which possess merit. There is as yet no standard, but perhaps not so many different designs are used as was the case several years ago. In selecting a design, simplicity, rugged construction, ease of adjustment, and small liability of parts working loose, with cheap production, will be the criteria.

CHAPTER XXVI

CROSSHEADS

Notation.

p = maximum unbalanced pressure per square inch acting on the piston.

P = maximum nominal pressure on crank pin per square inch of projected area.

P_1 = same on crosshead pin.

P_S = same on crosshead shoe.

P_X = total maximum unbalanced pressure on piston.

P_P = total pressure on piston at any part of its stroke.

P_N = total normal pressure on guide at any part of stroke due to piston thrust only.

p_A = total normal pressure on guide due to piston thrust and inertia forces, as given in Formula (53), Chap. XVI.

P_R = total resultant pressure on crosshead pin due to piston thrust and inertia of moving parts.

S = bending stress in crosshead pin in pounds per square inch.

D = diameter of cylinder in inches.

d = diameter of crosshead pin (in bearing) in inches.

l = length of crosshead pin bearing in inches.

A = projected area of crosshead pin bearing in square inches = dl.

L = length of crosshead shoe in inches.

w = width of crosshead shoe.

A_S = area of crosshead shoe = wL.

M = bending moment on crosshead pin.

n = ratio of length of connecting rod to length of crank.

f = factor of safety.

f_3 = factor of judgment (see Chap. XXI).

183. The crosshead is virtually a knuckle joint between the piston rod and connecting rod. In order to take the thrust due to the angularity of the connecting rod, shoes are provided which slide on the guide. Usually these shoes are made adjustable to provide for wear between crosshead and guide or between piston and cylinder; however, some builders, feeling that the adjustment is liable to be tampered with and claiming

that the bearing pressure is such as to make the wear negligible, omit the adjustment.

It does not seem practicable to make locomotive crossheads adjustable; when it is necessary to take up wear, new gibs are furnished or shims are used.

Material.—Cast iron has been much used for crossheads, but many are now made of steel castings. The shoes may be of cast iron even when the body is of steel. The shoes are sometimes faced with brass, but usually with babbitt metal. The pin is commonly an open hearth steel forging and fitted to the crosshead with a taper fit. For other steels see Par. 159, Chap. XXI.

Strength.—There are comparatively few strength computations for the cross head. The longitudinal stress is a reversed stress for double-acting engines (see Chap. XXI, Par. 166). The factor of judgment should provide for the possibility of water in the cylinder for steam engines, being from 1.25 to 1.5. The area through the weakest section should be such that the stress will not exceed the safe working stress. For cast iron this may be from 900 to 1200 lb. per sq. in., and for a steel casting, from 3000 to 4000 lb. Semi-steel is sometimes used; this is generally stronger than cast iron, but some builders allow the same working stress as for good cast iron.

184. Crosshead Pin.—In proportioning the wearing portion of the crosshead pin, some ratio of length to diameter must be assumed. If in a steam engine with over-hung crank, the length of the pin is equal to the diameter, it is convenient to make the length of the crosshead pin the same as that of the crank pin, and its diameter inversely proportional to the pressure per sq. in. of projected area allowed on the pins. Then if P_1 is the pressure per sq. in. on the crosshead pin, and P the pressure on the crank pin; if l and d are length and diameter respectively of the crosshead pin, the projected area is:

$$A = dl = \frac{P}{P_1} l^2 \tag{1}$$

If p is the maximum unbalanced pressure per sq. in. in the cylinder and D is the cylinder diameter in inches, equating the maximum piston thrust with the total pin pressure gives:

$$\frac{\pi p D^2}{4} = PA = Pl^2 \tag{2}$$

or:

$$l = D\sqrt{\frac{\pi}{4}\frac{p}{P}} = 0.887D\sqrt{\frac{p}{P}} \tag{3}$$

Then:

$$d = \frac{P}{P_1} l \tag{4}$$

These equations are convenient for steam engines for the conditions assumed. For center-crank engines the equations may be used by assuming tentatively a side-crank engine and designing the crosshead pin; then the crank pin may be given a separate treatment.

Bearing design is discussed in Par. 52, Chap. XI, and bearing pressures for different cases given in Table 19 of the same chapter.

If the crosshead or piston were assumed perfectly rigid, the crosshead pin would be a beam fixed at both ends and loaded with a load somewhere between a uniform and a concentrated load. It is probably safer, especially with trunk pistons, to assume the pin as supported at the center of each hub. The load may safely be taken as concentrated at the center of the pin; but as this is extreme, it is sometimes necessary, in order to keep the pin diameter from being excessive, to make different assumptions—that a certain fraction of the length of the pin carries the entire load uniformly distributed. Calling this fraction q, and P_X the total maximum pressure, a general expression may be found. From the general equation for bending moment, and referring to Fig. 390, the bending moment is:

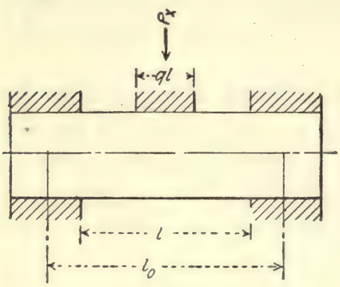


Fig. 390.

$$M = \frac{P_X}{2}\cdot\frac{l_0}{2} - \frac{P_X}{2}\cdot\frac{ql}{2} = \frac{P_X}{4}(l_0 - ql)$$

Equating this with the modulus of section gives:

$$0.098Sd^3 = \frac{P_X}{4}(l_0 - ql) \tag{5}$$

As:

$$P_X = \frac{\pi p D^2}{4}$$

Equation (5) may be written:

$$S = \frac{2pD^2(l_0 - ql)}{d^3} \tag{6}$$

or if S is assumed:

$$d = \sqrt[3]{\frac{2pD^2(l_0 - ql)}{S}} \tag{7}$$

The length l_0 may be taken to the center of the pin hubs, or it may be varied according to judgment. For crossheads, the author has often made the hub length one-half the pin length (in the bearing); taking l_0 to the center then makes $l_0 = 1.5l$. The value of q may be taken as

0.5. If the pin is first proportioned for bearing surface it must be checked for stress, or *vice versa*.

For steam engines with cut-off as great as one-half stroke, p is at least equal to maximum unbalanced steam pressure whether inertia is taken into account or not, as near mid-stroke inertia is zero. With internal-combustion engines, the maximum gas pressure acts only near dead center. The maximum pressure including inertia may not be near dead center if the inertia is great, but this should be found from a diagram. If inertia is considered in design, it should not include the connecting rod for calculations on the crosshead pin and it is well to check for pin stress with the maximum gas pressure alone, as this condition obtains at starting.

More correctly the load on the pin is the resultant of P_X, F_P and p_A, the last two being given by Formulas (53) and (32), Chap. XVI. The effect of p_A is small (See also Fig. 204 and Table 56, Chap. XVI).

See Par. 166, Chap. XXI for factor of safety. A factor of judgment of from 1.25 to 1.75 should be employed when neglecting inertia, etc.

185. Crosshead Shoe.—It is customary to limit the maximum pressure per sq. in. on the crosshead shoe. Values of this limiting pressure are given in Table 19, Chap. XI. The pressure on the guide P_N for any crank angle is given by (12), Chap. XVI, and is:

$$P_N = \frac{P_P \sin \theta}{n\sqrt{1 - \left(\frac{\sin \theta}{n}\right)^2}} \tag{8}$$

where θ is the angle the crank makes with the line of stroke, measured from the head-end dead center; n is the ratio of connecting rod to crank, and P_P is the total piston pressure at any point in the stroke.

For steam engines with a cut-off as long as one-half stroke, $P_P = P_X$ and $\sin \theta = 1$; this gives the maximum guide pressure. Then (8) becomes:

$$P_N = \frac{P_X}{n\sqrt{1 - \frac{1}{n^2}}} \tag{9}$$

If $n = 4$, and the quantity in the radical is ignored, the error will be about 3 per cent. With larger values of n the error is less.

For internal-combustion engines the maximum value of P_N is not so apparent, as P_P drops rapidly during the stroke, and when $\sin \theta$ is maximum, P_P is much less. A few trials will locate the angle at which $P_P \sin \theta$ is maximum.

More correctly, the value p_A given by Formula (53), Chap. XVI gives

the pressure on the guide, but as P_N is apt to give a greater value it is usually safe. For horizontal engines the weight of the crosshead and a portion of the connecting rod and piston rod should be added to p_A. This may result in a value greater than P_N but in view of the arbitrary values for allowable pressure P_S, varying greatly as given by different authorities, too great refinement seems out of place. P_N, P_X and p should properly include the inertia of the reciprocating parts (not including the connecting rod), but this is often neglected in strength calculations (see Chap. XVI, and Chap. XXI, Par. 166).

As a suggestion in proportioning crosshead shoes for steam engines, the author has used the following empirical formula for the length of shoes for a number of years in Corliss engine design:

$$L = 0.75D + 5 \tag{10}$$

This is in inches. The width w was taken one-half of the length. The area would be $1.5L$, and the bearing pressure per sq. in. on any shoe is:

$$P_S = \frac{P_N}{A_S} = \frac{P_N}{wL} \tag{11}$$

This also applies to trunk pistons which serve as crossheads; w is then replaced by D.

186. Application of Formulas.—In designing the pin for a steam engine, the bearing pressure is usually assumed. Let a pin be designed for the standard 20-in. Corliss engine of Chap. XII, Par. 64. The steam pressure p is 125 lb. Let $P_1 = 1200$ and $P = 1000$ (for the crank pin). Then from (3):

$$l = 0.887 \times 20\sqrt{\frac{125}{1000}} = 6.25 \text{ in.}$$

The nearest safe pin in Table 89 is 6½ in. long, and the diameter is 5½ in. and this will be used. Assume $b = l$ in Table 89 (Par. 187). Taking $l_0 = 1.51$ and $q = ½$, (6) gives:

$$S = \frac{2 \times 125 \times 400 \times 6.5}{5.5^3} = 3920.$$

Neglecting the angularity of the connecting rod, a ¾ cut-off occurs when the crank is at point 8, Table 57, Chap. XVI. The value of F_P (inertia of piston, piston rod and crosshead) at this point is 6450 lb. Added to 39,250 (being P_X) gives 45,700 lb. The value of p_A from the same table is 1306 lb. The resultant is:

$$P_R = \sqrt{45{,}700^2 + 1306^2} = 46{,}200 \text{ lb.}$$

Then:

$$P_1 = \frac{46{,}200}{5.5 \times 6.5} = 1290 \text{ lb.}$$

This is but slightly larger than 1200, as the pin diameter was increased to 5½ in., but had l been taken so that P_1 was exactly 1200 lb. for steam pressure only, the ratio of actual to assumed pressure on the pin would be:

$$\frac{P_R}{P_X} = \frac{46{,}200}{39{,}250} = 1.18$$

The actual stress is:

$$S = 3920 \frac{P_R}{P_X} = \frac{3920 \times 46{,}200}{39{,}250} = 4620 \text{ lb.}$$

The standard factor of safety for reversed stress in Par. 166, Chap. XXI is 6; taking an elastic limit of 38,000 as given in Table 73, Chap. XXI, the factor of judgment is:

$$f_3 = \frac{38{,}000}{6 \times 4620} = 1.37$$

This shows that the pin is safe.

For a Diesel engine assume the pin length to be $0.55D$. Assume a 10 in. cylinder and a pressure of 500 lb. Then $l_o = 8.25$. The factor of safety from Table 82, Chap. XXI is 3. If the factor of judgment is 1.25, total factor of safety is 3.75; calling this 3.8 gives an allowable working stress of 10,000 lb. Taking $q = 0.5$ as before, (7) gives:

$$d = \sqrt[3]{\frac{2 \times 500 \times 100 \times 5.5}{10{,}000}} = 3.81; \text{ say } 3\tfrac{7}{8} \text{ in.}$$

The bearing pressure per sq. in. of projected area is:

$$P = \frac{500 \times 78.54}{5.5 \times 3.875} = 1845 \text{ lb.}$$

which is practically the value allowed in Table 19, Chap. XI. Inertia would reduce this some after the engine is started.

In detailing the piston and connecting rod end it might be found that slightly different proportions would give better results, but the size of pin found, 3⅞ by 5½ in., seems reasonable.

Designing the crosshead shoe for the 20 in. Corliss engine, with a rod 6 cranks long gives, from (9).

$$P_N = \frac{0.7854 \times 20^2 \times 125}{6\sqrt{1 - \frac{1}{36}}} = 6630.$$

From (10):

$$L = 15 + 5 = 20.$$

From (11):

$$P_S = \frac{6630}{20 \times 10} = 33.15 \text{ lb.}$$

The maximum value of p_A from Table 57, Chap. XVI is 5432 lb., being less than P_N. Adding the weight of the crosshead and part of the connecting rod and piston rod brings this up to 7000 lb. This gives:

$$P_S = \frac{7000}{200} = 35 \text{ lb.}$$

The difference is negligible. The result is still less than the allowable pressure given in Table 19, Chap. XI.

In the internal-combustion engine the maximum guide pressure occurs earlier in the stroke. In the automobile engine, the data for which is given in Chap. XVI, the maximum pressure from (8) occurs at crank position 3, and P_P is 31.8 lb. per sq. in. including the effect of piston inertia. The cylinder being $3\frac{1}{4}$ in. in diameter, the total maximum guide pressure

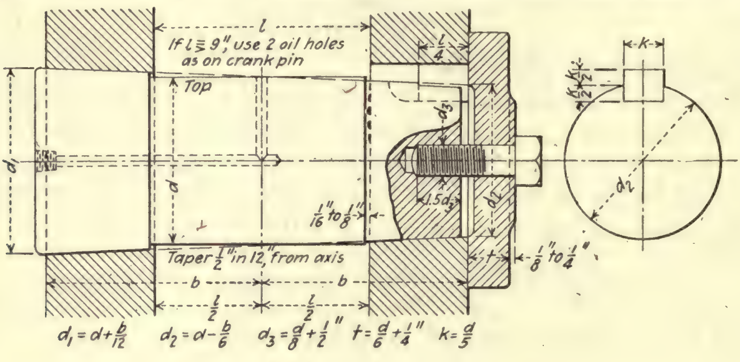


FIG. 391.

is 264 lb. The piston is 4 in. long, making a projected area of 13 sq. in. This gives a pressure of 20.3 lb. per sq. in. The rings extend about 1 inch from the head end. If this portion is deducted the unit pressure is 27 lb. The actual unit pressure is somewhere between these two values. In Table 19, Chap. XI, the allowable pressure for a trunk piston in an internal combustion engine is 21 lb. As economy of space and extreme lightness are requirements of an automobile engine, higher pressures are more likely to be used than in engines for industrial service.

187. Designs From Practice.—Fig. 391 shows a crosshead pin used by the author on Corliss engine work, and Table 89 gives data for pins from 3 to $8\frac{1}{2}$ in. in diameter. The notation applies only to the table. These pins were designed from Formulas (3) and (4), taking $p = 125, P = 1000$ and $P_1 = 1200$. It was also assumed that $b = l$. Crosshead pins for standard steam engines for the pressures just named are included with crank pins in Table 91, Chap. XXVII.

Figure 392 is one form of crosshead used on the Bass Corliss engine, built by the Bass Foundry and Machine Co., Fort Wayne, Ind. The shoe adjustment is such that the adjusting nut draws in the direction the

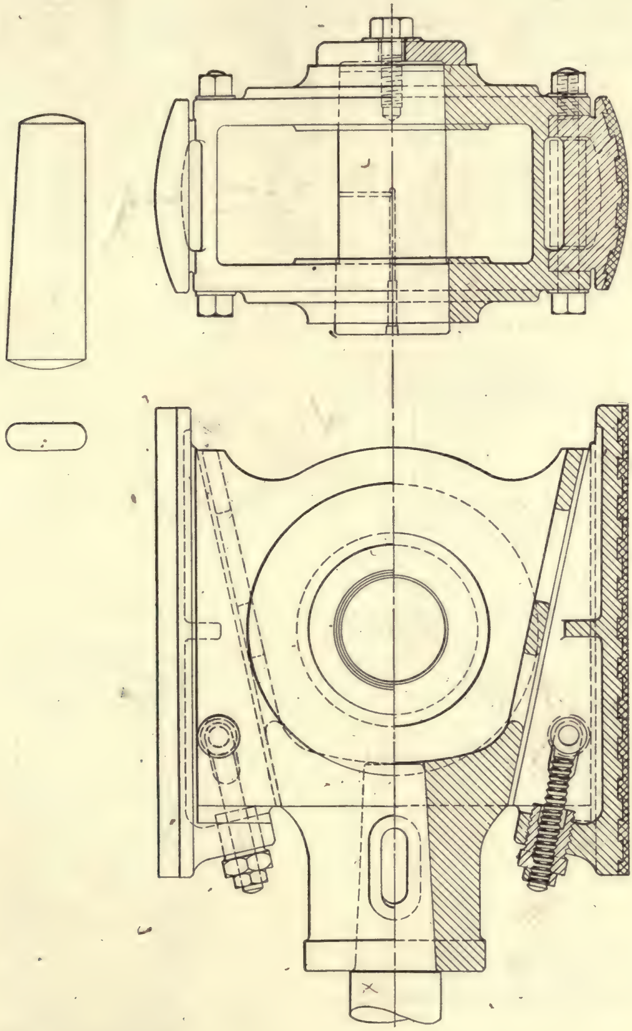


FIG. 392.—Bass-Corliss crosshead.

shoe slides, preventing a bending strain on the bolt. It is a turned crosshead, sliding in bored guides. The body of the crosshead is a steel casting and the shoes cast iron, babbitted.

Figure 393 is the McIntosh and Seymour crosshead, used on Type *F*

TABLE 89

d, in.	l, in.	If $b = l$		d_3, in.	t, in.	k, in.
		d_1, in.	d_2, in.			
3	3½	3³⁄₁₆	2.60	⅞	⅞	⅝
3½	4	4	3.33	1	1	¾
4	4½	4⅜	3⅝	1	1	⅞
4½	5½	4¹⁵⁄₁₆	4.02	1⅛	1¼	⅞
5	6	5½	4½	1⅛	1¼	1
5½	6½	6	4.91	1¼	1½	1⅛
6	7½	6⅝	5⅜	1¼	1½	1¼
6½	8	7⅛	5.79	1⅜	1⅝	1⅜
7	8½	7¹¹⁄₁₆	6.27	1⅜	1¾	1½
7½	9	8¼	6¾	1½	1¾	1½
8½	10	9⁵⁄₁₆	7.64	1⅝	2	1¾

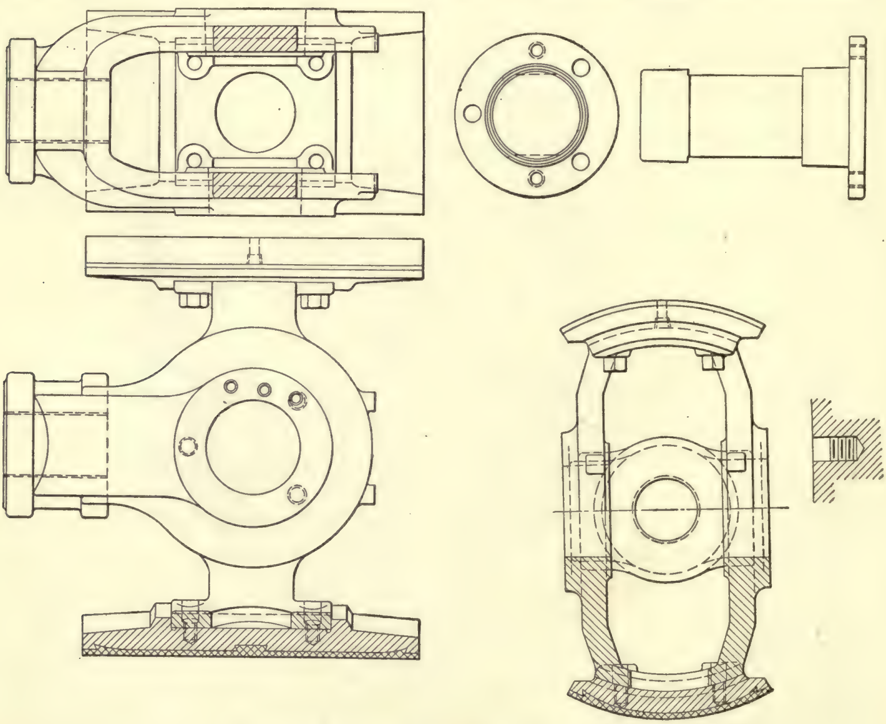


FIG. 393.—McIntosh and Seymour crosshead.

steam engine built by the McIntosh and Seymour Corporation, Auburn, N. Y. The pin is flattened top and bottom, and the brasses of the connect-

ing rod boxes are cut away so that there is a small wipe-over in the position of extreme angularity of the rod. The pin is held in place by three tap bolts. When it is desired to withdraw the pin, these bolts are removed and the pin forced out by two set screws in the pin flange.

The body of the crosshead is a steel casting and the shoes are cast iron faced with babbitt metal. The shoes have no adjusting wedges, but shims or liners may be used in case of wear.

CHAPTER XXVII

CRANKS

188. Introduction.—In this chapter it is intended to cover the over-hung crank. This is usually a casting of iron or steel forced on the shaft, and usually the crank pin is forced in. When pin and shaft diameters are large relative to the stroke, a steel casting with crank and pin cast integral is sometimes used; this is then forced onto the shaft.

The problem is to obtain proper bearing surface for the pin, ample strength and stiffness of pin and crank, and the securing of the pin and shaft to the crank.

The center crank and multi-cylinder crank will be treated under shafts in Chap. XXVIII.

In general, the over-hung crank is used only for steam engines and large double-acting gas engines. The factor of safety ($f_A f_T$) in Table 82 of Chap. XXI for these may be taken as 5 and 4.2 respectively, for ductile materials. Going on the assumption of Par. 159, Chap. XXI, that for reversed stress the compressive stress of cast iron may be taken as one-fifth of its actual value, f_A, the standard factor of Chap. XXI may be taken as 7.2 for both steam and gas engines of this type. The product $f_A f_T$ will then be 7 for the steam engine and 5 for the gas engine. A factor of judgment may be used in addition to these if desired. The value 7 was taken for steam as the value f_A for ductile materials in Table 82, Chap. XXI is 5—not a completely reversed factor.

Inertia and variation of pressure is provided for by these factors, as explained in Par. 166, Chap. XXI, so in applying them, the force acting is assumed to be that produced by maximum unbalanced steam or gas pressure only.

Notation.

d = diameter of crank pin (in bearing) in inches.

d_1 = diameter of crank pin fit.

d_H = outside diameter of pin or shaft hub, in inches.

l = length of crank pin bearing in inches.

l_o = length of moment arm for computing strength of pin or crank arm, in inches.

D = diameter of cylinder in inches.
D_S = diameter of standard cylinder when some standard pressure is assumed, as explained in Par. 63, Chap. XII.
t = thickness of hub considered as a thick cylinder.
b = effective breadth of crank arm in inches, as shown on Fig. 396.
h = depth of arm section in inches.
A = area of arm section in square inches.
I = moment of inertia of arm section.
c = distance from neutral axis to extreme fiber, in inches.
S = stress in pounds per square inch.
S_B = bending stress in arm.
S_D = direct stress in arm.
S_C = crushing stress in key.
P_X = total maximum unbalanced pressure on piston in pounds, due to steam or gas pressure only.
P_T = turning effort at crank pin (tangential) in pounds.
P_P = total unbalanced pressure at any part of stroke in pounds, including inertia.
P = maximum allowable pressure on pin per square inch of projected area.
P_M = mean pressure per square inch of piston area per cycle, including inertia of reciprocating parts and connecting rod. This must be taken from a complete stroke diagram for the cycle. P_M may be found from diagrams of Par. 105, Chap. XVI, both in magnitude and direction (in line of stroke only).
p = maximum unbalanced pressure per sq. in. in cylinder.
T = tons per in. of diameter per in. of length required to complete pressed fit.
N = r.p.m. of engine shaft.
k = l/d.
C = a constant in wear formula.
μ = coefficient of friction.

189. Crank Pin.—The over-hung crank pin may be designed independent of the shaft as it transmits no power from one part of the shaft to another. Fig. 394 shows two methods of pin design. In Fig. 394*A* the pin is reduced from $\frac{1}{8}$ to $\frac{1}{4}$ in. in diameter—depending upon the size—where it enters the crank. In Fig. 394*B* a collar is turned on the pin and the portion in the crank may be equal or greater in diameter than the wearing part. In this pin the moment arm l_O is greater than one-half the bearing length l. Should the diameter d be much less than the

diameter of fit in the crank, it should be checked for strength by using a moment arm $l/2$.

A pin may be designed for strength and checked for bearing pressure and wear; or it may be designed for bearing pressure and checked for wear and strength. The latter method will be used here.

As the method of assuming allowable bearing pressures is usually based upon direct piston thrust, the additional load on the pin due to angularity of the connecting rod may be neglected, greatly simplifying calculations. Let P_X be the total maximum unbalanced pressure on the piston, P the pressure per sq. in. of projected area of the pin, and A the projected area of the pin journal in sq. in. The general equation is then:

$$P_X = PA \tag{1}$$

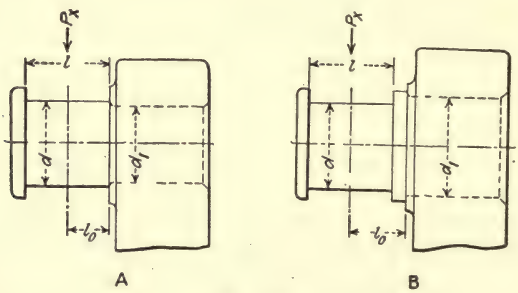


FIG. 394.

If p is the maximum unbalanced pressure per sq. in. on the piston:

$$P_X = \frac{\pi D^2 p}{4}$$

also

$$A = dl:$$

Substituting in (1) gives:

$$dl = \frac{\pi D^2 p}{4P} \tag{2}$$

Let $l = kd$, then:

$$d = \frac{D}{2}\sqrt{\frac{\pi p}{kP}} = 0.887D\sqrt{\frac{p}{kP}} \tag{3}$$

The value of P may be selected from Table 19, Chap. XI, and k may be assumed, or it may be calculated from (4), Chap. XI; this formula, however, must be used with judgment, and it is better used as a check after selecting some value of k found to be satisfactory in practice.

The pin may now be checked for strength. The modulus of section of a circular section in bending is:

$$\frac{\pi d_1^3}{32}.$$

Then if S is the stress:

$$\frac{\pi d_1^3 S}{32} = P_X l_o.$$

From which, combining with (1):

$$S = \frac{32 P_X l_o}{\pi d_1^3} = \frac{10.2 k P d^2 l_o}{d_1^3} \tag{4}$$

It may safely be assumed that d_1 and d are equal; then the value found may be taken as the smaller of the two, and (4) may be written:

$$S = \frac{10.2 k l_o P}{d} \tag{5}$$

It is usually assumed that the load P_X acts at the center of the pin; this would be theoretically correct if it were taken either as a concentrated or distributed load. For Fig. 394*A*, l_o would equal $l/2$; for Fig. 394*B*, the thickness of the collar must be added to this.

Except in locomotive practice, Fig. 394*A* is the most usual design; then $l_o = l/2$, and a special formula for this becomes:

$$S = \frac{5.1\ klP}{d} = 5.1\ k^2 P \tag{6}$$

It is good practice, and now quite common, to have k equal to unity; then (6) becomes:

$$S = 5.1\ P \tag{7}$$

This makes a good rigid pin with small deflection.

Perhaps the most direct method of determining the pin dimensions is as follows. Formula (6) may be written:

$$k = \sqrt{\frac{S}{5.1\ P}} \tag{8}$$

The maximum pressure P may be taken from Table 19, Chap. XI (p_M in this table), and S may be determined by using the factor of safety given in the introduction. Assuming the elastic limit as 38,000, these data, with the resulting values of k are given in Table 90 for two values of the factor of judgment f_3 (see Chap. XXI, Par. 166).

Selecting k with the corresponding value of P from Table 90, (3) will give the diameter; the length is obviously equal to kd. These dimensions are usually rounded up to eighths, quarters or even halves of an inch in the larger sizes, and if desired may be checked for new values of S and P, but this is seldom necessary.

Steam Engine.—Assuming k as unity, Table 91 has been computed for steam engine crank pins of the style shown in Fig. 394*A*. It is assumed that the maximum unbalanced pressure p is 125, and that P is 1000 lb.

TABLE 90

f_3	Steam engines				Internal-combustion engines			
	f.	S	P	κ	f	S	P	κ
1.00	5.00	7600	1000	1.22	4.20	9050	1400	1.13
			1200	1.12			1700	1.02
1.25	6.25	6100	1000	1.10	5.25	7230	1400	1.01
			1200	1.00			1700	0.92

TABLE 91

D_S, in.	Crosshead pin		Crank pin	
	d, in.	l, in.	d, in.	l, in.
10	3	3½	3½	3½
12	3½	4	4	4
14	4	4½	4½	4½
16	4½	5½	5½	5½
18	5	6	6	6
20	5½	6½	6½	6½
22	6	7½	7½	7½
24	6½	8	8	8
26	7	8½	8½	8½
28	7½	9	9	9
30	8½	10	10	10

The pressure p $(=125)$ may be considered the standard pressure acting in standard cylinders as explained in Par. 63, Chap. XII.

From (3), $d = 0.313D_S$. The dimensions of the crosshead pin are also given, based on a pressure of 1200 lb. per sq. in. of projected area. The length is taken equal to that of the pin, as explained in Chap. XXVI; the diameter is then ⅚ the diameter of the crank pin.

Figure 395 and Table 92 are for standard crank pins used by the author for Corliss engines. The notation only applies to the figure and table should it disagree with that used elsewhere in this chapter.

While these tables were used for several years on successful engines, they are given mainly to illustrate convenient methods of arranging design data. The selection of dimensions for other steam pressures is explained in Par. 63, Chap. XII.

TABLE 92

d, in.	d_1, in.	b, in.	d_2, in.	l_1, in.	c, in.	d_3, in.	Oil holes
3½	4⅝	⅝	1	1½	1¼	½	1–¼″
4	5¼	¾	1	1½	1½	½	1–¼″
4½	5⅞	¾	1⅛	1¾	1⅝	⅝	1–¼″
5½	7¼	⅞	1¼	1⅞	2	⅝	1–¼″
6	7⅞	1	1¼	1⅞	2¼	⅝	1–⅜″
6½	8½	1⅛	1⅜	2⅛	2⅜	¾	1–⅜″
7½	9¾	1¼	1½	2¼	2¾	¾	1–⅜″
8	10½	1⅜	1½	2¼	3	¾	1–½″
8½	11	1⅜	1⅝	2½	3⅛	⅞	1–½″
9	11¾	1½	1⅝	2½	3⅜	⅞	2–⅜″
10	13	1⅝	1¾	2¾	3¾	⅞	2–⅜″

The method of finding the mean pressure per sq. in. of piston area, P_M acting on the pin is described in Par. 104, Chap. XVI. For steam

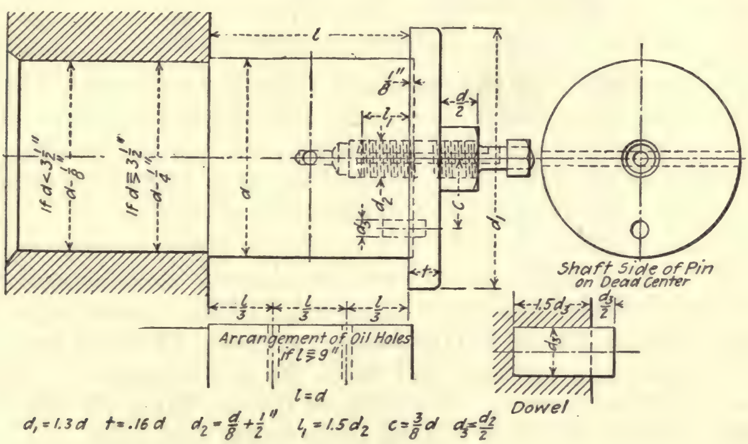


FIG. 395.

engines this is practically the m.e.p. For engines with long-range cut-off it approximates the maximum pressure in the cylinder and may be safely taken as such.

Adopting the notation of this chapter, Formula (1), Chap. XI may be written:

$$\frac{\pi D^2 P}{4kd^2} \cdot \frac{\pi d N}{12} = C \tag{9}$$

The value of C is taken from Table 93 of the same chapter. Taking this value and solving for N, the corresponding rotative speed may be found.

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With $p = P_M = 125$, $P = 1000$ and $k = 1$; the values used for Table 91 substituted in (3) and the value of d so found substituted in (9), we have:

$$N = \frac{4.3\ C}{P_M D}\sqrt{\frac{kp}{P}} = \frac{2430}{D_S} \tag{10}$$

TABLE 93

D_1, in.	N	D_1, in.	N	D_1, in.	N
2	1220	12	202	32	76
3	810	14	174	34	71
4	610	16	152	36	67
5	487	18	135	38	64
6	405	20	122	40	61
7	347	22	110	42	58
8	304	24	101	44	55
9	270	26	93	46	53
10	243	28	87	48	50
11	221	30	81	50	48

It is interesting to note these results in Table 93, which compare favorably with conservative speeds often used. Higher speeds than those given in the table have been used for large engines, but it is stated in Par. 52, Chap. XI, that the value of C, which was taken as 200,000, may be doubled with excellent design and conditions of operation.

Internal-combustion Engine.—For these, the mean pressure on the pin must be used for P_M in (10), and this depends upon the indicator and inertia diagrams as already explained. Güldner assumes for his standard diagram 128 lb. for the expansion stroke, 18 lb. for each idle stroke (exhaust and suction) and 32 lb. for the compression stroke. The mean of these is 49 lb. The value of C from Table 19, Chap. XI is 90,000. Taking $k = 1$, $P = 1400$ and $p = 400$, (10) becomes:

$$N = \frac{4220}{D_S} \tag{11}$$

Then from (3) for the same assumptions:

$$d = 0.473 D_S \tag{12}$$

Güldner takes S as 12,000. For the factor of safety already used S_e must be 50,000 if $f_3 = 1$, or 63,000 if $f_3 = 1.25$. With other data as before, this gives, from (8):

$$k = \sqrt{\frac{12{,}000}{5.1 \times 1400}} = 1.3 \text{ nearly.}$$

and from (3):

$$d = 0.415D_S \tag{13}$$

This increases the allowable value of N by 14 per cent.

From (6), the value of S when d is as given by (12) is 7150—practically the same as in Table 90 when f_3 is 1.25. The best value of S may be determined only when the material is known. It is better to take the value of P_M from diagrams drawn for a particular design—at least for a particular type.

By using a greater value of P the ratio k is reduced, decreasing the moment arm for the crank and shaft. If the product kP is constant, (3) shows that the pin diameter is not changed. It may also be seen from (8)

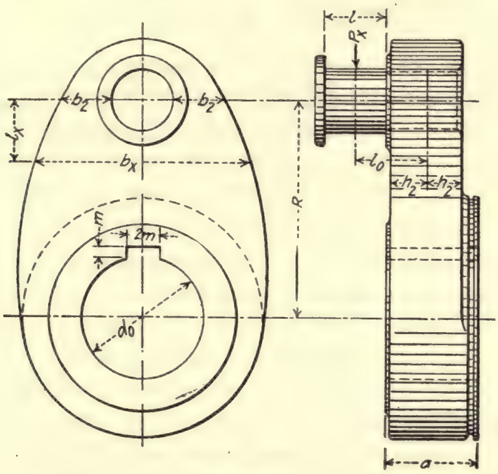

Fig. 396.

that this makes PS constant, and these relations may be convenient in determining conditions.

In all pin design there should be a fillet where the pin changes diameter. If the pin is cast integral with the crank there should be a fillet where they join.

Crank Arm.—A crank arm of substantial design is shown in Fig. 396. This is a plain crank having all the requisites to transmit power from the crank pin to the shaft. Sometimes a counterbalance is added, and sometimes a disc crank is made by adding rim and web, but these should not be depended upon to add to the strength of the crank and may be omitted for the present discussion.

In a crank of this general design the greatest liability to failure is

through the pin hole when the crank is on one of the dead centers, and this will first be considered.

Assuming a perfectly rectangular section, the bending stress is:

$$S_B = \pm \frac{6P_P l_O}{bh^2}$$

The direct stress is:

$$S_D = \pm \frac{P_P}{bh}$$

These act at the same time at the dead center, P_P being the total force including steam or gas pressure and all inertia effects. The sign is plus for tensile stress and minus for compressive. The total stress is:

$$S = S_B + S_D = \frac{P_P}{bh}\left(\pm \frac{6l_O}{h} \pm 1\right) \tag{14}$$

At the beginning of the stroke both signs are minus on the pin side, or face of the crank, and for the back side the first sign is plus and the second, minus. At the end of the stroke both signs are plus on the face, while on the back the first is minus and the second plus.

From the equations of Par. 166, Chap. XXI, the worst condition for given indicator and inertia diagrams may be determined.

Cast Iron Crank.—For the conditions assumed in Table 82, Chap. XXI, for the factor 5 for steam engine cranks there is nearly full reversal, and the factor 7 may be assumed for cast iron as previously explained. There is an initial stress in the metal around the pin presumably higher than the liveload stress. The factor for this may be 4 as for static stress. Due to this initial stress and the fact that the conditions assumed in determining the factor 7 only obtain for very large overloads; also because the resulting dimensions are greater than those of many actual cranks, the factor of safety may be taken as 5. This factor will be applied by making P_P in (14) equal to P_X and taking both signs positive. Then the general formula is:

$$S = \frac{P_X}{bh}\left(\frac{6l_O}{h} + 1\right) \tag{15}$$

The diagrams of Fig. 186, Chap. XVI, drawn to scale for the 20 in. Corliss engine previously referred to, when analyzed by the method of Par. 166, Chap. XXI give a factor of 4.3 for ¼ cut-off. For ¾ cut-off the factor is 5.55. These are both for cast iron. Then in view of the initial stress in the crank-pin eye due to forced fit, the factor 5 seems safe. It is likely that the iron used for cranks would usually have a tensile strength greater than 16,000 lb., but this value will be used here.

An old rule was to make the length of crank fit ¾ of the shaft diame-

ter. Another rule was to make the diameter of the shaft ½ the cylinder diameter. This makes the length of the crank ⅜ the cylinder diameter. This may be done satisfactorily in steam engine practice by taking the cylinder diameter as that of the standard cylinder D_S carrying some standard pressure. For any other pressure (and this may be used for internal-combustion engines if desired) the length of the fit may be found as for other standard dimensions as explained in Par. 63, Chap. XII, or for compound engines in Par. 72, Chap. XIII.

If h, the arm depth, is made $0.32D_S$, there is room for an oil groove on the hub, even on the smallest sizes. Then taking P_X as referred to the standard cylinder D_S, and substituting the value of h in (15), we have:

$$b = \frac{46.2pD_S}{S}\left(\frac{l_o}{D_S} + 0.0532\right) \tag{16}$$

which is a special formula when $h = 0.32D_S$.

As it is well to confine the use of cast iron cranks to steam engines not carrying high pressures, another special formula may be given, based upon the pin dimensions of Table 91. Then let $p = 125$, $S = 3200$ and $l_o = 0.32D_S$, for which:

$$b = 0.672D_S = 2.1d \tag{17}$$

This dimension may be obtained as shown in Fig. 396, and while rupture might not occur in line with the pin diameter, the method has proven satisfactory in practice.

Steel Cranks.—For steam engines carrying high pressures and for gas engines, cranks should be steel castings As dimensions are not so excessive there is no difficulty in applying the formula with the factors already assumed in the introduction, and used for the pin. Formulas (14) to (16) also apply to steel cranks

From Table 73, Chap. XXI, the elastic limit is given as 30,000 lb. Taking S as 6000 and other data as before, a special formula for steam engines is:

$$b = 0.36D_S = 1.12d \tag{18}$$

For gas engines, calculations show a massive construction when the pin is to be forced in. It is no doubt the best practice to cast the pin with the crank. As an example, the proportions of (12) will be assumed in which $k = 1$. Let a standard pressure p be 400; then S is 7150. Let it be assumed that the length of the hub fit is $0.5D_S$ and that $h = 0.42D_S$. Then as $d = 0.473D_S$, l_o may be taken as $0.45D_S$. Substituting these in (16) gives:

$$b = 1.3D_S = 2.7d \tag{19}$$

which is excessive.

The crank arm is sometimes cut away as shown in Fig. 398. This is permissible, especially in steel castings, but is not logical design. It gives a poor section for torsional stiffness when the crank center is normal to line of stroke; it also necessitates joining a light and heavy section if the walls are made thin. It may lighten the crank if the stroke is long, and it reduces the moment arm by moving the neutral axis toward the pin; but it also reduces the section modulus and should be carefully checked for tension on the side away from the pin. To find the stress in this section the general formula is:

$$S \cdot \frac{I}{c} = P_X l_O \tag{20}$$

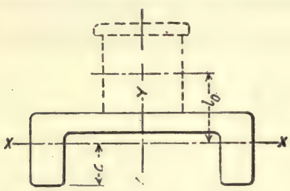

Fig. 397.

where I is the moment of inertia about axis XX and c is the distance to the extreme fiber as shown in Fig. 397.

Well designed cranks seldom need checking when in a position normal to line of stroke. They are then subjected to torsion and bending. The bending moment is:

$$M_B = P_P l_X$$

where l_X is the distance from the pin center to any section as shown in Fig. 396. The modulus of section for this case is about axis YY. The twisting moment is:

$$M_T = P_P l_O$$

where l_O is taken from Fig. 396. If the section is rectangular the maximum shearing stress is at the center of the surface of the long side and may be found from Par. 163, Chap. XXI. At the center of the short side the stress is proportional inversely to the stress at the long side, as the lengths of the sides of the rectangle. These stresses may be combined by (20), Chap. XXI. As stated in Par. 166, Chap. XXI, the factor of safety for this position may be taken as 3 for ductile materials and 6 for brittle material, such as cast iron.

190. Hubs.—The subject of pressed fits is treated in Par. 178, Chap. XXIV, so the discussion will not be repeated.

Let T be tons per inch of diameter per inch of length required to complete the fit; let μ be the coefficient of friction and t the thickness of hub. Then from (28), Chap. XXIV:

$$q = \frac{2000T}{\pi\mu S} \tag{21}$$

and from (29), same chapter:

$$t = \frac{d}{2}\left[\sqrt{\frac{1 + 0.7q}{1 - 1.3q}} - 1\right] \tag{22}$$

The outside diameter is then:

$$d_H = d + 2t \tag{23}$$

For cast iron cranks, taking $\mu = 0.25$, $T = 0.8$ and $S = 4000$, assuming static loading:

$$q = 0.51$$

and

$$t = 0.5d.$$

Then:

$$d_H = 2d \tag{24}$$

The allowable static stress in a steel casting is 15,000 lb. If T is 1.25, the outside diameter could be less than $1.3d$. If the stress is taken at 6250 when T is 1.25, $d = 2d$ as just found for cast iron. There are

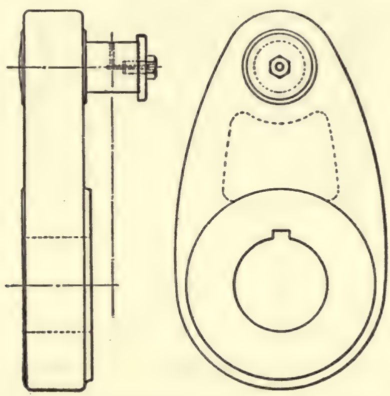

FIG. 398.—Nordberg crank and pin.

other considerations besides strength in crank design, such as the necessity of a diameter sufficient to cover the bearing, but the formulas may be used as a check.

Crank pins are forced in and riveted over cold as shown in Fig. 396. There is no tendency for the pin to turn so no key is needed, but as all the power of the engine is transmitted through the shaft fit, the shaft is keyed to the crank with one or two keys as a precaution against slipping.

In Fig. 396, if the fit were not tight enough to keep the crank from slipping on the shaft, the surface ma of the keyway must sustain the entire load. If S_C is the crushing stress in the key, the moment of resistance of the key is approximately.

$$\frac{d_O}{2} ma S_C$$

This must equal the twisting moment $P_T r$, where r is the radius of the crank circle in inches. Then:

$$m = \frac{2P_T r}{d_O a S_C} \tag{25}$$

Should the key take the entire load the standard factor of safety would be 3 for repeated load. It is probable in most cases that the fit would hold without a key, so if desired, a factor of judgment less than unity may be applied.

For steam engines the maximum value of P_T may be taken as P_X. For internal-combustion engines, Güldner finds that the maximum P_T is $0.5P_X$ for his standard reference diagram.

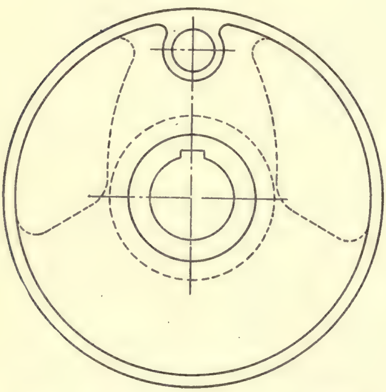

Fig. 399.—Bass-Corliss disc crank.

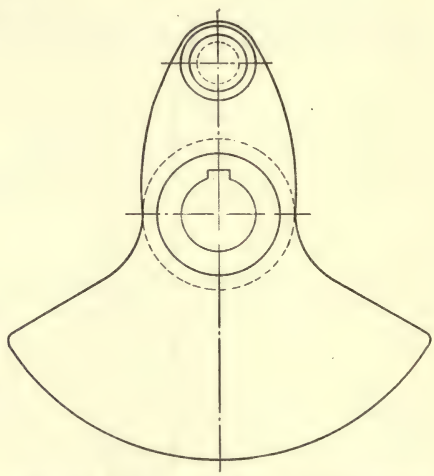

Fig. 400.—Plain crank with counterbalance.

By assuming values in terms of D_S, a special formula may be derived. In doing so it is best to retain d_O and S_C.

For the steam engine, as already assumed, $a = 0.375D_S$. Also let $r = 1.2D_S$ which covers most cases. Assume p as 125. Then:

$$m = \frac{630D_S^2}{d_O\,S_C} \tag{26}$$

For the internal-combustion engine we may take: $a = 0.5D$, $r = 0.8D_S$ and $p = 400$. Then:

$$m = \frac{500D_S^2}{d_O\,S_C} \tag{27}$$

These two special formulas may be used for preliminary calculations.

If two keys are used they are placed 90 degrees apart. The value of m for each key may be taken as one-half that for a single key, or perhaps

some greater. This arrangement weakens the shaft less than a single key.

The factor of judgment may be taken as 0.75, making a total factor of 2.25, which, with an elastic limit of 38,000 gives $S_C = 17{,}000$.

191. Crank Designs.—Fig. 398 shows a crank used on steam engines built by the Nordberg Manufacturing Co., Milwaukee, Wis. The hub projection next to the bearing is omitted, except that necessary for facing. A small amount may be placed on the opposite side so long as it does not interfere with the connecting rod. This arrangement either shortens the moment arm used in determining the shaft strength, or makes the crank arm thicker, or both. A thicker arm reduces the difficulties encountered in determining b.

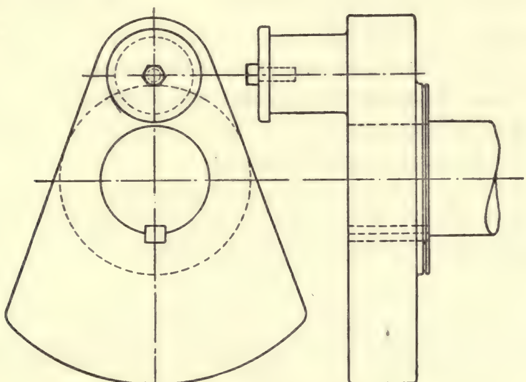

FIG. 401.—Gas engine crank.

The oil groove must be omitted in this design, but this matters little if oil shields are used.

Figure 399 is the design of a disc crank used by the Bass Foundry and Machine Co., Fort Wayne, Ind. The disc portion is considered by some to have a more finished appearance, the rim usually being polished.

A simple arm crank with counterbalance is shown in Fig. 400. It is proportioned for the 20 by 48 in. Corliss engine designed through the book. The crank of Fig. 396 is for the same engine and is calculated for cast iron, but Fig. 400 is for steel.

Figure 401 shows a gas engine crank with pin cast integral, and is therefore of steel casting.

CHAPTER XXVIII

SHAFTS

Notation.

D = diameter of cylinder in inches.

D_S = diameter of cylinder when some standard pressure is used (see Par. 63, Chap. XII and Par. 72, Chap. XIII).

d_O = diameter of pin or shaft determined for stress relations. Used especially for shaft fit in hub.

d_P = diameter of crank pin.

d_S = diameter of shaft at any part considered.

h = thickness of crank arm, measured axially.

b = width of crank arm.

r = radius of crank circle in inches.

l_O = moment arm of crank pin in inches.

l_A = moment arm of crank arm in inches.

l_M = moment arm of main journal in inches.

l_P = length of crank pin in inches.

l_S = length of main journal in inches.

p = maximum unbalanced steam or gas pressure in pounds per square inch.

P = maximum pressure on crank pin per square inch of projected area.

P_M = mean pressure per square inch of piston area, including inertia, per cycle.

P_X = maximum total unbalanced steam or gas pressure in pounds.

P_P = total unbalanced force, including inertia, acting in line of stroke

P_L = thrust along connecting rod in pounds, including inertia of reciprocating parts and of rod itself.

P_T = maximum turning effort in pounds.

P_R = force in pounds acting along crank, toward shaft center.

P_B = maximum belt pull in pounds.

R = reaction on bearings in pounds. Subscripts correspond with those of force P_X, P_T, etc.

R_M = mean reaction on bearing during cycle, in pounds.

W = weight of flywheel in pounds.

M_B = bending moment.
M_T = twisting moment.
M_R = equivalent bending moment.
Δ = total deflection.
δ = deflection per inch of length.
I = moment of inertia.
E = modulus of elasticity.
S = stress in general, direct or combined.
S_P = stress in crank pin.
S_A = stress in crank arm.
S_M = stress in main journal.
S_E = elastic limit.
S_S = shearing stress.
S_D = direct stress in crank arm.
f = factor of safety.

C and K are constants in wear formulas in Chap. XI.

Other notation on figures. All dimensions are in inches, and forces in pounds.

192. Shaft Types.—Two general types of crank shaft are in use; the *side-crank*, shown in Fig. 402 and the *center-crank*, shown in Fig. 403.

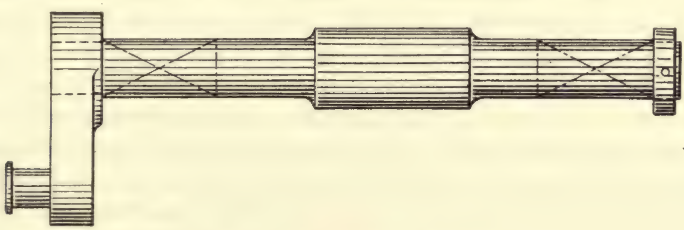

Fig. 402.—Side-crank shaft.

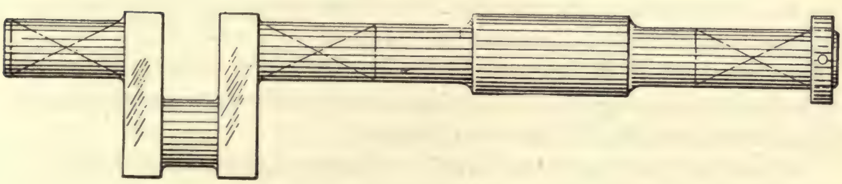

Fig. 403.—Center-crank shaft.

Center-crank shafts are also constructed with no outer bearing for small engines, the wheel being overhung and close to the bearing. Sometimes two wheels are employed, one containing the governor.

In this country the use of the center-crank is mostly confined to engines of small size and to multi-cylinder engines, being a necessity for the latter. Large engines, both steam and internal-combustion, are nearly always provided with the side-crank shaft. On the continent, both large and small internal-combustion engines are built with the center-crank shaft.

The advantage claimed for the center-crank is an even distribution of pressure on two bearings, and the advantage from the standpoint of strength of a beam supported at both ends, over a cantilever.

It is obvious that an outer bearing is necessary with a side-crank shaft. For many classes of service it is also necessary with a center-crank shaft, especially for large sizes with heavy wheels, in which case there are three bearings to keep in alinement. Failure to do so results in excessive strain, so that engineers in this country are inclined to use the side-crank shaft where feasible, and although heavier, stresses may be calculated with greater accuracy.

In all cases of design, the point in each bearing from which to measure lengths for bending moment calculations is indeterminate, and certain assumptions have to be made. For the side-crank shaft with two bearings, calculations are comparatively simple, as is also the case with center-crank shaft with no outer bearing. For three bearings, and for multi-throw shafts, the calculations become much involved if an attempt at rigid analysis is made. The Theorem of Three Moments (Clapyron's formula) is sometimes applied to this case and referred to as an exact method, but the laws of bending of center-crank shafts are very complex; in fact, part of the deflection is due to torsion of the crank arms. Were these complications due to shaft form taken into account, assumptions must still be made due to the resistance to bending offered by the bearings, and the nature and exact location of the reactions. It is therefore certain that any such method is far from exact, and in all probability gives no better results than the simple methods employed with discretion, and some consideration of successful practice.

For side-crank shafts an analysis which has given satisfactory results in practice has been employed, and this is checked for various other factors which have a limiting influence on design.

For two-bearing center-crank shafts a simple analysis is also made for crank pin, arm and shaft journal.

For center-crank shafts with outer bearing the portion of the shaft between main and outer bearings is designed as for a side-crank engine, while the cranks are designed as for a two-bearing, center-crank shaft, ignoring the remainder of the shaft. The bending moment due to wheel

load would be practically zero at the point of maximum bending due to piston thrust, and it is unlikely that two maximum values are found simultaneously at the same point. It may be that wheel load, etc. will require a larger diameter of the main journal than given by the simple center-crank analysis; should this be greater than the required crank pin diameter, they should be made equal.

With multi-throw crank shafts, the crank next to the delivery end of the shaft is designed as a single-throw shaft, with the addition of any turning effect from the other cylinders. This is best determined from a combined turning-effort diagram, omitting the end cylinder diagram. It is usually comparatively small, especially if of the 4-cycle type, and may be provided for by multiplying the moments found by the simple method by a factor a little greater than unity—possibly 1.1.

On account of greater uncertainty respecting strains in center-crank shafts, lengths for determining moments are measured more nearly from the centers of bearings; at the crank pin, however, the load due to connecting rod thrust is more symmetrical about the center of the pin, and may be assumed as a uniform load for a portion of the pin near the center.

Material.—Shafts for large engines are often open hearth steel forgings, the elastic limit of which may conservatively be taken as 38,000, as given in Table 73, Chap. XXI. Higher grades of steel are sometimes used, and properties of some of the alloy steels are tabulated in Chap. XXI. Allowable bearing pressures sometimes limit the stress, so that high elastic limits are not required unless desired for their higher factor of safety. In some designs however, especially where few bearings are used for a multi-throw shaft, the ratio of bearing pressure to stress is low, so that higher permissible stresses are a great advantage, enabling the use of smaller diameters.

The side-crank shaft will be first treated, both for steam and internal-combustion engines, after which the center-crank shaft will be analyzed, largely in connection with 4-cycle internal-combustion engines, where it is most used.

As for crank pins, the factor of safety may be determined by the aid of Par. 166, Chap. XXI. From Table 82 of Chap. XXI, the factor for single-acting engines may be taken for usual conditions as 3.45; for double-acting steam engines as 5 and for double-acting internal-combustion engines as 4.2. These factors are based upon maximum steam or gas pressure only, and apply to crank positions on or near dead center. As explained in Par. 166, Chap. XXI, the stresses are practically repeated for crank positions giving large turning efforts, and the factor may be taken

as 3, with perhaps a factor of judgment of from 1.1 to 1.2. In order to reduce deflection it is better to use a higher factor for shafts with considerable distance between bearings, so in such cases the factor will be as in Table 82, Chap. XXI.

193. Side-crank Shaft.—Except when very heavy wheels and long shafts are used, the diameter of a side-crank shaft may often be determined by combining the bending and twisting moments due to the piston thrust. From Formula (11) and Table 53 of Chap. XVI it may be seen that when the connecting rod is but four cranks long, the pressure along the rod is but little more than 3 per cent. greater than the piston thrust; if friction were considered the difference would be still less. Then for this discussion it may be considered sufficiently accurate to neglect the angularity of the connecting rod.

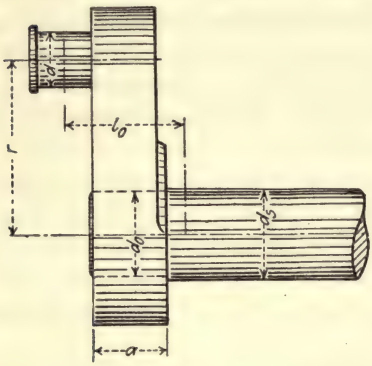

Fig. 404.

The older rules assumed the moment arm l_o of Fig. 404 to extend to the center of the bearing. For long lines of shafting, or where ball-and-socket bearings are used, this method is probably correct; but when ball joints are used in engine bearings, they do not allow freedom in the line of thrust. It is then probable that the point of maximum bending is near the crank end of the bearing. The author has assumed for many years that it is at the point where the shaft enters the crank. The shaft is practically always reduced to form a shoulder here; then if this reduced diameter be solved for at this point, and the diameter in the bearing increased $\frac{1}{8}$ to $\frac{1}{4}$ in. to form a shoulder, the equivalent moment arm for the larger diameter will extend a short distance along the bearing. By this method it is not necessary to assume stress greater than that indicated by the laws of fatigue, which was necessary with the old assumption.

Steam Engine Shaft.—For engines cutting off steam later than one-half stroke, which is now usual when overloads are being carried, the maximum bending and twisting moments occur when the crank is normal to the line of stroke. The bending moment is then:

$$M_B = P_X l_o$$

and the twisting moment:

$$M_T = P_X r.$$

The value to be used in these formulas is not strictly P_X, but P_L, the force acting in the direction of the connecting rod. As this differs little from P_P, the thrust in line of stroke (including inertia), and it is desired to base the factor of safety upon P_X, the latter may be used.

From (22), Chap. XXI, these may be combined thus:

$$M_R = 0.35M_B + 0.65\sqrt{M_B^2 + M_T^2} \tag{1}$$

Equating with the modulus of section of the shaft gives:

$$d_O = \sqrt[3]{\frac{32M_R}{\pi S}} \tag{2}$$

For the standard cylinder diameter D_S carrying standard unbalanced unit pressure p, the length and diameter of the crank pin was taken as $0.32D_S$ in Chap. XXVII. The crank hub was taken as $0.375D_S$. Then for the design shown in Fig. 404:

$$l_O = \frac{0.32D_S}{2} + 0.375D_S = 0.535D_S \tag{3}$$

Let $r = 1.2D_S$, and as $p = 125$ (Chap. XXVII), and $f = 5$, by substituting these values in (1) and (2), d_O may be found in terms of D_S, or:

$$M_R = 98.6D_S^3$$

and

$$d_O = D_S\sqrt[3]{\frac{1005}{S}} \tag{4}$$

Taking the elastic limit S_E as 38,000, $S = 7600$ and:

$$d_O = 0.51D_S \tag{5}$$

Formulas (4) and (5) are special and apply for the assumptions made. Their use, however, will give good practical results.

Between the bearings there is combined bending and twisting. The twisting moment is the same as for the case just considered, and the bending may be caused by weight of wheel and shaft, weight of gear or armature, belt pull, or magnetic pull of armature. Any of these forces may be referred to the bearing by taking moments about the other bearing; then a resultant may be found graphically as in Fig. 405, in which R_W is the reaction due to the weight and R_B the reaction due to the belt pull. Total reaction R may include all forces acting between the bearings.

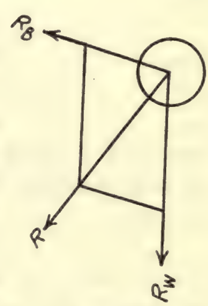

FIG. 405.

The central part of the shaft is usually enlarged to take the wheel or any other desired mountings, and the smaller diameter only requires cal-

culation for strength and stiffness. Referring to Fig. 406, the moment arm l_O may be taken to the center of the bearing in this case, as there is sometimes freedom of movement of the bearing in this plane. The bending moment is:

$$M_B = Rl_O$$

From (1) the equivalent bending moment is:

$$M_B = 0.35Rl_O + 0.65\sqrt{(Rl_O)^2 + (P_Xr)^2} \tag{6}$$

If, after adding the shoulder to d_O found from (2), the value of M_R from (6) gives a greater result in (2) than this, it must be used.

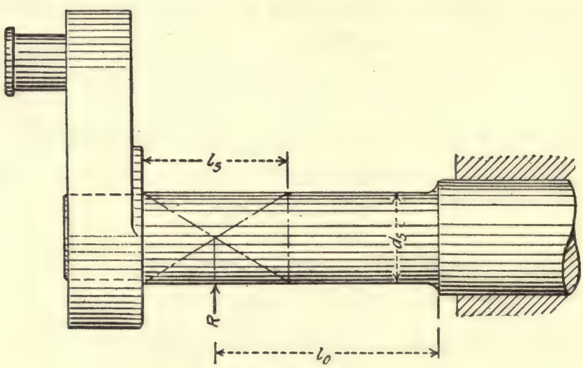

FIG. 406.

The deflection may be taken as for a cantilever of length l_O. Then:

$$\Delta = \frac{Rl_O{}^3}{3EI} = \frac{6.8Rl_O{}^3}{Ed_S{}^4} = \frac{\delta l_O}{12} \tag{7}$$

where δ is the deflection per ft. of length. Table 73, Chap. XXI gives: $E = 29{,}000{,}000$. A value of δ sometimes given is 0.003. Then from (7):

$$d_S = 0.175\sqrt[4]{Rl_O{}^2} = 0.175\sqrt[4]{R}\sqrt{l_O} \tag{8}$$

All three of these methods should be used and the largest diameter adopted. By using a lower stress in the first method, a diameter will be obtained which may be ample for nearly all conditions which may arise, but it gives a larger shaft than is required in a good many cases, increasing the cost.

A further check should be made by the formulas of Par. 52, Chap. XI, and by the bearing pressure in Table 19 of the same chapter. The weight of crank and part of connecting rod should properly be included, also the

mean piston thrust. The resultant mean pressure on the bearing may be found for each stroke of the cycle and the average of these taken as the value of P in Chap. XI.

It is not claimed that these values give limits beyond which it is never permissible to go, but it is well to keep within or near these figures if possible.

A common ratio of length to diameter of steam engine bearings is 2. A bearing length may sometimes be made twice the diameter of shaft by the first method even though the actual diameter must be greater according to the second or third methods, providing the check on bearing pressure and wear by Chap. XI is satisfactory.

In some cases the shaft is enlarged immediately outside the bearing. The crank fit and bearing may then be proportioned for the piston load, and the considerations of friction and wear treated in Par. 52, Chap. XI.

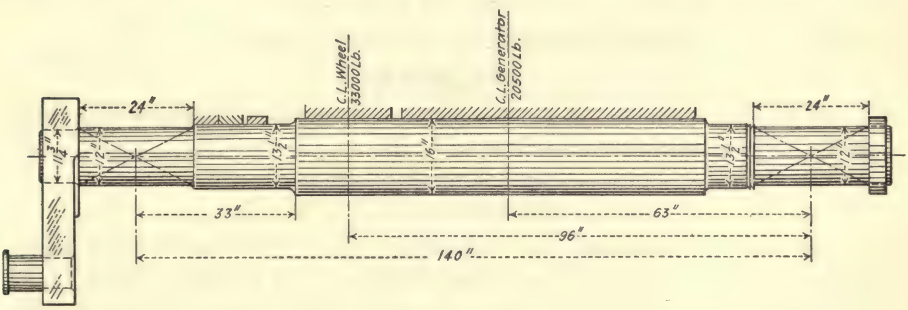

Fig. 407.

As an example of application, assume the 20 by 48 in. Corliss engine mentioned in Chap. XII and elsewhere through the book, to drive a 300 kw. railway generator. The data for this generator was taken from an old table, but will answer for the problem. The weight of the armature is 20,500 lb. The wheel, determined by the method of Chap. XVIII, weighs 33,000 lb. The two eccentric hubs, governor pulleys, wheel hub, and limiting lines of generator are shown in Fig. 407

As $f = 6$ between bearings, $S = 6330$.

The reaction of the wheel at center of main bearing is:

$$R_W = \frac{33{,}000 \times 94.5}{137} = 22{,}750 \text{ lb.}$$

Of the generator:

$$R_G = \frac{20{,}500 \times 61.5}{137} = 9200 \text{ lb.}$$

The weight of shaft may be neglected at first. The bending moment due to total reaction R (= 31,950, or say 32,000 lb.) is:

$$M_B = 32{,}000 \times 31.5 = 1{,}010{,}000 \text{ lb.}$$

The maximum turning effort is 39,250 (approximately). The twisting moment is:

$$M_T = 39{,}250 \times 24 = 945{,}000 \text{ lb.}$$

From (1) or (6):

$$M_R = 1{,}380{,}000.$$

This is more conveniently found from (24), Chap. XXI. Now from (2):

$$d = \sqrt{\frac{32 \times 1{,}380{,}000}{\pi \times 6330}} = 13 \text{ in.}$$

This gives some idea of the necessary diameter, but we know it will be some greater. In the shaft fit (5) gives:

$$d = 0.51 \times 20 = 10.2 \text{ in.}$$

With a $\frac{1}{8}$-in. shoulder the diameter of bearing would be:

$$d = 10.5 \text{ in.}$$

With this diameter as a basis let k, the ratio of length to diameter, be 2. The length may be taken as 21 in.

Checking for deflection by (8), with reaction R gives:

$$d = 0.175\sqrt[4]{32{,}000}\sqrt{31.5} = 13.2 \text{ in.}$$

As the weight of the shaft was not included it is better to take the diameter between bearing and hub fit as 13.5 in. It is desirable when taper keys are used in the flywheel hub, to increase the diameter so that the key will clear the smaller portion of the shaft. If straight keys are set in the shaft this is not necessary. In this case we will make the diameter through wheel and generator fit 16 in., an increase of $2\frac{1}{2}$ in. From the generator table, the range of shaft diameter is from 13 to 16 in., so this is allowable.

The reaction of the shaft at the main bearing is 4000 lb. The reaction of the crank is 3700 and of the connecting rod 675 lb. The total reaction producing bending moment is due to wheel, generator and shaft, the effect of the latter being safely taken as a concentrated load at the center; this reaction is nearly:

$$R = 36{,}000 \text{ lb.}$$

The reaction causing bearing friction is the resultant of this plus crank and rod effect, the mean load due to piston thrust and inertia. The latter at maximum cut-off is 32,500, and as this acts normal to the other, the mean reaction causing wear is:

$$R_M = \sqrt{40{,}400^2 + 32{,}500^2} = 51{,}800 \text{ lb.}$$

The main bearing may have any dimensions from 10.5 by 21 in. to 13.5 by 27 in., as far as strength is concerned. By trying several values by the formulas of Par. 52, Chap. XI, we will take 12 by 24 in. and check. This does not change the reactions perceptibly, but lengthens the moment arm l_o to 33 in.

For strength:

$$M_B = 36{,}000 \times 33 = 1{,}190{,}000$$
$$M_T = 39{,}250 \times 24 = 945{,}000.$$

From (1) or (6):

$$M_R = 1{,}520{,}000.$$

From (2)

$$d_S = \sqrt[3]{\frac{32 \times 1{,}520{,}000}{\pi \times 6330}} = 13.45 \text{ in.}$$

which is safe, as 13.5 in. was taken.

For deflection:

From (8):

$$d_S = 0.175\sqrt[4]{36{,}000}\sqrt{33} = 13.9 \text{ in.}$$

This is a little larger than the dimension taken, but the moment arm is probably less than 33 in., and there is some variation allowed in the deflection factor used, which was a mean value.

Bearing pressure.

$$P = \frac{51{,}800}{12 \times 24} = 180 \text{ lb.}$$

Wear.

From (4), Chap. XI:

$$C = \frac{0.262 \times 51{,}800 \times 100}{24} = 56{,}300.$$

From (6), Chap. XI:

$$K = \frac{0.512 \times 51{,}800}{24}\sqrt{\frac{100}{12}} = 3190.$$

The values of C and K are greater than the tabular values, but these factors are more or less arbitrary and may be increased under good conditions. In determining P, C and K, the maximum load pressure was assumed in the cylinder.

The shaft is drawn to scale in Fig. 407. There must be fillets in all places where the shaft changes diameter. A smaller diameter might have been used if stronger material had been assumed, but the modulus of elasticity is practically the same for all different steels, and this is what influences the deflection.

The eccentrics and straps, governor pulley, etc. were neglected. The

shaft, crank and connecting rod are often omitted; if bearing pressure and the constants for wear are based upon such omissions it matters little.

The 10 in. of length between the outer bearing and the swell could no doubt be reduced, equalizing the load more nearly between the two bearings. It is possible that a more detailed study would show the same for the 21 in. at the crank end, shortening the shaft slightly.

Internal-combustion Engine Shafts.—The maximum bending and torsional strains do not occur at the same time in these shafts, and d_O should be computed for maximum bending stress at dead center, and for combined bending and twisting at the position of maximum turning effort.

At the dead center (1) gives:

$$M_R = M_B.$$

Then this may be used in (2). At the position of greatest turning effort:

$$M_B = P_L l_O$$

and

$$M_T = P_T r$$

where P_T is the turning effort and P_L the thrust along the rod. P_L will be used for this part of the discussion, although the numerical value of P_P may actually be used with small error. The position of P_L and the value of P_T may be found from the crank-effort diagram. Güldner finds this position 40 degrees from the head-end dead center for his reference diagram, with a value of $P_L = 0.7P_X$, and $P_T = 0.5P_X$.

In Fig. 197, Chap. XVI, the maximum turning effort is at 45 degrees. $P_L = 0.53P_X$ and $P_T = 0.43P_X$. P_X is the total maximum gas pressure, while P_P is the total pressure at any point including the effect of inertia. The value of P_L and P_T taken from Chap. XVI include the inertia, while Güldner's values do not; they would apply at starting and are safer.

Combining the twisting and bending moments in (1), d_O may be found from (2). The analysis for stress, deflection and wear between the bearings are the same as for the steam engine shaft.

Assuming standard pressure, proportions of crank pin and length of shaft fit, a special formula may be derived as was done for the steam engine. In deriving a special formula for the internal-combustion engine side-crank in Chap. XXVII, it was assumed that the length of crank fit a was $0.5D_S$; then taking k as unity, (12) of Chap. XXVII was derived; or, letting d_P be the pin diameter (and also the length for this case):

$$d_P = l_P = 0.473D_S.$$

Then:

$$l_O = 0.737D_S$$

Further assume that $P = 400$, $f = 4.2$ and $S = 9000$. At dead center:

$$P_X = \frac{\pi}{4}D^2 \times 400$$

and

$$M_B = P_X l_O = \frac{\pi \times 400 \times 0.737}{4} \cdot D_S{}^2 = 232 D_S{}^2.$$

This is the only moment acting in this position. Taking Güldner's values at the position of maximum turning effort:

$$P_L = 0.7 \times \frac{\pi}{4}D_S{}^2 \times 400 = 220 D_S{}^2$$

$$P_T = 0.5 \times \frac{\pi}{4}D_S{}^2 \times 400 = 157 D_S{}^2.$$

Then:

$$M_B = 162 D_S{}^3$$

and

$$M_T = 126 D_S{}^3$$

From (1):

$$M_R = 182 D_S{}^3.$$

This is less than M_B for the dead-center position, so the latter must be used; then from (2):

$$d_O = 0.64 D_S \tag{9}$$

As this is much greater than the value given by (5) for the steam engine there is less likelihood of the computations between bearings giving a greater diameter than with the steam engine, although the wheel of the latter may not be as heavy. As mentioned in connection with steam engine shafts, the diameter may be increased outside the bearing if a larger diameter is required for the wheel load, etc.

Comparing with the problem of the steam engine, a 20-in. double-acting gas engine might have a 32-in. stroke. With the same piston speed it would run 150 r.p.m. and develop approximately one-half the power. A tandem engine would develop the same power as the steam engine, and aside from the increased inertia effects would produce no greater strains than the single-cylinder gas engine. The wheel would probably be no heavier as it runs at higher speed. From (9), the diameter of the shaft fit is:

$$d_O = 0.64 \times 20 = 12.8; \text{ say } 12\tfrac{7}{8} \text{ or } 13 \text{ in.}$$

In the bearing the diameter could be 13½ in. Otherwise it is probable that about the same dimensions could be used. The generator from the tables used before weighs but 17,000 lb. and the greatest shaft fit is 14 in.

It is likely that the shaft might be turned to this diameter at the generator end and still have sufficient stiffness.

194. Center-crank Shafts.—For either steam or gas engines the center-crank shaft can have no advantage in a problem like that of the preceding paragraph. If a center-crank is specified, the pin diameter may be determined in a manner which will be given later. The arms may be checked for bending on dead center, and for tension and bending when normal to the line of stroke. In this latter position, due to the uncertain effect of the extra bearing, it is safe to ignore it, as it can only cause torsion of the pin by undue distortion and bearing friction.

In multi-cylinder marine steam engines taking power from one end of the shaft—say at engine No. 1. The bending moment on the pin of No. 1 caused by the maximum turning effort of this engine, may be combined with the twisting moment of all the other engines. For the end journal of No. 1, the bending moment caused by the thrust of this engine must be combined with the maximum turning moment of *all* engines. This can be determined accurately only by combined turning-effort diagrams at maximum load.

The formulas developed for the multi-throw crank for internal-combustion engines may be applied generally to steam engines, the chief difference being in the ratio of twisting to bending, and the position of the crank when principal calculations for the different portions are made.

Internal-combustion Engines.—Calculations show that the crank pin is subject to maximum stress intensity when on dead center, and that forces other than the direct piston thrust affect it but little. In this position the moment arm of the main journal is short, so that it receives its maximum stress at or near the position of maximum twisting moment, the stress being combined bending and torsional stress.

The maximum stress in the arms is at dead center position usually, but they may be checked for combined bending and twisting in the position of maximum turning moment.

The following analysis will cover these points in as simple a manner as possible, and are considered as reliable as a more complicated method.

In treating the subject, a number of general formulas are given for the various straining actions; these may be used as a check upon design after the proportions are assumed. The more important formulas are marked *, and from these, design formulas will be derived in terms of factors which may be assumed, or taken from practice. These are usually sufficient for design if proper values are assigned.

Provision is made in the more general formulas for an overhung wheel and pull of belt. Their resultant P_G may be resolved into any required

components, the sign of which may be easily determined. Should there be an outer bearing, the effect of P_G may be taken as one-half.

To get a better idea of the forces and reactions dealt with in the general formulas, the skeleton drawing of Fig. 408 is given. The reactions are all shown acting at the same point in each bearing, although

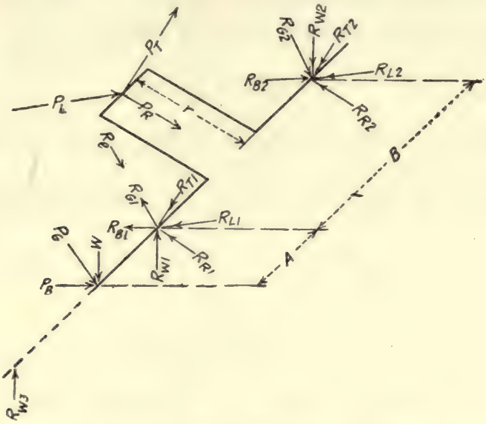

FIG. 408

this need not always be so assumed; the point is often indeterminate and will be given in what follows without reference to the length of the bearing.

The twisting moment due to other cylinders is also given in the

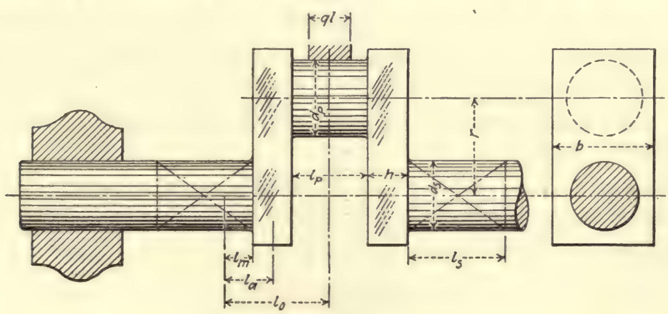

FIG. 409.

general formulas, although for most multi-cylinder internal-combustion engines this is not great. It may be studied from the combined crank-effort diagrams of Par. 176, Chap. XVI.

In order to give further notation, Fig. 409 is drawn. The thrust of the connecting rod P_L is assumed in Fig. 409 as a load distributed over

the fraction q of the central portion of the crank pin, in which q may be any desired value from zero to unity. Both Figs. 408 and 409 contain notation and must be consulted in the following formulas.

To make it more convenient to use P_G in certain equations, it may be referred to bearing No. 1 as an equivalent reaction R_E which acts in an opposite direction from the actual reaction R_{G1}. This force will give the same moment at a given section actually given by P_G. From Fig. 408:

$$R_{G2} = \frac{P_G A}{B} \tag{10}$$

The distances A and B may be taken to the center of the bearing or to some assumed point given by l_O of Fig. 409.

At any section between R_E and the connecting rod load:

$$M_B = R_{G2}(B - l_O) = \frac{P_G A(B - l_O)}{B} = R_E l_O.$$

Then:

$$R_E = \frac{P_G A(B - l_O)}{B l_O} \tag{11}$$

This may now be combined with R_T, R_{R1} or R_{L1}, and l_M or l_A may be used instead of l_O where moments are desired at points from which these measurements are taken.

The added subscript N refers to components of R_G or R_E normal to the crank, and P refers to their component parallel to the crank. The crank considered is either for a single cylinder, or for the engine nearest to where the load is applied, and will be referred to as No. 1.

The force P_L acting along the rod will be used in the equations, but as previously shown P_P may be taken as its numerical value without serious error.

For P_L, P_T and P_R see Par. 99, Chap. XVI, the notation for these quantities being the same.

Crank at Dead Center.—$P_L = P_X$ and $R_L = R_X$. Also $f = 3.45$ for single-acting engines and 4.2 for double-acting engines.

Crank pin. From P_X:

$$M_B = R_{X1} l_O - \frac{P_X}{2} \cdot \frac{q l_P}{4} \tag{12}^*$$

From P_G:

$$M_B = R_E l_O \tag{13}$$

The resultant of these may be found graphically and used for the bending moment. For vertical engines with cylinder over crank, (12) and (13) are of opposite sign and in line. From P_G:

$$M_T = R_{G2N} r \tag{14}$$

From the turning effort of all cranks when No. 1 is on dead center:

$$M_T = \Sigma P_T r \tag{15}$$

The sum of (14) and (15) may be combined with the resultant of (12) and (13) by means of (1); then:

$$M_R = \frac{\pi d_P{}^3 S}{32} \tag{16)*}$$

Crank arm. From P_X:

$$M_B = R_{X1} l_A \tag{17)*}$$

$$S = \frac{6M_B}{bh^2} \tag{18)*}$$

From the turning effort of all cranks when No. 1 is on dead center:

$$M_B = \Sigma P_T\left(r - \frac{d_P}{2}\right) \tag{19}$$

$$S = \frac{6M_B}{hb^2} \tag{20}$$

The moment given by (19) is always normal to that given by (17).
From P_G:

$$M_B = R_{G2N}\left(r - \frac{d}{2}\right) \tag{21}$$

$$S = \frac{6M_B}{hb^2} \tag{22}$$

Also from P_G:

$$M_B = R_{EP} l_A \tag{23}$$

$$S = \frac{6M_B}{hb^2} \tag{24}$$

From turning effort of all cylinders when No. 1 is on dead center:

$$M_T = \Sigma P_T l_A \tag{24a}$$

From P_G:

$$M_T = R_{G2N} l_A \tag{25}$$

For the algebraic sum of (24*a*) and (25), assuming that h is less than b, the stress at center of the shorter side from Par. 163, Chap. XXI is:

$$S_S = \frac{M_T(3b + 1.8h)}{b^3 h} \tag{26}$$

At the center of the long side:

$$S_S = \frac{M_T(3b + 1.8h)}{b^2 h^2} \tag{27}$$

For the direct load due to P_X:

$$S_D = \frac{R_{X1}}{bh} \tag{28)*}$$

This is not of great importance, but is combined with (17) in the design formula, so is marked.

In combining arm stresses three combinations may be made:

(1) The algebraic sum of (18), (20), (22), (24) and (28).

(2) Combine the algebraic sum of (18) and (24) with (27), by the use of Formula (20), Chap. XXI.

(3) Combine the algebraic sum of (20) and (22) with (26), by (20), Chap. XXI. Each of these values must come within the allowable stress.

Main journal, at juncture with arm.

From P_X:

$$M_B = R_{X1}l_M \tag{29}$$

From P_G:

$$M_B = R_E l_M \tag{30}$$

From turning effort of all cranks when No. 1 is on dead center:

$$M_T = \Sigma P_T r \tag{31}$$

The resultant of (29) and (30) may be combined with (31) by means of (1); then S or d_S may be found from (16).

Crank in Position of Maximum Turning Effort.—Factor of safety may be 3.3 for both single- and double-acting engines except when wheel is overhung, then 4.2 should be used. For the main journal, on account of uncertainty of the point of application of load, a factor of judgment of about 1.5 may be used.

Crank pin. From P_L:

$$M_B = R_{L1}l_O - \frac{P_L}{2} \cdot \frac{ql_P}{4} \tag{32}$$

From P_G:

$$M_B = R_E l_O \tag{33}$$

From turning effort of all cranks but No. 1, when No. 1 is at its position of maximum turning effort:

$$M_T = \Sigma P_T r \tag{33a}$$

From component of P_G normal to crank:

$$M_T = R_{G2N} r \tag{34}$$

The resultant of (32) and (33) may be combined with the algebraic sum of (33*a*) and (34) by (1); then S or d_P may be found from (16).

Crank arm.

From P_R, the component of P_L:

$$M_B = R_{R1}l_A \tag{35}$$

For the effect of turning effort, No. 1 should be omitted. It will be in the position of its maximum effort, however. With this difference, (18) and (27) apply to the arms in this position. Formula (28) applies by substituting R_{R1} for R_{X1}. Then by taking (35) instead of (17), the analysis for crank on dead center applies to this case for the arms.

Main journal at juncture with arm.

From P_L:

$$M_B = R_{L1}l_M \tag{36*}$$

From P_G:

$$M_B = R_E l_M \tag{37}$$

From the turning effort of *all* cylinders when crank No. 1 is in position of maximum turning effort:

$$M_T = \Sigma P_T r \tag{38*}$$

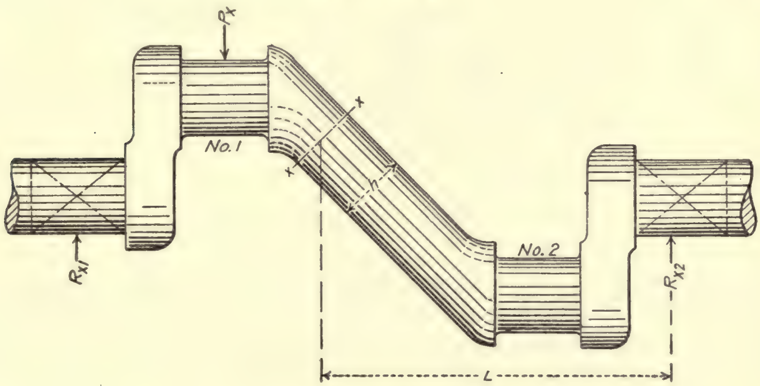

FIG. 410.

The resultant of (36) and (37) may be combined with (38) by means of (1); then S or d_S may be found from (16).

Some of the moments counteract the others, especially due to P_G. This will depend upon the direction of belt pull, etc., and if this is to be taken into account in designing stock engines, the worst condition should be taken. Bearing pressures and wear may be checked by Par. 52, Chap. XI.

The analysis is much the same for the arrangement shown in Fig. 410. The reaction R_1 will be greater for crank No. 1; therefore the bending moments will be greater. There might be advantage in using higher grade material for a crank of this design to keep dimensions smaller.

If the positions of the bearings may be located with reference to cylinder No. 1, reactions R_{X1} and R_{X2} may be found and calculations made as

for a symmetrical crank, special formulas for which will now be derived. Then R_{X1}, instead of one-half of P_X would be a larger fraction. As this relation would not likely be standard for engines of different size, special formulas would perhaps not be worth while in this case.

Special Design Formulas for Symmetrical Crank.—Referring to Fig. 409, let $l_P = kd_P$, $h = xd_P$, $b = yd_P$ and $l_M = md_P$. Then:

$$l_A = l_M + \frac{h}{2} = \left(m + \frac{x}{2}\right) d_P$$

and

$$l_O = \frac{l_P}{2} + h + l_M = \left(\frac{k}{2} + x + m\right) d_P.$$

Also

$$P_X = \frac{\pi D^2 p}{4} \tag{40}$$

Crank Pin.—Using only (12) for crank at dead center:

$$R_{X1} = \frac{P_X}{2} = \frac{\pi D^2 p}{8} \tag{41}$$

$$P = \frac{P_X}{l_P d_P} = \frac{\pi D^2 p}{4kd_P{}^2} \tag{42}$$

Substituting in (12) gives:

$$M_B = \frac{\pi D^2 p}{8}\left[x + m + \frac{k}{2}\left(1 - \frac{q}{2}\right)\right] d_P \tag{43}$$

We may solve for d_P from (16) and (43), or for S if dimensions are known; but it is more convenient to consider first the relation of S to P, as a compromise must sometimes be made between them. Then taking S_P from (16) and P from (42):

$$\frac{S_P}{P} = \frac{16}{\pi} k\left[x + m + \frac{k}{2}\left(1 - \frac{q}{2}\right)\right] \tag{44)*}$$

By assuming k, x, m, and q, the ratio S_P/P may be found and the maximum limit of one fixes the value of the other. Then d_P may be found most conveniently from (42); or,

$$d_P = D\sqrt{\frac{\pi p}{4kP}} \tag{45)*}$$

Crank Arm.—Combining S from (18) and (28) with other substitutions gives:

$$S_A = \frac{\pi D^2 p}{8xyd_P{}^2}\left[6\left(\frac{m}{x} + \frac{1}{2}\right) + 1\right] \tag{46)*}$$

Substituting the value of d_P from (45) gives:

$$\frac{S_A}{P} = \frac{k}{2xy}\left[6\left(\frac{m}{x} + \frac{1}{2}\right) + 1\right] \tag{47)*}$$

All factors but y have been determined already, and this may now be determined from (47), or S_A checked for a given value of y. P should be retained as already determined, but it may be that the value of S from (44) was smaller than it need be, in order to keep P within limit; then a higher value of S might be used in (47) if found desirable.

Main journal at juncture of arm. Maximum stress is taken at position of maximum turning effort of crank No. 1, for which (36) and (38) apply.

Then:
$$R_{L1} = \frac{P_P}{2}$$

P_T may be taken from No. 1 crank only. The bending moment is:

$$M_B = \frac{P_P}{2} md_P$$

and the twisting moment is:

$$M_T = P_T r.$$

They may be combined from (1), and d_S found from (2). By taking Güldner's values for his standard reference diagram:

$$P_P = 0.7P_X$$

and

$$P_T = 0.5P_X$$

Also assume $r = 0.8D$. An approximate formula may now be derived by substitution as before. Without going through the derivation:

$$d_S = D\sqrt[3]{\frac{mp}{S_M}\sqrt{\frac{p}{kP}}\left[1 + 1.86\sqrt{1 + \frac{1.66kP}{m^2p}}\right]} \qquad (48)^*$$

The value of P is for the crank pin. The value of d_S in practice ranges from 0.8 to $1.0d_P$, usually being the latter in automobile engines. One well-known maker of multi-cylinder gas engines makes d_S and d_P the same on their 4-cylinder engines, and d_S about $0.8d_P$ on their 2-cylinder engines.

195. Application of Formulas.—Formulas (44) to (48) may be used for design—at least for preliminary calculations. Should there be a heavy overhung wheel it is best to check for its effect; but space will not be taken to check through an example, and it probably is not usually necessary. When extreme lightness is not desired, a factor of judgment may be employed which will cover minor straining actions.

Güldner says that the factor x should be from 0.6 to 0.7. It is sometimes outside these limits. The factor k ranges from 0.7 to 1.4, the more common range being from 1.0 to 1.2. The factor q is usually ignored, the load being considered as concentrated at the center of the pin. It is

probably more nearly the truth to consider it as a uniform load over a portion of the pin next the center, say one-half of the length.

The factor m is usually such that the moment arms l_M, l_A, and l_O extend to the center of the bearing, but this is probably not a correct measure of the bending moment. It is probably nearer the crank arm, but if considered too short in computation for diameter, the shaft may not have the required stiffness. For preliminary work where center distances are not known, m may be taken as 0.5; should the finally determined distance to the center of the bearing be greater than this, it does not seem likely that the pin would be made weaker by lengthening the bearing.

The factor y must be greater than unity, and should be relatively large as x is made small. A minimum value of 1.2 may be assumed.

By assuming these factors, special formulas may be derived for standard engines of cylinder diameter D_S, or the factors may be measured from actual engines from which stresses and bearing pressures may be readily determined; examples of both kinds will be given.

Gas Engine.—Assume a single-acting gas engine with a maximum pressure of 400 lb. per sq. in. gage. Let $x = 0.6$, $k = 1.2$, $m = 0.5$ and $q = 0.5$. Also let $f = 3.45$, which, if $S_E = 38{,}000$, gives $S = 11{,}000$. Then (44) gives for the pin:

$$\frac{S_P}{P} = 9.5$$

or,

$$P = \frac{11{,}000}{9.5} = 1160 \text{ lb.}$$

From (45):

$$d_P = 0.475D_S.$$

For the arm, assume $y = 1.2$; then from (47):

$$S_A = 7550$$

and

$$f = 5.$$

For the main journal the maximum stresses are more uncertain and it is well to use a factor of judgment; this will be made 1.5, giving $f = 5$ and $S = 7600$. Then from (48):

$$d_S = 0.43D_S$$

or

$$\frac{d_S}{d_P} = 0.91.$$

The value of l_O is $0.81D_S$.

By taking $p = 360$, $S = 14{,}000$ and $l_O = 0.9D_S$, Güldner obtains:

$$d_P = 0.45D_S.$$

Had he used $p = 400$, this would have been $0.475D_S$ as just found with a stress of 11,000 lb. Güldner assumes the load concentrated at the center of the pin and the moment arm l_O measured from the center of the bearing. It is no doubt safer to make these assumptions, but probably not so near the truth.

In the problem just considered, a higher elastic limit would permit a higher bearing pressure on the pin and still be within the limit. This might also be accomplished by decreasing the value of k.

Diesel Engine.—By scaling the drawings of a certain Diesel engine the following values were found: $k = 1$, $x = 0.5$, $y = 1.35$, $m = 0.65$, $l_O = 1.51d_P$ and $d_P = 0.606D_S$. The value of q will be taken as zero. Then assuming p as 500, (42) gives:

$$P = 1070 \text{ lb.}$$

Also (44) gives:

$$S_P = 8.4 \times 1070 = 9000 \text{ lb.}$$

Taking S_E as 38,000, $f = 4.22$; this gives a factor of judgment over 3.45 of 1.22.

From (47):

$$S_A = 8570 \text{ lb.}$$

and

$$f = 4.43$$

This gives a factor of judgment of 1.28.

From (48), if $S_M = S_P$:

$$d_S = 0.564D_S = 0.93d_P.$$

These diameters were made the same, which gives:

$$S_M = 7300$$

or,

$$f = 5.2.$$

This gives a factor of judgment of 1.5.

These values may be taken as fairly representative. The measurements from scale drawings from three makes of Diesel engines give values of d_P from 0.57 to $0.606D_S$.

Automobile Engine.—Values from a well-known automobile engine are: $k = 1.2$, $x = 0.427$ and $d_P = 0.49D_S$. Assume $m = 0.5$ and $q = 0.5$. Then from (42), substituting the value of d_P:

$$P = 2.73p.$$

From (44):

$$S_P = 8.4P = 23p.$$

From (47):

$$S_A y = 15.5P = 42.3p.$$

As y is not known, it will be solved for, assuming $S_A = 11{,}000$. Then:

$$y = \frac{p}{260}.$$

These values are given in Table 94 for different pressures.

TABLE 94

p	P	S_P	y
300	820	6,900	1.16
350	960	8,050	1.35
400	1,100	9,200	1.54
450	1,230	10,350	1.73

If y is less than the tabular values, S_A is greater than 11,000.

As d_S is equal to d_P, S_M is less than S_P and is not given. Had q been

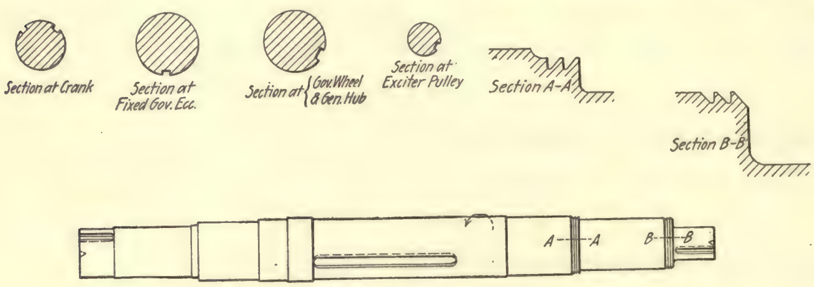

FIG. 411.—McIntosh and Seymour crank shaft.

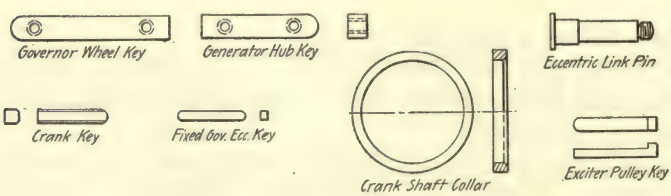

FIG. 412.—McIntosh and Seymour shaft details.

neglected and l_o taken to the center of the bearing, the stresses would have been greater. For the examples given it may be seen that moderate bearing pressures and stresses obtain.

For both crank pin and main journal, the wear may be checked by the formula:

$$C = \frac{P_M N D}{4.3}\sqrt{\frac{P}{kp}} \qquad (49)$$

This was derived from (1), Chap. XI, the allowable values of C being given in Table 19 of that chapter.

196. Designs from Practice.—Fig. 411 shows the

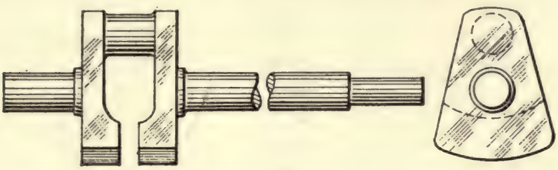

FIG. 414.—Crank with counterbalance.

design of a shaft used on the McIntosh and Seymour Type F steam engine. The outer bearing is a ring-oiled bearing. Grooves are turned in the shaft at each end of the outer bearing to prevent oil from traveling along the shaft to generator or exciter, the latter being located on the end of the shaft extending beyond the outer bearing. Details of these grooves, with other details, are shown in Figs. 111 and 412.

The center-crank shafts used as illustrations thus far have been drawn to scale. A 6-cylinder automobile engine shaft with seven bearings is shown in Fig. 413.

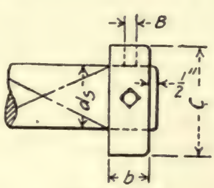

FIG. 415.

A crank shaft with counter balance forged integral —taken from Güldner—is shown in Fig. 414.

Collars.—In Fig. 407 a collar was shown on the end of the shaft beyond the outer bearing. A standard collar used by the author on Corliss engine shafts is shown in Fig. 415; dimensions are given in Table 95, taken from the following formulas:

$$b = \frac{d_s}{8} + 2 \text{ in.} \quad c = 1.25 d_s + 1.5 \text{ in.}$$

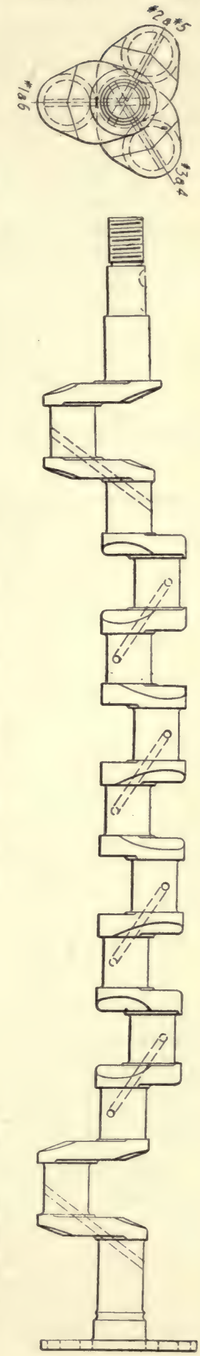

FIG. 413.—Franklin crank shaft.

TABLE 95

d_S	b	c	B
6	3	9	7/8
7	3	10 1/4	7/8
8	3	11 1/2	7/8
9	3	13 1/4	7/8
10	3 1/2	14	1
11	3 1/2	15 1/4	1
12	3 1/2	16 1/2	1
13	3 1/2	17 3/4	1
14	4	19	1 1/4
15	4	20 1/4	1 1/4
16	4	21 1/2	1 1/4
17	4	22 3/4	1 1/4
18	4 1/2	24	1 1/4

CHAPTER XXIX

FRAMES

Notation.

P_P = total pressure in line of stroke, including inertia.
P_N = total pressure on guide, normal to line of stroke.
P = force in general, in pounds.
P_X = total maximum pressure on piston due to steam or gas pressure only.
p = maximum unbalanced pressure per square inch in cylinder.
S = stress per square inch in general.
S_B = bending stress.
S_D = direct stress.
M = bending moment.
A = area of section in square inches.
I = moment of inertia.
c = distance from neutral axis to extreme fiber, in inches.
D = diameter of cylinder in inches.
D_S = diameter of cylinder when some standard pressure is assumed (see Par 63, Chap. XII).
d = diameter of bolt; in some cases at root of thread if this is the weakest section.
n = number of bolts involved in a discussion.
μ = coefficient of friction.

197. Stresses in Engine Frames.—While serving the same general purpose, the requirements of engine frames differ widely, depending upon the type of engine and class of service. In large heavy-duty engines, there must be mass to furnish stability and absorb vibration. Cast iron is the material best adapted to such frames. In marine engines, lightness is important, and such frames are often built of steel bars.

For center-crank engines the forces act symmetrically, and frame stresses are comparatively easy to determine. Let Fig. 416 be a diagram of a vertical engine. This is taken as there are no restraining forces due to foundation, acting on the frame proper. The force P_P is the direct force tending to sever the frame. The guide pressure P_N tends

to push the frame from its vertical position; this acts at the distance l_X from any point, producing a bending moment $P_N l_X$ at that point. This may be resisted by the cylinder bolts if a trunk piston is used, by the foundation bolts, by the bolts holding the frame together, and by any section of the frame. The sum of the stress produced by P_P and P_N is the total stress, and its maximum value depends upon the cut-off in a steam engine. Inertia and gravity of course have their effect as may be seen in Chap. XVI.

Neglecting the stress due to screwing up, the stress in the bolts is usually repeated, as they are in tension only. But it usually is reversed in frame sections as may be seen from Par. 166, Chap. XXI.

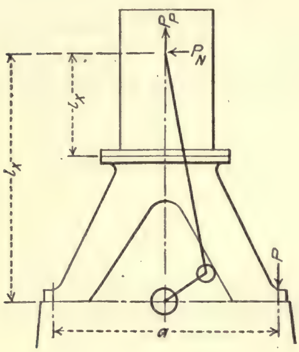

FIG. 416.

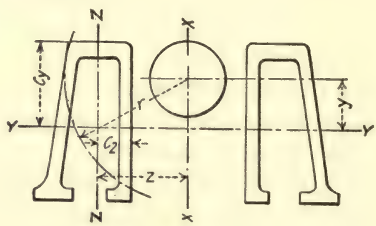

FIG. 417.

An application was given in connection with cylinder bolts in Par. 171, Chap. XXII. Considering the bolts which hold the A-frame to the base in Fig. 416 the tension in the bolts on the right side is:

$$\frac{P_N l_X}{a}.$$

The tension due to P_P is:

$$\frac{P_P}{2}.$$

The total tension in the bolts at the right side is therefore:

$$P = \frac{P_N l_X}{a} + \frac{P_P}{2} \tag{1}$$

Next assume a section of a horizontal, cast iron frame, neglecting the effect of the foundation. This is shown in Fig. 417. The bending moment is due to P_N and is $P_N l_X$ as before. The section is symmetrical

about axis xx but not about axis yy; furthermore, the direct load P_P is applied at a distance y from the neutral axis of the section. This produces a bending moment P_Py. The total bending moment on the section then is:

$$M = P_N l_X + P_P y \tag{2}$$

The bending stress is:

$$S_B = \frac{Mc}{I}.$$

The direct stress is:

$$S_D = \frac{P_P}{A}.$$

Or, the total stress is:

$$S = \frac{Mc}{I} + \frac{P_P}{A}. \tag{3}$$

From Par. 166, Chap. XXI, the factor of safety for single-acting engines, for a cast iron frame is 6; for double-acting steam engines it is 12, and for double-acting gas engines 10.8. A factor of judgment may be used if desired. Frames symmetrical about one axis may thus be checked with a reasonable amount of satisfaction.

Side-crank Frames.—Assume a side-crank engine with a section like one-half of Fig. 417. There is now added a bending moment $P_N z$ to be resisted by the modulus of section about axis zz. The maximum stress would probably be at the upper right-hand corner of the section. The bending moment about axis yy is given by (2) and may be denoted by M_Y; about axis zz the bending moment is:

$$M_Z = P_P z \tag{4}$$

The total stress is then:

$$S = \frac{M_Y c_Y}{I_Y} + \frac{M_Z c_Z}{I_Z} + \frac{P_P}{A} \tag{5}$$

With the form of section sometimes used, and with the load applied eccentrically about two axes, it is questionable whether the stress relations given by (5) hold with great accuracy.

Besides the bending and direct stresses in a side-crank frame, there is a torsional stress produced by P_N. It is absolutely useless to attempt even an approximation to finding the torsional stress in such a section. An ample factor of judgment must be used to allow for this, possibly as high as 2.

Still further considering the forces acting on a side-crank frame,

Fig. 418 shows a diagram in plan. The notation is all contained in the diagram and applies only to it. Equating moments:

$$Pa = P_1 b$$

or,

$$P_1 = \frac{Pa}{b}$$

Equating couples:

$$P_1 l_1 = P_2 l_2.$$

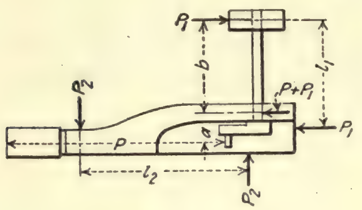

Fig. 418

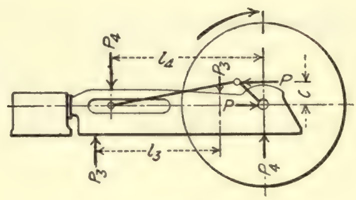

Fig. 419.

These may be considered as resultant couples, the actual forces being reactions at the foundation bolts, and friction. They are indeterminate if the frame is attached to the foundation at more than two points, which is always the case. The bearing jaw receives the force $P + P_1$, but tension in the frame is only P when P_1 is applied as shown. If P_1 were applied near the cylinder, the tension in the frame would be $P + P_1$.

Figure 419 is a side elevation of the same engine.

Equating couples:

$$Pc = P_3 l_3.$$

Or, as $Pc = P_4 l_4$ also

$$P_4 l_4 = P_3 l_3.$$

Force P_3 may be supplied by weight of engine and by downward pull of the foundation bolts; it is also indeterminate if the frame is held to the foundation at more than two points. If a belt wheel or gears are used, other couples may be introduced, increasing the resultant couple $P_3 l_3$.

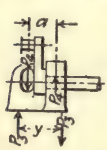

Fig. 420.

Figure 420 is an end elevation. Equating couples:

$$P_4 a = P_3 y.$$

The couple $P_4 a$ produces combined bending and twisting, the amount depending upon the manner of holding the frame to the foundation, and the rigidity of the latter.

In Figs. 418 to 420, P and P_4 are forces acting within the engine, due to the resistance of the engine shaft to turning; forces P_1, P_2 and P_3 are

reactions on the frame by the foundation. If transmission is by belt, ropes or gears, other reactions occur and other couples are formed.

A consideration of the foregoing will show how impossible it is to

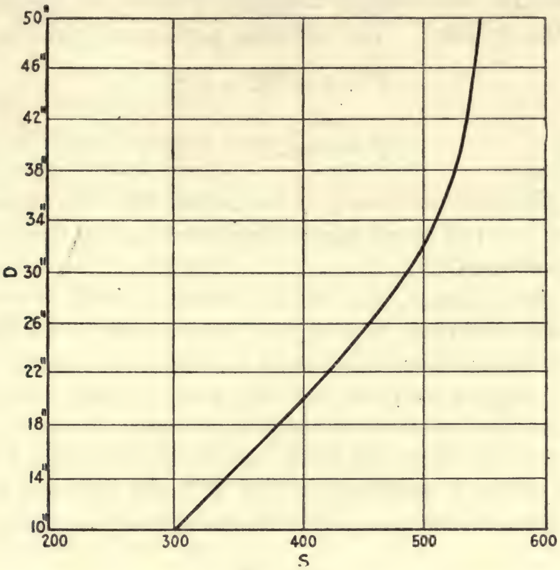

FIG. 421.

analyze stresses in frames of this type with any degree of accuracy. For this reason but little analysis is usually attempted, a common method being to divide the maximum total pressure by a low stress to determine the required area. A factor of judgment of from 2.5 to 5 is sometimes used with this method, the larger values being for the smaller engines. The chart of Fig. 421 was constructed partly from data found in Kent's Mechanical Engineers' Pocket-book and credited to F. A. Halsey. The values of S are not strictly the stress, but merely a factor of design. It may be called the nominal stress. This chart was used by the author in Corliss engine design for several years. It of course applies to the smallest section subject to the complex loading. For the neck of the frame carrying the flange which connects with the cylinder, the applied force is more direct and a higher stress may be used. A factor of judgment of from 1.25 to 1.5 may be used with the factors obtained from Par. 166, Chap. XXI.

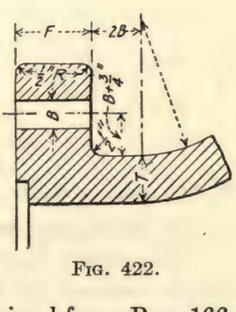

FIG. 422.

Ribs on flanges are objectionable, and it is better to increase the thickness of a flange and leave it plain. Figure 422 gives a flange and neck design which has been satisfactory on Corliss engines. The following empirical formulas were used, B being the diameter of the stud and D the diameter of the cylinder. The notation applies only to Fig. 422.

$$F = 0.075D + 0.75 \tag{6}$$

and

$$T = 0.045D + 0.85 \tag{7}$$

Formula (7) gives stresses nearly as low as Fig. 421 for a steam pressure of 125 lb. per sq. in.; but much higher pressures may be used, the stress increasing proportionally.

In heavy-duty frames, the mean stress in a section of the frame between the guides and main bearing may be made much less than given by Fig. 421, especially for the style shown in Figs. 426 and 430. The weakest portion is the top wall and the side wall next the engine center line, and these should be somewhat thicker. It may be well to include in the area assumed to carry the load, only the area inclosed within a certain circle of radius r, as shown in Fig. 417, the choosing of this radius to be based upon experience. Then the necessary area would be:

$$A = \frac{P_x}{S} \tag{8}$$

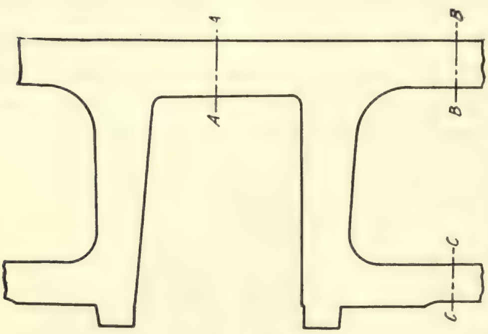

Fig. 423.

To sum up, this method is not very scientific, but it is probable that many engines are successfully running today with frames which did not receive even the analysis given here. It perhaps is a little more satisfactory if results are checked by the combined direct and bending stress given by (5). In this the twisting moment is neglected and a factor of judgment must be allowed; if this is as high as 2, the check may be assumed satisfactory.

Locomotive frames are made of steel castings in this country, and the simple formula (8) is used in determining main dimensions. As already stated, S cannot be considered as the actual stress, nor is it as applied to locomotive frames. A portion of a frame is shown in Fig. 423.

Through AA, $$A = \frac{P_X}{2500} \text{ to } \frac{P_X}{2700}.$$

Through BB, $$A = \frac{P_X}{3000} \text{ to } \frac{P_X}{3200}.$$

Through CC, $$A = \frac{P_X}{4300} \text{ to } \frac{P_X}{4500}.$$

198. Main Bearings.—General bearing dimensions are discussed in Chap. XI and Chap. XXVIII. There is comparatively little which permits calculation in the main bearing of an engine, but a simple discussion will be given of a few items.

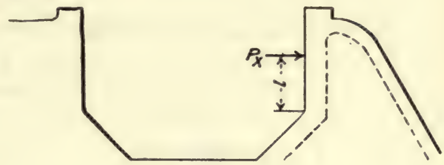

Fig. 424.

The general construction may be seen from some illustrations given in the following paragraph.

Bearing Jaw.—The jaw may be considered as a cantilever with a maximum load P_X acting at the center of the shaft, and a moment arm l as shown in Fig. 424; the arm l must be taken so as to give a maximum stress. The modulus of section is I/c, c being taken to the inside, or tension side of the jaw. Then:

$$P_X l = \frac{SI}{c}$$

or,

$$S = \frac{P_X lc}{I} \tag{9}$$

A strong cap is provided which no doubt adds greatly to the strength of the jaw, but it is well to neglect it in the check.

Wedge Adjusting Bolts.—The hook bolt is sometimes used, and a simple analysis will be given. A wedge and bolt are shown in Fig. 425. No engine adjustments should be made while the engine is running, so it will be assumed that the bolt has only to hold the wedge in place.

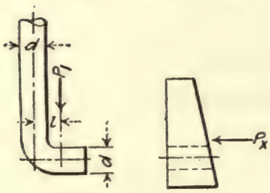

Fig. 425.

Let k be the taper of the wedge in inches per foot, μ the coefficient of friction and n the number of bolts holding the wedge. Also let S_B be the bending stress and S_D the direct stress. Other notation is on Fig. 425.

The maximum total load on the bolt is:

$$P_1 = \frac{kP_X}{12n} - \frac{2\mu P_X}{n} = \frac{P_X}{n}\left(\frac{k}{12} - 2\mu\right) \tag{10}$$

This is not strictly accurate as one of the two pairs of friction surfaces is slanting. The bending moment caused by P_1 is:

$$M = P_1 l = \frac{\pi d^3 S_B}{32}.$$

Let $l = qd$; then:

$$S_B = \frac{32 q P_1}{\pi d^2}.$$

The direct stress on the bolt is:

$$S_D = \frac{4P_1}{\pi d^2}.$$

Then:

$$S = S_B + S_D = \frac{4P_1}{\pi d^2}(8q + 1) = \frac{4P_X}{\pi n d^2}\left(\frac{k}{12} - 2\mu\right)(8q + 1) = \frac{KP_X}{nd^2} \quad (11)$$

Then:

$$d = \sqrt{\frac{KP_X}{nS}} \quad (12)$$

For average conditions it may safely be assumed that: $q = 0.75$, $k = 1.5$ and $\mu = 0.036$; then $K = 0.4725$.
Also

$$P_X = \frac{\pi D^2 p}{4}.$$

It may further be assumed that $n = 2$. Then for ordinary use:

$$d = 0.43D\sqrt{\frac{p}{S}} \quad (13)$$

Assuming a standard pressure of 125 lb. as has been done in other parts of the book:

$$d = \frac{4.93D_S}{\sqrt{S}} \quad (14)$$

If $S = 7000$, $d = 0.059D_S$.
If $S = 10{,}000$, $d = 0.0493D_S$.

For internal-combustion engines, if $p = 400$:

$$d = \frac{8.6D_S}{\sqrt{S}} \quad (15)$$

This would give excessive sizes, and no doubt a T-head or threaded bolt would be used. For these types, $q = 0$; then $K = 0.0675$. Then if $n = 2$:

$$d = 0.163D\sqrt{\frac{p}{S}} \quad (16)$$

Then if $p = 400$ and $S = 7000$:

$$d = 0.039D_S.$$

If $S = 10{,}000$:

$$d = 0.0326D_S.$$

When direct load only is applied, d is the diameter at root of thread.

In some bearings the wedge bolts are in compression, but they are usually so short that a strut formula is unnecessary.

Cap Bolts.—In vertical engines, and when inclined bearings are used, a portion of the piston thrust is carried by the bolts. In automobile engines with the cap on the bottom of the bearing, the entire piston thrust is taken by the bolts as a repeated load. In the common form of horizontal engine bearings, no load is theoretically applied to the cap bolts. The simplest way to dispose of the matter is to assume that they carry the entire piston thrust. This gives good results in steam engine practice. If the bolts are unnecessarily large for gas engines, a factor of judgment less than unity may be employed, or the diameter may be arbitrarily chosen by experienced designers.

Using the same notation as for the wedge bolts:

$$\frac{\pi d^2 nS}{4} = \frac{\pi D^2 p}{4}.$$

From which:

$$d = D\sqrt{\frac{p}{nS}} \tag{17}$$

In this case d is the diameter at root of thread.

In some engines $n = 2$, but more commonly 4 bolts are used.

For steam engines, if $p = 125$, $n = 2$ and $S = 7000$:

$$d = 0.095D_S.$$

If $n = 2$ and $S = 10{,}000$:

$$d = 0.079D_S.$$

If $n = 4$ and $S = 7000$:

$$d = 0.067D_S.$$

If $n = 4$ and $S = 10{,}000$:

$$d = 0.056D_S.$$

For internal-combustion engines, if $p = 400$, $n = 4$ and $S = 10{,}000$:

$$d = 0.1D_S.$$

For automobile engines, if $p = 400$, $n = 4$ and $S_E = 60{,}000$ and $f = 5$, then $S = 12{,}000$ and:

$$d = 0.092D_S.$$

199. Designs from Practice.—The general proportions of frames may be seen from builders' catalogues, and books which are largely descriptive. A few examples of bearing design are given in this paragraph.

Figure 426 is the bearing built by the Bass Foundry and Machine Co., Ft. Wayne, Ind. There is a double wedge adjustment and hook bolts

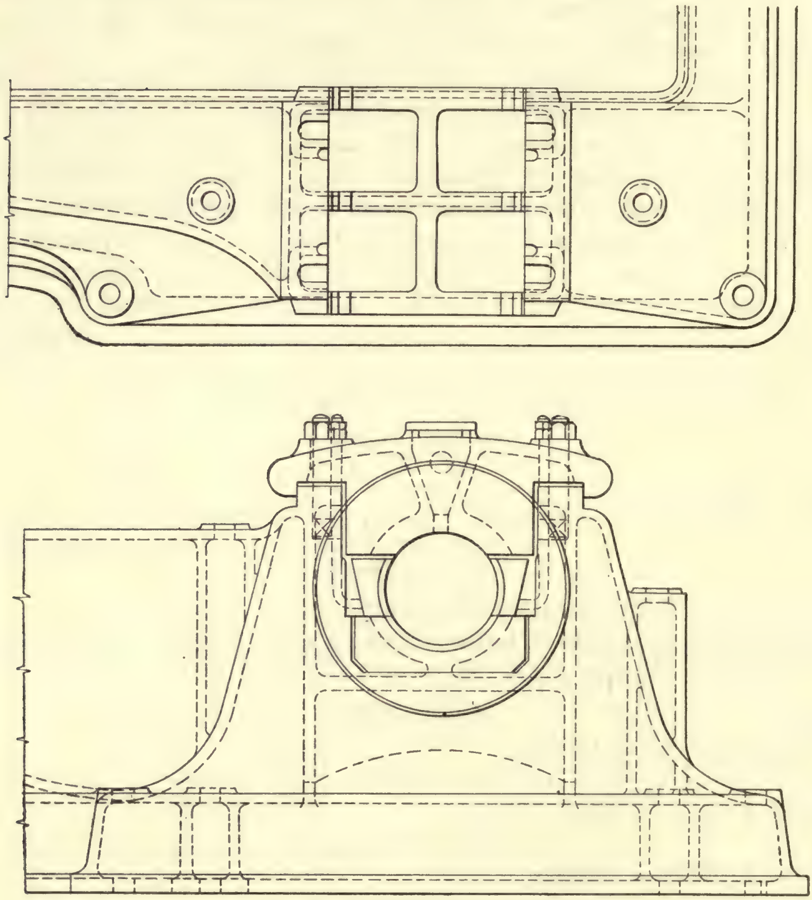

FIG. 426.—Bass-Corliss main bearing.

are used for wedges. T-head bolts, set in pockets are used for cap bolts, and it is claimed that these are less apt to break than studs screwed into the frame. The top box is not babbitted unless desired by the purchaser.

Figure 427 is the main bearing used on the Lentz poppet-valve engine,

built by the Erie City Iron Works. It is a chain-oiled bearing with double wedge adjustment.

Figure 428 shows a bearing used on the horizontal Diesel engine built

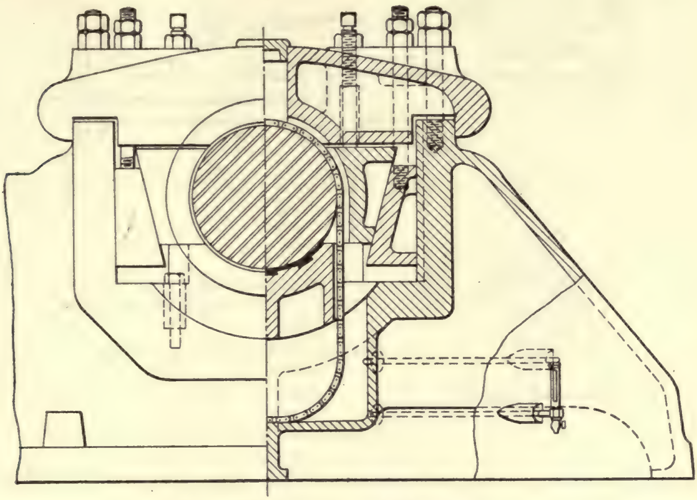

Fig. 427.—Lentz main bearing.

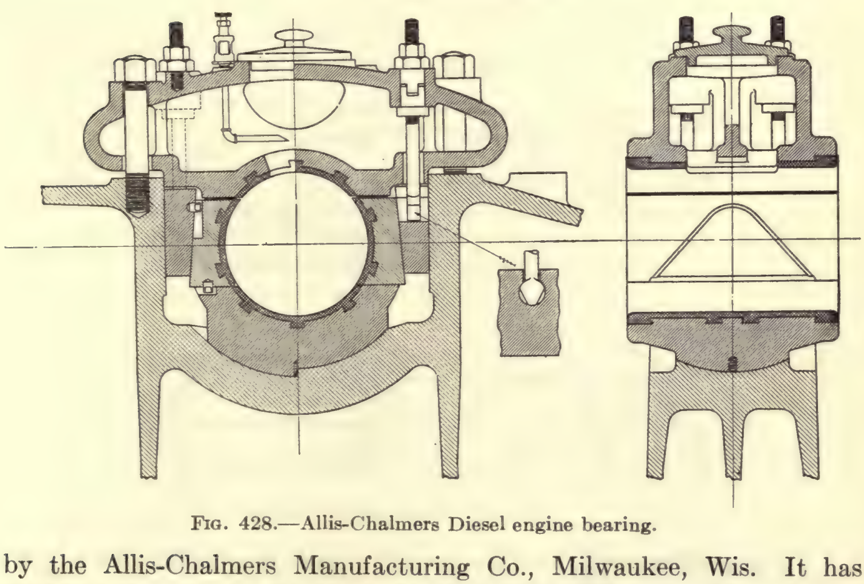

Fig. 428.—Allis-Chalmers Diesel engine bearing.

by the Allis-Chalmers Manufacturing Co., Milwaukee, Wis. It has double wedge adjustment, the wedges being forced downward by a special design of wedge bolts.

In most of the designs shown, the bottom box may be rotated about the shaft for removal without disturbing the shaft further than to jack

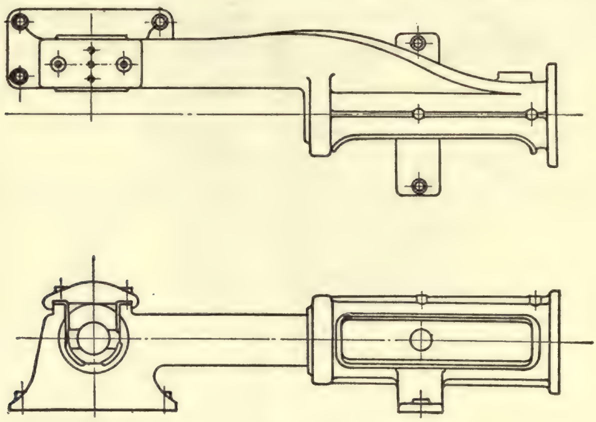

Fig. 429.—Girder frame.

it up slightly. With the exception of Fig. 426, there is no provision for changing the position of the shaft center, the wedges being only to take up wear. In Fig. 426, a certain amount of adjustment of the shaft center may be made (the line showing this was omitted in the drawing). In some bearings a vertical adjustment is also provided.

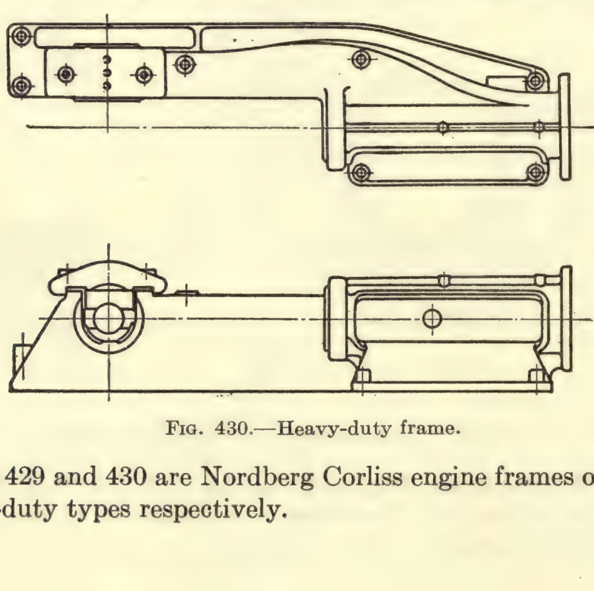

Fig. 430.—Heavy-duty frame.

Figures 429 and 430 are Nordberg Corliss engine frames of the girder and heavy-duty types respectively.

CHAPTER XXX

FLYWHEELS

200. Introduction.—Chapter XVIII was devoted to the determination of the weight of wheel required to keep speed or displacement within fixed limits. In this chapter the various straining actions will be discussed with a view to determining proper dimensions to make the wheel safe against rupture.

Notation.

P = force in pounds applied at radius R, producing bending moment in arms.

P_T = maximum turning effort in pounds.

P_F = sum of components of force F_2, normal to hub division.

P_R = resultant of forces acting on shear hub bolts.

w = weight of material in pounds per cubic inch.

W_R = weight of rim in pounds.

C = centrifugal force of rim in pounds.

T = tension in rim in pounds.

F_1 = tension in one arm at rim, in pounds.

F_2 = tension in one arm at hub, in pounds.

S = stress in general in pounds per square inch.

S_T = stress due to tension in rim.

S_{F1} = stress in arm at rim due to centrifugal force.

S_{F2} = stress in arm at hub due to centrifugal force.

S_{BR} = bending stress in rim.

S_{BA} = bending stress in arm at hub

S_R = combined stress in rim.

S_A = combined stress in arm at hub.

S_1 = tensile stress in link, or in hub bolts; or in rim bolts at root of thread.

S_S = shearing stress in shear hub bolts.

S_C = compressive stress.

T_D = force in tons per inch of diameter required to complete forced fit.

T_{DL} = same as T_D, but per inch of diameter per inch of length.
R = radius in feet, of center of rim.
R_O = radius in feet, of outside of rim.
D = outside diameter of wheel in feet ($= 2R_O$).
r_1 = radius in feet, of inside of rim.
r_2 = radius in feet, of outside of hub.
L = length of arm in feet.
a = length of crank in feet.
h_R and b_R = depth and breadth of rim section in inches, respectively.
h_{A1} and b_{A1} = depth and breadth of arm at rim, in inches.
h_{A2} and b_{A2} = depth and breadth of arm at hub, in inches.
y = height of crown in inches.
d = diameter of hub in inches.
l = effective length of hub in inches, omitting central core.
d_T = diameter of rim or hub bolts at root of thread.
d_S = diameter of hub shear bolts.
l_S = minimum length of shear bolt in inches, taking total force.
m = mean depth of key seat in inches.
A_R = area of rim section in square inches.
A_A = mean area of arm section in square inches.
A_{A1} = area of arm section at rim.
A_{A2} = area of arm section at hub.
A_{A3} = area of arm section midway between rim and hub.
A_T = area of total link section at one joint, in square inches, or of rim or hub bolts at root of thread.
A_C = total link area at one joint taking compression.
z_R = modulus of section of rim.
z_{A1} = modulus of section of arm at rim.
z_{A2} = modulus of section of arm at hub.
I = moment of inertia of section.
c = distance of extreme fiber from neutral axis.
M_A = bending moment on one arm.
N = number of arms in wheel.
N_V = virtual number of arms assumed to carry entire load.
n = number of hub bolts or rim bolts at one joint.
V = velocity in feet per minute.
t = time in seconds required to bring wheel to rest with a constant force P applied at rim center.
k_R = ratio of width to depth of rim.
k_A = ratio of width to depth of arm
μ = coefficient of friction.

201. Hoop Stress.—It is shown in works on mechanics that unit radial pressure applied to an indefinitely thin circular ring has the same effect on the section of the ring as the same unit pressure acting normal to a diameter. Then if the total centrifugal force of such a ring with a radius of R ft. is C, the unit force is:

$$\frac{C}{2\pi R}.$$

The force normal to a diameter tending to rupture the ring is then:

$$\frac{C}{2\pi R}\cdot 2R = \frac{C}{\pi}.$$

The hoop tension is one-half of this, or:

$$T = \frac{C}{2\pi} \qquad (1)$$

and if the area of the section is A_R, the hoop stress is:

$$S_T = \frac{T}{A_R} = \frac{C}{2\pi A_R} \qquad (2)$$

If V is the velocity of the hoop in ft. per min.:

$$C = \frac{WV^2}{3600gR} = \frac{\pi w A_R V^2}{150g} \qquad (3)$$

Then from (2):

$$S_T = \frac{wV^2}{9650} \qquad (4)$$

Flywheels are usually of cast iron, for which $w = 0.26$; then:

$$S_T = \frac{V^2}{37{,}000} \qquad (5)$$

From this the stress may be determined or the allowable rim velocity found.

An old rule for the maximum velocity of cast iron flywheel rims is "a mile a minute," and this is pretty generally followed today, although 6000 feet per min. is sometimes allowed. If V is 5280, (5) gives: $S_T =$ 752 lb. This is a low stress, but in case of overspeeding from any cause, (5) shows that the stress for a given wheel increases as the square of the rotative speed, and the factor of safety should properly be based upon the speed. The velocity V is usually taken at the outside of the rim, but may be taken at the center of depth, at radius R.

The foregoing formulas apply only to very thin rings, but are used for flywheel rims of the usual radial thickness, assuming a uniform stress distributed over the section. That the error increases greatly as the ratio of the rim thickness to wheel diameter increases may be seen in the following chapter, but it may be safely neglected in flywheel design.

The formulas apply also to unrestrained rings which maintain a circular form at all speeds. This is not the condition of the flywheel, the arms modifying the stress relations to a considerable extent. Prof. Unwin, in his Machine Design, gives a rather elaborate analysis of the stresses produced by simple rotation, taking account of the influence of the arms, upon the assumption that the arms extend from the wheel center to the center of the rim section.

A number of approximate analyses have been given with a view to simplification; they usually depend upon some constant which is determined by guess, or by comparison with Unwin's analysis for a given condition.

Several years ago the author substituted more readily usable terms in Unwin's formulas, making tables of the more unwieldy quantities, and taking variable angles at values which give maximum stresses. Then a factor was introduced to make some allowance for actual arm length upon the supposition that relatively short arms stretch less and produce higher stresses. With these changes, the Unwin formulas are more easily applied than most of the approximate methods, with the probable advantage of greater accuracy. These formulas are given in the next paragraph. No allowance for bending of the arms is included in this analysis, but this will be treated in paragraphs which follow.

202. Unwin's Formulas.—All values in Unwin's analysis depend upon the arm tension, and this will be given first; as determined by the assumptions made:

$$F = \frac{wA_AA_RV^2}{2275(NA_A + 2\pi A_R)} \tag{6}$$

This is Unwin's value, and is at the outer, or rim end of the arm. In this formula, w is the weight per cu. in., A_A the mean arm area and A_R the area of rim section—both in sq. in. N is the number of arms, equally spaced, and V the rim velocity in ft. per min. Let subscript 1 refer to the under side of the rim at radius r_1, and 2 to the outside of the rim at radius r_2; let 3 refer to a section midway between, at a radius:

$$\frac{r_1 + r_2}{2}.$$

These radii are all in feet. Then the mean area of the arm is, very closely:

$$A_A = \frac{A_{A1} + A_{A2} + A_{A3}}{3} \tag{7}$$

An arbitrary modification may now be made by multiplying (6) by a factor:

$$\sqrt{\frac{R}{L}}$$

where R is the radius of the wheel in feet (to center of rim, according to Unwin), and L is the arm length in feet (from hub to rim). Then where the arm joins the rim:

$$F_1 = \frac{wA_AA_RV^2}{2275(NA_A + 2\pi A_R)}\sqrt{\frac{R}{L}} \tag{8}$$

The stress at this point is:

$$S_{F1} = \frac{F_1}{A_{A1}} \tag{9}$$

The tension at the hub due to centrifugal force of an arm, assuming the center of gravity at the center of the arm length, is:

$$F_A = \frac{wA_ALV^2}{9650R^2} \cdot \frac{r_1 + r_2}{2}.$$

The total tension at the hub is:

$$F_2 = F_1 + F_A = \frac{wA_AV^2}{2275}\left[\frac{A_R}{NA_A + 2\pi A_R}\sqrt{\frac{R}{L}} + \frac{L(r_1 + r_2)}{8.5R}\right] \tag{10}$$

The stress at the hub is:

$$S_{F2} = \frac{F_2}{A_{A2}} \tag{11}$$

The maximum hoop tension in the rim is at the arm, and is:

$$T = \frac{wA_RV^2}{9650} - \frac{F_1}{2}\cot\frac{180}{N} \tag{12}$$

The hoop stress is:

$$S_T = \frac{T}{A_R} \tag{13}$$

The values of $\cot\frac{180}{N}$ are given in Table 96 for different number of arms.

TABLE 96

N	4	6	8	10	12
$\cot\frac{180}{N}$	1.000	1.732	2.414	3.078	3.732

The maximum bending stress in the rim is at the arm, on the under side, and is:

$$S_{BR} = \frac{6F_1R}{z_R}\left[\frac{N}{\pi} - \cot\frac{180}{N}\right] \tag{14}$$

where z_R is the modulus of section of the rim. The values of the quantities in brackets are given in Table 97.

TABLE 97

N	4	6	8	10	12
$\frac{N}{\pi} - \cot\frac{180}{N}$	0.273	0.177	0.132	0.105	0.087

By comparing (4) with (12) and (13) it may be seen that the maximum hoop stress is reduced by the arms. However, the bending stress must be added to obtain the maximum rim stress; or,

$$S_R = S_T + S_{BR} \tag{15}$$

In like manner the maximum arm stress is given by:

$$S_A = S_{F2} + S_{BA} \tag{16}$$

S_{BA} is the bending stress in the arm at the hub and is given by (27), to be derived later.

Should it be desired to use Unwin's equations as originally given, the factor $\sqrt{R/L}$ may be omitted in (8) and (10).

203. Forces Producing Bending Stresses in Arms.—The maximum bending moment on the arms will either be due to the maximum turning effort on the crank, or to the inertia of the wheel when the crank is on dead center. It is not likely that these will work together, as they would tend to offset each other. This combined effect might be felt in the teeth of a gear on the shaft, or on the shaft itself if the wheel is between the crank and the gear.

Let P_T be the maximum turning effort on the crank in lb. and a the crank radius in feet. The maximum force applied at radius R_o is:

$$P = \frac{P_T a}{R_o} \tag{17}$$

If a heavy rim is brought to rest in t seconds by an assumed uniform force P applied at radius R, the force required is:

$$P = \frac{W_R V}{60gt} = \frac{W_R V}{1930t} \tag{18}$$

The value from (17) is caused by belt or ropes and acts in a direction opposite to rotation; while the force in (18) is overcoming a resistance to gear or generator and acts in the direction of rotation. In (17), R_o is the outside radius of the wheel. In (18), V may be taken as the velocity at radius R.

The results of Formulas (17) and (18) depend upon the value assumed for t, and the number of arms assumed to carry the load. The value of t may be taken as about 3, although this is not based upon any actual tests. It checks fairly well with heavy wheels used for rolling mills and elec-

trical machinery. The number of effective arms is discussed in the next paragraph.

204. Number of Arms Carrying Load.—There have been various ideas as to the maximum load carried by one arm, or virtually, the number of arms assumed to carry the entire load. It is certain there is no exact solution to the problem. The nominal stresses allowed by the Lewis gear tooth formula are high. In using this formula for large cast gears some years ago, the author devised a formula for allowable stress in which two different constants were used, one for the arms being 25 per cent. greater than that for the teeth. For use with these formulas a method of determining the number of arms was devised, assuming that the arm opposite the tooth engaged was idle, and the load taken by the other arms increasing in direct proportion as they approached the teeth which were in mesh. Without taking space for the derivation, the virtual number of arms taking the entire load, according to this assumption is:

$$N_V = \frac{N}{4} + \frac{1}{2} \tag{19}$$

This may be used for gears when the allowable stress in arms and teeth is nearly the same for the same material. Some authorities assume the rim so stiff that the load is distributed equally among the arms. The actual condition is no doubt somewhere between these extremes.

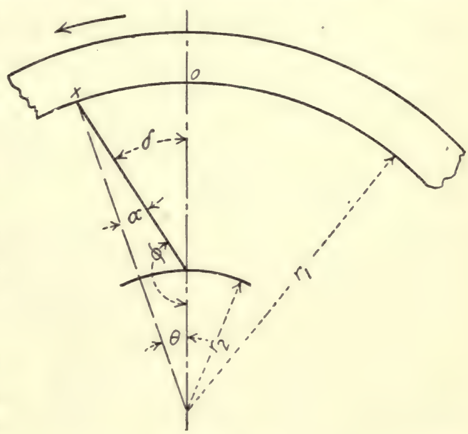

Fig. 431.

Formula (19) was modified for flywheels as follows:

(1) For belt or rope wheels when the value given by (17) is greater than that of (18), multiply (19) by 2.

(2) For belt or rope wheels when (17) gives a smaller value than (18), multiply (19) by 2.5.

(3) For flywheels used for regulation only, $N_V = N - 1$. Then use (18).

These rules must be used with judgment, as it is not claimed that they are applicable to all cases.

205. Bending Moment in Arms.—Assume an arm hinged at the under side of the rim at radius r_1; and at the outside of the hub at radius r_2 as shown by diagram in Fig. 431. If the rim revolves while the hub remains

stationary, so that the rim end of the arm moves through a very small angle δ from its original position relative to the hub, it moves through the angle α from its original position relative to the rim. From Fig. 431:

$$\alpha + \theta + \phi = 180 = \delta + \phi$$

or,

$$\alpha + \theta = \delta$$

and

$$\phi = 180 - (\alpha + \theta).$$

From the properties of triangles:

$$\frac{r_2}{\sin \alpha} = \frac{r_1}{\sin \phi} = \frac{r_1}{\sin [180 - (\alpha + \theta)]} = \frac{r_1}{\sin (\alpha + \theta)} = \frac{r_1}{\sin \delta}.$$

For the very small angles involved:

$$\frac{\alpha}{\delta} = \frac{\sin \alpha}{\sin \delta} = \frac{r_2}{r_1} \tag{20}$$

If the rim and hub are rigid, the bending moments at the two ends of the arm are proportional to these angles; then for equal bending stress at the two ends:

$$\frac{z_{A1}}{z_{A2}} = \frac{\alpha}{\delta} = \frac{r_2}{r_1} \tag{21}$$

where z_A is the modulus of section of the arm. If the arm sections at the two ends are similar and h_A is the depth of section in inches in the direction of motion:

$$\frac{h_{A2}}{h_{A1}} = \sqrt[3]{\frac{r_1}{r_2}} \tag{22}$$

Table 98 shows this relation.

TABLE 98

r_1/r_2	1.500	1.750	2.000	2.500	3.000	4.000	5.000	6.000
h_{A2}/h_{A1}	1.145	1.205	1.260	1.358	1.442	1.587	1.710	1.820

A rule sometimes used for wheel arms is to make the section modulus at the hub twice that at the rim, and this usually gives a well-appearing design. With these proportions, when bending only is considered and when r_1/r_2 is greater than 2—which is nearly always the case—the rim end is stronger than the hub end. It is always safe to take z_{A2}/z_{A1} as 1.5, the minimum value of Table 98, and this gives a good appearance.

It is obvious that there is a point of contra-flexure at which the bending moment is zero, and that this moves out toward the rim as the ratio α/δ, and therefore r_2/r_1 is made smaller. The fact that the rim is not rigid, and that the arm is more flexible toward the rim due to the reduced

depth, makes it apparent that the point of zero bending moment is still nearer the rim than (20) indicates. It is therefore obviously safe to assume the arm as a cantilever loaded at the end, which may be considered as at the inside of the rim at radius r_1. The modulus of section may be taken at the radius r_2, which is at the outside of the hub. The moment arm may be taken to the rim center for (18) and to the outside for (17). The bending moment, taking P from (17) or (18) is then:

$$M_A = 12(R - r_2)\frac{P}{N_V} \tag{23}$$

The maximum value of P/N_V must be used. For a heavy rim with belt try both (17) and (18). For heavy-rimmed balance wheel (18) is used. The values of P and N_V may be found from Pars. 203 and 204 respectively, and R in (23) taken as R, R_O or r_1 as desired.

For the arm at the hub:

$$S_{BA}z_{A2} = M_A \tag{24}$$

From this either S_{BA} or z_{A2} may be determined. Values of z_{A2} will be given in Par. 207.

206. Working Stresses.—A flywheel may be considered as subject to repeated loads which never have as great a range as from zero to full load; or if reversed, the percentage of maximum load is small, and cast iron is much stronger in compression as explained in Par. 159, Chap. XXI. Safety decreases, however, as speed increases, so if a repeat factor is taken at a comparatively low speed, the stress at this speed may be considered the maximum limit. The factor may then be increased, or the allowable stress decreased gradually up to the maximum allowable speed.

For cast iron, which is most commonly used, let the minimum factor be 5; if the ultimate strength is taken as 16,000 lb., this makes S equal to 3200. Let the velocity at stress 3200 be 1000 ft. per min. and assume the stress to vary as the cube root of the velocity inversely.
Then:

$$S = \frac{32{,}000}{\sqrt[3]{V}} \tag{25}$$

This is an entirely empirical formula and may be taken as the maximum limit of allowable stress in rim or arms for velocities at either center or outside of rim from 1000 to 6000 ft. per min. where the materia is cast iron. Subtracting from this the hoop stress given by (13), or approximately and safely by (5), leaves the allowable bending stress in the rim which limits the value found by (14). Subtracting the value given by (11) from that of (25) gives the allowable bending stress in the arm near the hub.

The rim section is determined by consideration of weight, size of belt,

or the size and number of ropes. In determining arm dimensions there are various empirical formulas, but they are not suitable for general application. If a number of assumptions are made in (10), a simple formula may be derived which will assist in determining arm dimensions, and if desired, these may be more carefully checked. Then let $A_A/A_R = 0.25$, $R/L = 1.25$, $L = 0.8R$ and $r_1 + r_2 = 1.125R$. If $z_{A2} = 2z_{A1}$ for similar sections, $A_{A2} = 1.29A_A$. For cast iron, $w = 0.26$.
Then:

$$S_{F_2} = \frac{V^2}{87{,}500}\left[\frac{45}{N + 25} + 1\right] \tag{26}$$

and

$$S_{BA} = S - S_{F2} \tag{27}$$

To facilitate calculation Table 99 is given.

TABLE 99

V	S	S_{F_2}			
		$N = 6$	$N = 8$	$N = 10$	$N = 12$
4000	2015	448	435	415	405
4500	1940	570	550	525	510
5000	1870	700	680	650	630
5280	1840	785	760	725	705
5500	1810	850	820	785	765
6000	1760	1050	980	940	910

It is convenient and safe to take V at the outside of the rim in finding S and S_{F2}. It is probably safer not to design wheels for a velocity much less than 4000 ft. per min. even though they are to run slower than this; this is especially applicable to large wheels.

207. Rim and Arm Sections.—The form of rim section is determined by the service required. The faces of belt wheels are crowned as shown in Fig. 437, which is made for four belts of different widths. There is no fixed rule for the height of the crown, although formulas are sometimes given—more usually for small shop pulleys. A formula given in Leutwiler's Machine Design and credited to Mr. C. G. Barth is:

$$y = \frac{b_R^{2/3}}{32} \tag{28}$$

where b_R is width of face and y the height of crown in inches.

Formulas for the ratio of wheel face to belt width are given for small pulleys, but are generally unsatisfactory for large wheels. From ½ to 1½ in. may be allowed, depending upon the size of the wheel and condition of service.

There are a number of designs of grooves used on rope wheels, but they are usually in agreement on the angle of groove, which is 45 degrees. The sides against which the rope bears are usually straight, but sometimes the sides are curved; it is claimed for the latter that the rope is more apt to turn in the groove, causing more even wear and prolonging its life. The multiple system is mostly used for main drives and is therefore of most in-

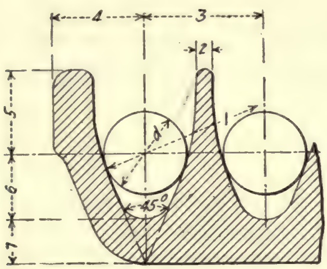

Fig. 432.—Engineers' standard groove.

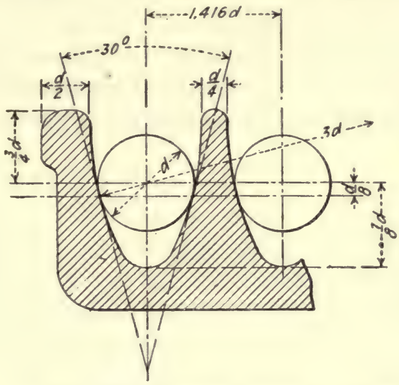

Fig. 433.—Cresson standard groove.

terest to engine designers. Figure 432 shows a groove known as the Engineers' Standard, designed by the Jones and Laughlin Co. Table 100 gives dimensions for ropes of different size.

Table 100

d	1	2	3	4	5	6	7
$\frac{3}{4}$	$2\frac{15}{16}$	$\frac{1}{4}$	$1\frac{3}{8}$	$\frac{15}{16}$	$\frac{3}{4}$	$\frac{5}{8}$	$\frac{3}{8}$
$\frac{7}{8}$	$2\frac{15}{16}$	$\frac{1}{4}$	$1\frac{1}{2}$	$1\frac{1}{16}$	$\frac{7}{8}$	$\frac{3}{4}$	$\frac{7}{16}$
1	$3\frac{7}{8}$	$\frac{3}{8}$	$1\frac{7}{8}$	$1\frac{1}{4}$	1	$\frac{13}{16}$	$\frac{1}{2}$
$1\frac{1}{8}$	$3\frac{7}{8}$	$\frac{3}{8}$	2	$1\frac{3}{8}$	$1\frac{1}{8}$	$\frac{15}{16}$	$\frac{9}{16}$
$1\frac{1}{4}$	4	$\frac{3}{8}$	$2\frac{1}{8}$	$1\frac{1}{2}$	$1\frac{1}{4}$	1	$\frac{5}{8}$
$1\frac{3}{8}$	$3\frac{1}{8}$	$\frac{3}{8}$	$2\frac{1}{8}$	$1\frac{9}{16}$	$1\frac{3}{8}$	$1\frac{1}{8}$	$1\frac{1}{16}$
$1\frac{1}{2}$	$3\frac{5}{16}$	$\frac{3}{8}$	$2\frac{1}{4}$	$1\frac{11}{16}$	$1\frac{1}{2}$	$1\frac{1}{4}$	$\frac{3}{4}$
$1\frac{3}{4}$	$3\frac{3}{8}$	$\frac{3}{8}$	$2\frac{1}{2}$	$1\frac{15}{16}$	$1\frac{3}{4}$	$1\frac{7}{16}$	$\frac{7}{8}$
2	$3\frac{9}{16}$	$\frac{3}{8}$	$2\frac{3}{4}$	$2\frac{1}{8}$	2	$1\frac{5}{8}$	1

In the G. V. Cresson Co. catalogue of 1907, their standard for the American, or continuous system has a groove angle of 45 degrees, while for the English, or multiple system the angle is 30 degrees. The groove for the multiple system is shown in Fig. 433. The ratio of the tension on the tight side to that on the loose side is greater than when the

angle is 45 degrees; the power transmitted would therefore be greater for the same maximum tension. With the curved groove, an old rope, well fitted to the groove would settle deeper in the groove where the angle is greater, while a new rope will ride the groove at a smaller angle, thus tending to divide the work. The author used a groove much like the Cresson standard for a number of years, and so far as he knows it has always been entirely satisfactory, and especially well adapted to heavy work and large overloads.

As the rims of belt and rope wheels are relatively thin, flanges, and sometimes ribs are used to increase strength, as shown in Fig. 437. The modulus of section is for a section similar to a T; in taking:

$$z_R = \frac{I}{c} \tag{29}$$

c must be taken from the neutral axis to the edge of the ribs—toward the wheel center. The section should be taken near the arm, as the maximum bending moment is at this point. The modulus of section of a rope wheel rim may be found by the graphical method of Appendix 1. The section between two rope centers may be taken, the value of z_R being for the sum of these sections.

Heavy flywheels usually have a section so nearly rectangular that the section modulus may be that of a rectangle; or,

$$z_R = \frac{b_R h_R^2}{6} \tag{30}$$

The weight of the rim is determined by one of the methods of Chap. XVIII.

If this is W_R:

$$2\pi \times 12Rb_Rh_Rw = W_R.$$

If $b_R = k_Rh_R$:

$$h_R = \frac{W_R}{24\pi wb_RR} = \sqrt{\frac{W_R}{24\pi wk_RR}} \tag{30a}$$

R may be taken to give a certain velocity; then the outside diameter is:

$$D = 2R + h_R \tag{31}$$

If D is to have a certain value, R may be assumed, a tentative value of h_R found, and adjustment made by trial and error. In using (30a), some standard value of k_R may be taken, Halsey preferring ⅔. To check the weight when adjustments are made:

$$W_R = \pi(12D - h_R)h_Rb_Rw \tag{32}$$

For cast iron wheels, $w = 0.26$, and (30a) becomes:

$$h_R = \frac{1}{4.43}\sqrt{\frac{W_R}{k_R R}} \tag{33}$$

If $k = \frac{2}{3}$:

$$h_R = \frac{1}{3.6}\sqrt{\frac{W_R}{R}} \tag{34}$$

In some very heavy rims, when the diameter is limited by a gear shaft, as was the case with Fig. 436, a more efficient wheel is obtained by making b_R greater than h_R, so long as the latter is large enough to properly accommodate the links.

In determining the thickness of belt wheel rims, the first form of (30a) may be used, b_R having been determined from the required belt width; R may be taken as the outside radius. The value of h_R so found would give less than the required weight, and crowning reduces this still more by reducing the radius to the center of gravity; but the addition of ribs and flanges will probably compensate for this. If greater accuracy is desired, the weight may be carefully calculated after the section is designed. In finding the moment of inertia of the section it is best to neglect the crown, using only the thickness h_R found from (30a), and the ribs and flanges. The depth of rim flanges in inches may be made about $0.3D$, where D is outside diameter in feet. The thickness of the flange may be about 0.2 times the depth of the flange if this is not greater than the rim thickness.

Arm sections are usually approximate ellipses drawn by circular arcs. A theoretically exact section modulus may be found graphically by the method of Appendix 1. It is usually sufficiently accurate to assume an ellipse and this simplifies calculation. It will be assumed in this discussion that:

$$z_{A2} = 1.5 z_{A1} \tag{35}$$

Also let the sections taken along the arm be assumed similar. Then if $b_A = k_A h_A$;

$$z_{A2} = \frac{\pi b_{A2} h^2_{A2}}{32} = \frac{\pi k_A h^3_{A2}}{32} = 1.5\,\frac{\pi k_A h^3_{A1}}{32} \tag{36}$$

From this:

$$h_{A1} = h_{A2}\sqrt[3]{\frac{1}{1.5}} = 0.875 h_{A2} \tag{37}$$

Then:

$$A_{A2} = \frac{\pi k_A h_{A2}^2}{4} = 0.785 k_A h_{A2} \tag{38}$$

And:

$$A_{A1} = \frac{\pi k_A h_{A1}^2}{4} = 0.6 k_A h^2_{A2} \tag{39}$$

Also:

$$A_{A1} = 0.765 A_{A2} \tag{40}$$

To find the mean area:

$$h_{A3} = \frac{h_{A1} + h_{A2}}{2} = 0.9375 h_{A2}$$

and:

$$A_{A3} = 0.737 k_A h^2_{A2}$$

Then from (7), the mean area is:

$$A_A = 0.707 k_A h_{A2}^2 \tag{41}$$

A good arm proportion is when $k_A = 0.5$; then the following special formulas may be obtained:

At hub:

$$z_{A2} = 0.0492 h^3_{A2} \tag{42}$$

At hub:

$$A_{A2} = 0.392 h^2_{A2} \tag{43}$$

At rim:

$$A_{A1} = 0.392 h^2_{A1} \tag{44}$$

The mean area is:

$$A_A = 0.353 h^2_{A2} \tag{45}$$

Equating (42) with the bending moment given by (23) gives:

$$h_{A2} = 2.73 \sqrt[3]{\frac{M_A}{S_{BA}}} \tag{46}$$

S_{BA} may be obtained from (27).

208. Hubs.—The analysis of hubs given in Par. 3, Chap. XXVII may be used for wheel hubs and will not be repeated. The result given by (24) of that chapter, making the outside hub diameter twice the bore, is quite common practice. With this ratio, combining (21) and (22) of Chap. XXVII, gives:

$$T_{DL} = \frac{\mu S}{1250} \tag{47}$$

where T_{DL} is tons per inch of diameter per inch of length required to complete the fit if the hub were forced on, S the stress and μ the coefficient of friction. This formula will be used in connection with the discussion of hub bolts.

If l is the effective length of fit on the shaft and T_D is tons per inch of diameter:

$$T_D = T_{DL} l \tag{48}$$

To simplify equations, especially when l is not well known, T_D is sometimes used; when l is later determined, S may be checked by (47).

209. Hub Bolts.—Let it be assumed that the bolts are to hold the hub to the shaft with a pressure equivalent to a forced fit requiring T_D tons per inch of diameter to complete. Then if d is the diameter of the fit, d_T the diameter of the bolt at root of thread, S_1 the bolt stress and n the number of bolts:

$$\frac{\pi d_T^2 n S_1}{4} = \frac{2000 T_D d}{\pi\mu}.$$

Or:

$$d_T = 28.5\sqrt{\frac{T_D d}{\mu n S_1}} \tag{49}$$

Taking $T_D = 5$, $\mu = 0.25$, $n = 4$ and $S_1 = 10{,}000$:

$$d_T = 0.637\sqrt{d} \tag{50}$$

The bolts may be selected from the bolt table in Appendix 2.

The resultant of the force F_2 acts on the hub bolts and reduces the value of T_D; this resultant is:

$$P_F = F_2 \Sigma \sin\theta \tag{51}$$

where θ is the angle made by the arm with the diameter at right angles with the bolts Table 101 gives values of $\Sigma \sin\theta$.

TABLE 101

N	6	8	10
$\Sigma \sin\theta$	2.000	2.704	3.240

Then in reality the equation is:

$$\frac{\pi d_T^2 n S_1}{4} - P_F = \frac{2000 T_D d}{\pi\mu}$$

The value of T_D is then:

$$T_D = \frac{\pi\mu}{2000d}\left[\frac{\pi d_T^2 n S_1}{4} - P_F\right] \tag{52}$$

Formula (52) may be used as a check, in which T_D must be a positive quantity capable of holding the wheel firmly to the shaft. The original value of T_D used in (49) should be used for determining stress in (47) and (48), as the effect of the arms does not reduce the hub stress.

Hub bolts are usually steel bolts having a nut on each end (commonly called studs).

Shear bolts are sometimes used in large wheels, the hubs being forced on the shaft. Such a construction is shown in Figs. 436 and 437. This style of fastening cannot be designed satisfactorily without drawing to scale, and may require more than one trial. An algebraic solution is clumsy and liable to error; therefore a simi-graphic method is easier and better.

Figure 434 shows a diagram for one arm. The force P/N_V at the rim causes a direct force equally divided among the three bolts, and acting in the same direction, parallel to P. The force F_2 may be obtained from (10) and the resultant of these two forces found graphically as shown. Unless x is much greater than y, the bolt for which the diagram is drawn receives the maximum load. The force P_Y acting on this bolt may be found as follows:

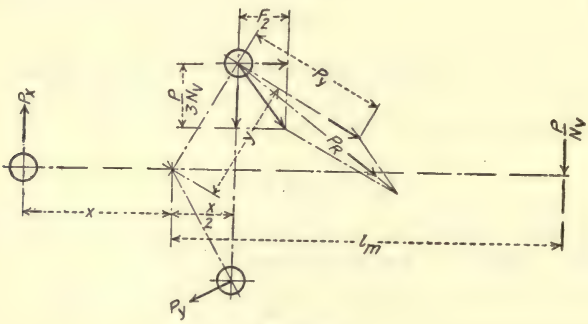

FIG. 434.

Equating moments about the geometrical center of the bolts:

$$P_X x + 2P_Y y = \frac{P}{N_V} \cdot l_M \cdot$$

Also:

$$\frac{P_X}{x} = \frac{P_Y}{y}$$

or:

$$P_X = \frac{P_Y x}{y}.$$

Substituting gives:

$$P_Y = \frac{P l_M}{N_V \left(\frac{x^2}{y} + 2y\right)} \tag{53}$$

This may be combined graphically with the resultant already found,

giving the final resultant P_R. This is resisted by the bolt in double shear. Then:

$$\frac{2\pi d^2{}_S S_S}{4} = P_R$$

Or:

$$d_S = 0.8\sqrt{\frac{P_R}{S_S}} \tag{54}$$

The length of fit in either hub or arm must be:

$$l_S = \frac{P_R}{S_C d_S} \tag{55}$$

This construction requires a driving fit of turned bolts in reamed holes.

As with the arms, the maximum assumed stress is probably seldom reached, so whether the stress is repeated or reversed, it is probably through a small range and of an intensity much less than the maximum. A factor of safety of 3 may be assumed, which, if the elastic limit in shear is taken as 29,000 as in Table 73, Chap. XXI, gives $S_S = 10{,}000$, nearly. S_C may be taken as 12,000, but with the usual construction this will not be reached. Shear bolts are also usually stud bolts.

210. Rim Bolts and Links.—These must hold the rim tension T given by (12), or more simply and safely, by (1); then:

$$A_T = \frac{T}{S_1} \tag{56}$$

where A_T is the total area of the link sections, or the area of bolts at root of thread. It may also be taken as the bearing area of the link head in which case $S_1 = S_C$, the compressive stress.

Neglecting the effect of arms:

$$A_T = \frac{wA_R V^2}{9650 S_1} \tag{57}$$

As the stress distribution in links is probably not uniform, it is safer to use a low stress. If this is taken as 5000 and the rim is cast iron, (57) becomes:

$$A_T = \frac{A_R V^2}{186{,}000{,}000} \tag{58}$$

The link may be proportioned for the maximum speed, or when $V = 5280$, and the dimensions retained for all speeds; then:

$$A_T = 0.15 A_R \tag{58a}$$

With I links, the bearing pressure may be twice as great as the tensile stress just assumed; the area for this is then:

$$A_C = \frac{T}{S_C} = 0.075 A_R \tag{59}$$

It must be remembered that (58) and (59) give the area of both links at any section. In providing for A_C, allowance must be made for clearance in the link pocket, as the projection at the link ends will not bear on the surface nearest the link due to this clearance.

For rim bolts:

$$A_T = \frac{\pi d_T^2 n}{4}$$

where n is the number of bolts and d_T the diameter at root of thread. Then (57) becomes:

$$d_T = \frac{V}{87}\sqrt{\frac{wA_R}{S_1 n}} \qquad (60)$$

For a cast iron wheel with a rim velocity of 5280 ft. per min., if $S_1 = 7000$:

$$d_T = 0.37\sqrt{\frac{A_R}{n}} \qquad (61)$$

If $S_1 = 10{,}000$:

$$d_T = 0.31\sqrt{\frac{A_R}{n}} \qquad (62)$$

Bolts may be selected from Appendix 2. As with the hub bolts, stud bolts are generally used, having a nut at each end.

211. Keys.—Standard keys are often used, a given width of key always being used with a given shaft diameter. While such keys are usually ample they may be much stronger than necessary and weaken the shaft unnecessarily. It is always well to check a key in engine work. With notation already used in this chapter, taking m as the mean depth of the keyway in either shaft or hub, the equation of moments gives:

$$S_C ml \cdot \frac{d}{2} = 12P \cdot \frac{D}{2}.$$

From which:

$$m = \frac{12PD}{S_C ld} \qquad (63)$$

The hub is sometimes cored for a part of its length, through which the key does not bear, therefore l must be taken as the actual bearing length. S_C may be taken as about 10,000 lb.

212. Methods of Construction.—Wheels are commonly made in one of the following ways:

1. *Cast in One Piece.*—Small wheels are made in this way. Sometimes the hub is split to relieve shrinkage strains; after boring to a snug fit it is clamped to the shaft by the hub bolts.

2. *Cast in One Piece and Split Apart.*—This method is sometimes used with small and medium-sized wheels. It cannot be considered very good engineering. Arms sometimes part from the rim when the wheel is split, which leads one to believe that large internal stress may exist.

3. *Cast in Halves with Planed Joints.*—This is a good construction if properly designed and cast, and is used for large wheels. Figure 435 is an example of this design. For belt and rope wheels it is probably better to have the split at the arms, making a double arm. Such wheels are much less apt to be broken in transportation, and are no doubt stronger when in use.

4. *With One or Two Arms Cast with a Segment of the Rim.*—This type usually has the arms fastened to the hub with shear bolts. A design with one arm cast with a segment is shown in Fig. 436.

5. *Built up from Separate Hubs, Arms and Rim Segments.*—Such a wheel is shown in Fig. 437. It is likely that this construction gives a minimum of internal stress when properly machined. It also permits ease of shipment.

A series of oft-quoted experiments were made by Prof. Benjamin on very small wheels of different design to determine their relative strengths, in which the one-piece wheel proved strongest. It does not seem as if this applies to large wheels, and it is very doubtful if the wheel of Fig. 437 would be as strong cast in one piece as it is at present, even though the feat were practicable and shipment possible.

The relative area of arm and rim section should doubtless be influenced by the mode of construction. With a wheel made by method (4), and even more by (5), the arm section may be as much smaller than the rim as desired if computation shows it strong enough. With the other methods this is not so, due to internal stress.

While cast iron is the most common material for wheels, semi-steel and sometimes steel castings are used. If the latter were more common, report of flywheel "explosions" would be more uncommon. Special wheels are built up of steel plates, and some are wound with steel wire.

A wheel was built by the Mesta Machine Co. a few years ago for a rim velocity of 10,000 ft. per min. Neglecting arm influence, the hoop stress from (5) would be 2700 lb. The wheel was cast from air-furnace iron with a tensile strength of 30,000 lb.

213. Application of Formulas.—The weight of a wheel for a 20 by 48 in. Corliss engine was determined in Chap. XVIII. It was to run 100 r.p.m. and the rim weighed 20,500 lb. With an outside diameter of 16 feet, the formulas of Par. 207 gives a section 15 in. deep and 10 in. wide. The latter measurement gives a weight a little in excess, and will be

modified slightly in the finished design as shown in Fig. 435. The shaft bore of 16 in. was determined in Chap. XXVIII, and the outside diameter of the hub is 32 in. The length of hub was taken as 18 in. This might perhaps be 24 in., but 18 will be used. The number of arms will be taken as 8.

From the above data, as many additional data will be taken as possible, and are:

$R = 7.37$; $L = 5.42$; $r_1 = 6.75$; $r_2 = 1.33$; $V = 4620$; $t = 3$.
From (19), Case 3: $N_V = 7$. $A_R = 150$, and from (30), $z_R = 375$.
From (18):

$$P = \frac{20{,}500 \times 4620}{1930 \times 3} = 16{,}400 \text{ lb.}$$

From (23):

$$M_A = \frac{12 \times 6.04 \times 16{,}400}{7} = 170{,}000.$$

From (25):

$$S = \frac{32{,}000}{\sqrt[3]{4620}} = 1820 \text{ lb.}$$

From (26):

$$S_{F2} = 573 \text{ lb.}$$

From (27):

$$S_{BA} = 1247 \text{ lb.}$$

From (46):

$$h_{A2} = 2.73\sqrt[3]{\frac{170{,}000}{1247}} = 14 \text{ in.}$$

From (42):

$$z_{A2} = 135.$$

From (43):

$$A_{A2} = 76.5.$$

From (37):

$$h_{A1} = 12.25 \text{ in.}$$

From (44):

$$A_{A1} = 58.7.$$

From (45):

$$A_A = 69.$$

From (8):

$$F_1 = \frac{0.26 \times 69 \times 150 \times 4620^2}{2275(552 + 940)}\sqrt{\frac{7.37}{5.42}} = 20{,}600 \text{ lb.}$$

From (9):

$$S_{F1} = \frac{20{,}600}{58.7} = 350.$$

From (10):

$$F_2 = \frac{0.26 \times 69 \times 4620^2}{2275}\left[\frac{150}{1492} + \frac{5.42 \times 8.08}{8.5 \times 7.37^2}\right] = 32{,}800 \text{ lb.}$$

From (11), the actual value of S_{F2} is:

$$S_{F2} = \frac{32{,}800}{76.5} = 428 \text{ lb.}$$

From (12):

$$T = \frac{0.26 \times 150 \times 4620^2}{9650} - \frac{20{,}600 \times 2.414}{2} = 86{,}000 - 24{,}900 = 61{,}100 \text{ lb.}$$

From (13):

$$S_T = \frac{61{,}100}{150} = 408 \text{ lb.}$$

From (14):

$$S_{BR} = \frac{6 \times 20{,}600 \times 7.37 \times 0.132}{375} = 320 \text{ lb.}$$

From (24), the actual value of S_{BA} is:

$$S_{BA} = \frac{170{,}000}{135} = 1260 \text{ lb.}$$

This is slightly greater than the assumed value.
From (15), the total rim stress is:

$$S_R = 408 + 320 = 728 \text{ lb.}$$

From (16), the total arm stress is.

$$S_A = 428 + 1260 = 1688 \text{ lb.}$$

This is about 87 per cent. of the allowed stress from (25).

Hub Bolts.

From (50):

$$d_T = 0.637\sqrt{16} = 2.55 \text{ in.}$$

The nearest safe standard bolt diameter from the bolt table of Appendix 2 is 3 in. The actual diameter at root of thread is 2.629 in.

From (51) and Table 101:

$$P_F = 32{,}800 \times 2.704 = 89{,}000 \text{ lb.}$$

From (52) the effective value of T_D is:

$$T_D = \frac{\pi \times 0.25}{2000 \times 16}\left[\frac{\pi \times 2.629^2 \times 4 \times 10{,}000}{4} - 89{,}000\right] = 2.975.$$

This is tons per inch of diameter. The total force is:

$$16 \times 2.975 = 47.5 \text{ tons,}$$

which far exceeds the weight of the wheel, and would therefore hold it if the shaft were vertical.

From (48):

$$T_{DL} = \frac{2.975}{12} = 0.248 \text{ tons per in. of diameter per in. of length.}$$

From (47), the hub stress due to this force is:

$$S = \frac{1250 \times 0.248}{0.25} = 1248 \text{ lb.}$$

It must be assumed that the total hub stress is 4000 lb., the value used in determining hub thickness, the difference being due to P_F.

Rim Links.—Assuming the links to take the entire load, (58) gives:

$$A_T = \frac{150 \times 4620^2}{186{,}000{,}000} = 17.2.$$

The area of one link section is 8.6 sq. in. and the section may be made 2¼ by 3¾ in. As the bearing pressure is twice as great, the head may be 5⅝ in. wide; allowing ¼ in. on each side for clearance, the head width may be 6¼ in. wide. This reduces the rim section by the area:

$$2 \times 6.75 \times 2.25 = 30.3 \text{ sq. in.}$$

The original rim area has been assumed as 150 sq. in., but is really about 135 sq. in. The actual hoop stress is then about 30 per cent. greater than the computed hoop stress, but the bending stress is affected but little as the metal is removed from the center. The bending moment midway between the arms is one-half as great as at the arms. The bending stress may be assumed as about $0.6S_{BR}$, or 190 lb.; the hoop stress may be taken as 30 per cent. greater than S_T, or 530 lb., the sum being 720 lb., which is practically the same as S_R previously found.

If there were two rim bolts, and they were assumed to carry the entire load, their diameter would be 3 in. In this type of wheel the bolts are used for convenience of construction, so they may be made smaller than this, perhaps 2 in. in diameter.

Keys.—It has been assumed that the hub is cored out at the middle third of its length, giving an effective length of 12 in. If a single key is used, (63) gives for the mean depth of keyway:

$$m = \frac{12 \times 16{,}400 \times 16}{10{,}000 \times 12 \times 16} = 1.64 \text{ in.}$$

This may be made 1¾ in. and the width of the key 3½ in. If two keys spaced 90 degrees apart are used, they may be half as large, or, as the

author has usually done, they may be made from $\frac{5}{8}$ to $\frac{3}{4}$ as large—say $1\frac{1}{4}$ deep by $2\frac{1}{2}$ in. wide.

Total Weight.—The weight of the wheel is approximately 32,675 lb. Before this example was computed, Table 62, Chap. XVIII was computed, giving 33,000 lb. as the weight; this is 1.6 times the rim weight.

Figure 435 is a scale drawing of a portion of the wheel, showing construction. The method of gradually enlarging the arm as it joins the rim has been used by the author on many heavy wheels, although not original with him. Holes (not shown) are cored in the rim are for jacking the

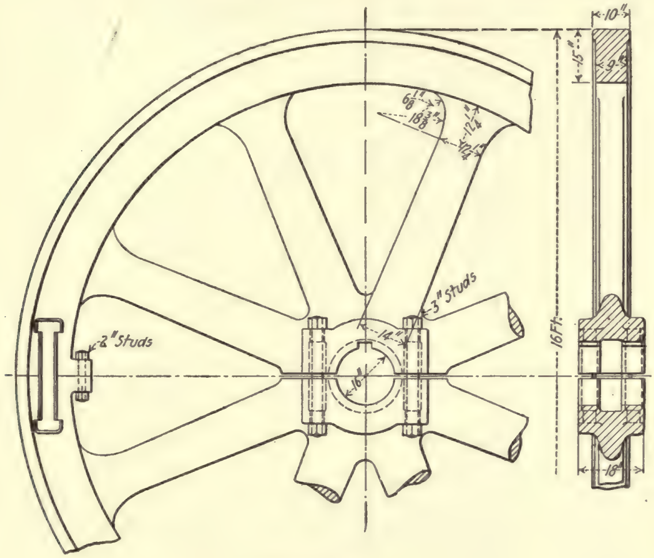

Fig. 435.

wheel around for the purpose of valve setting or to get the crank from dead center. They are spaced about 6 in. center to center.

214. Designs from Practice.—Figure 436 shows a wheel built by the Bass Foundry & Machine Co. It is $18\frac{1}{2}$ ft. in diameter and weighs 140,000 lb. It runs 75 r.p.m., giving a rim speed of only 4350 ft. per min. It was built for a geared rolling mill drive and its diameter was limited by the countershaft carrying the gear. To increase its efficiency, the rim was made wide. It is an example of an arm and rim section being cast together, the arm being fastened to the hub by shear bolts. Both links and bolts are used at the rim, the former being designed to carry the entire load.

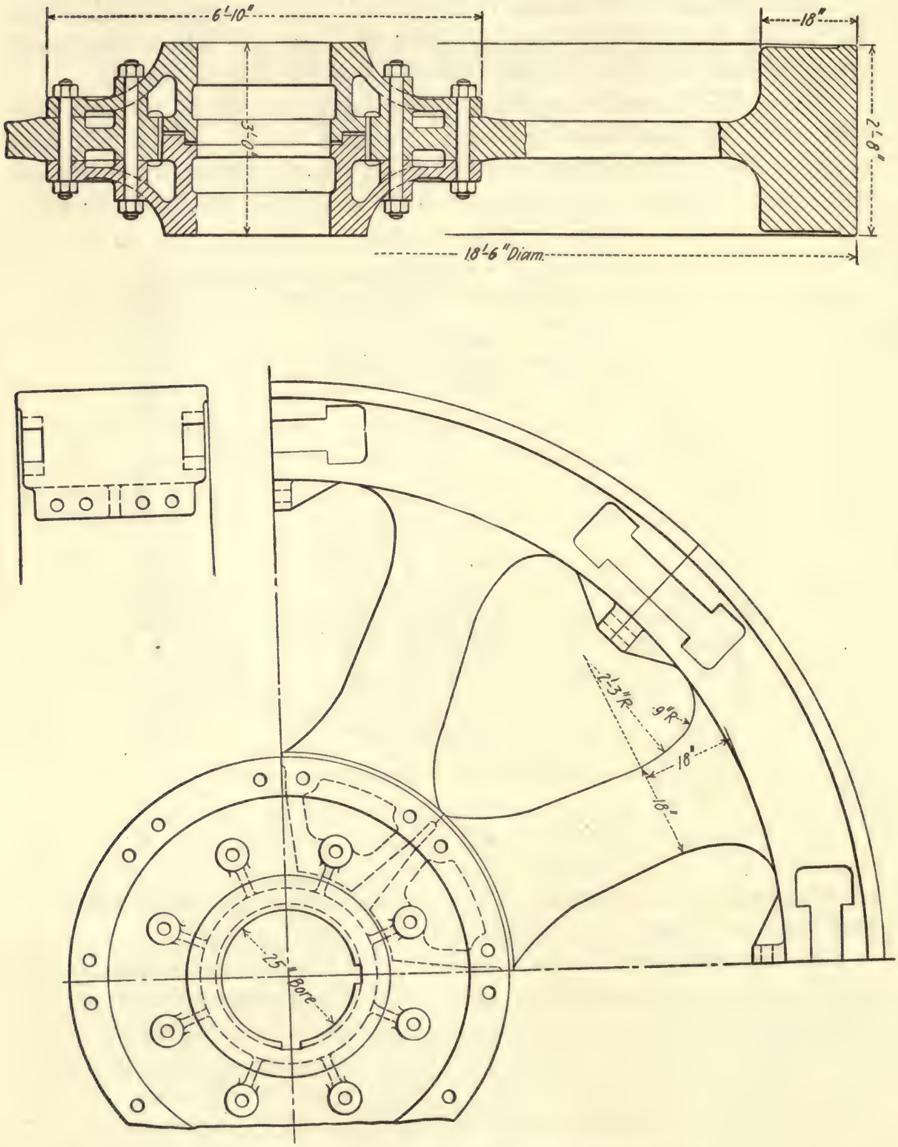

FIG. 436.—Bass-Corliss rolling-mill engine wheel.

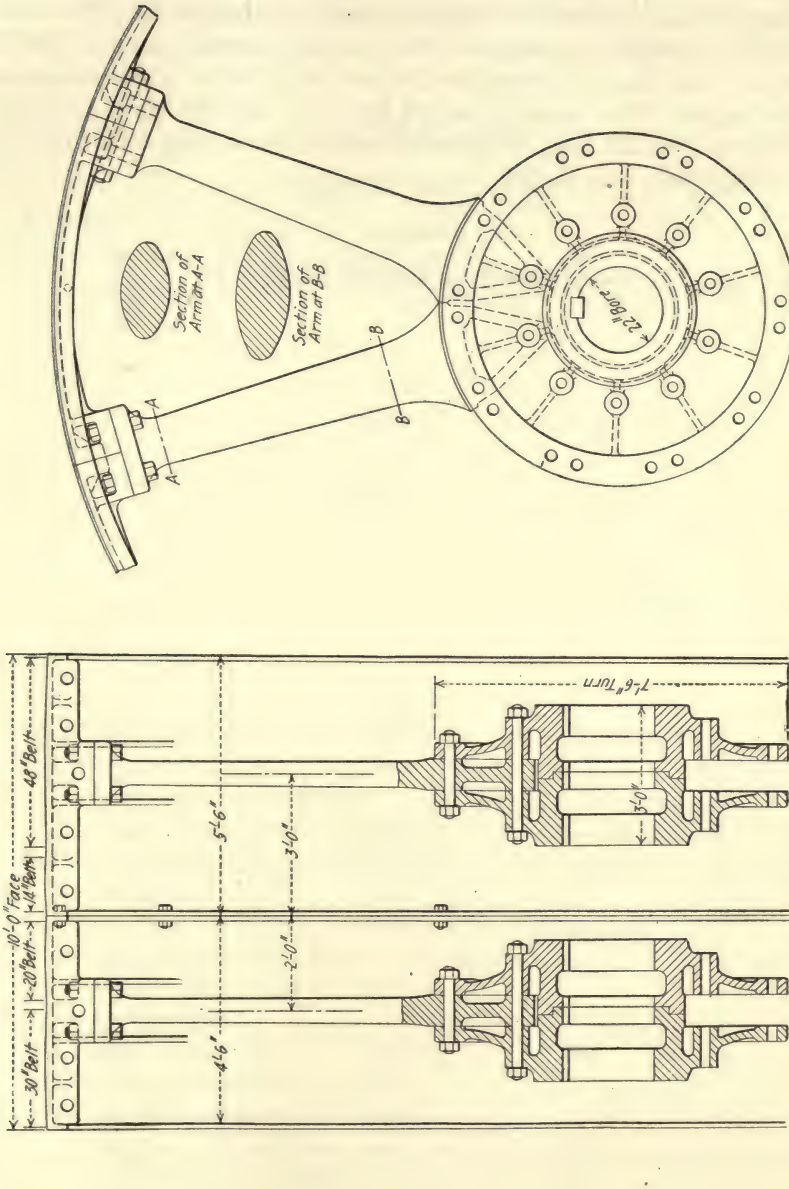

FIG. 437.—Bass-Corliss belt wheel.

Figure 437 shows another Bass wheel. It is a double wheel, 24 ft. in diameter with a 10 ft. face crowned for four belts. It is a good example of a built-up wheel, each arm and segment being cast separately. The wheel weighs about 200,000 lb. and runs 80 r.p.m., giving a rim speed of 6000 ft. per min. The wheel is driven by a 28 and 56 by 60 in. cross-compound Corliss engine. The bearings are 18 by 36 in. and the shaft, made of crucible steel by the fluid compressed process, has a hole 8 in. in diameter through its entire length.

Reference

Halsey's handbook for machine designers.

CHAPTER XXXI

TURBINE WHEELS

Notation.

d = diameter of wheel bore in inches.

D_O = diameter of disc in inches measured to center of gravity of rim and blades.

r_O = radius of same = $D_O/2$.

r = radius in inches at any point.

t_O = thickness of disc at radius r_O.

t = thickness of disc at radius r.

W_R = weight of rim in pounds.

W_B = weight of blades, shrouding, etc. in pounds.

W = weight of rim, blades, etc. carried by disc.

P = load per inch of periphery due to W.

F = tangential force in pounds at any radius, due to transmission of power.

E = modulus of elasticity.

S = stress in general, in pounds per square inch.

S_O = assumed tangential stress where disc and rim join.

S_E = equivalent simple tangential stress where disc and rim join.

S_L = limiting tangential stress in rim, giving same strain as S_O in disc.

S_C = radial stress where disc joins rim, due to centrifugal force of W.

S_D = shearing stress produced by F at any radius r at which F is calculated.

S_H = hoop stress in thin revolving ring of diameter D_O.

S_T = total tangential stress at any radius r.

S_R = total radial stress at any radius r.

f_W = tangential force due to jet velocity.

f_P = axial force due to jet velocity.

M = bending moment on blades.

Z = modulus of section of blades.

n = number of blades receiving steam at one time. Also exponent.

l = moment arm of blade in inches.

m = the reciprocal of Poisson's ratio.

N = revolutions per minute.
V = velocity in feet per second.
a = area in square inches.
H = horsepower developed in blading of wheel.
q = fraction of the rim carried by itself.

215. In all impulse turbines the blades are carried upon disc wheels which must be designed so that the maximum stress will not exceed certain limits at the intended speed of rotation.

The hoop stress in a revolving ring is given by Formula (4), Chap. XXX. Taking the diameter in inches at the center of gravity of rim and blades, the formula may be written for steel:

$$S_H = \frac{V^2}{9.42} = 2\left(\frac{D_O}{10}\right)^2\left(\frac{N}{100}\right)^2 \tag{1}$$

For rim velocities ordinarily used for flywheels, the value of S_H is not excessive, but a velocity of 500 ft. per sec., not uncommon in turbine operation, gives: $S_H = 26{,}500$. This does not include the effect of the blades, which, when the radial thickness of the rim is small compared to the diameter, may be found by dividing the centrifugal force of the blades by $2\pi a$, where a is the area of the rim section in sq. in. If we consider a ring with an axial thickness t_O, and S_C the radial stress due to some external load such as the blades, the resulting tangential stress is:

$$\frac{S_C t_O D_O}{2a}$$

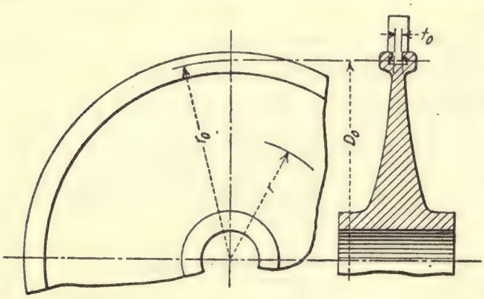

Fig. 438.

This is only strictly true if the radial thickness of the ring is very small relative to D_O, but as stated in Chap. XXX, may be assumed correct for usual flywheel proportions.

Assume for example that $S_C = 16{,}000$, $T_O = 1$, and $D_O/2a = 5$. Then for a rim velocity of 500 ft. per sec., the tangential stress due to rim load would be 80,000 lb.

It is obvious that such a stress is too great, and the radial thickness of the ring must be increased. The strength does not increase, however, in proportion to the added area, and the axial thickness must often be increased toward the center of the wheel as shown in Fig. 438.

The thickness t_O is determined from the radial load due to blading and

rim. The thickness at any other radius, for discs having a central hole, is commonly found from the exponential equation:

$$\frac{t}{t_o} = \left(\frac{r_o}{r}\right)^n \tag{2}$$

The principal stresses produced by the centrifugal force of the blading and by the rotation of the disc itself are radial and tangential, the latter being the most important. The effect of forcing the wheel upon the shaft may probably be safely neglected. This produces an initial stress, probably nearly static, and not increased by rotation; on the other hand, it probably decreases the range of stress due to starting and stopping.

Then the radial stress due to rim loading varies from S_C at the radius r_o to zero at the surface of the bore; and due to the disc itself it increases from zero at radius r_o to a maximum, and back to zero at the bore. The tangential stress is always greater than zero and is best shown by the chart of Fig. 440.

Probably no exact mathematical analysis has been devised for the case of the rotating disc; those which are used are greatly involved and the formulas are awkward and cumbersome, necessitating extensive use of trial and error. After a vain attempt at simplification it was decided to use a set of curves, the values of which were derived from another set of curves in Martin's excellent work. The mathematical treatment of the subject may be found in treatises on the steam turbine by Stodola and Jude, and in Morley's Strength of Materials.

216. It may be assumed that the blades, and a portion of the shaded area of the rim in Fig. 439, must be carried by the disc, involving one or more trial calculations, or, safely, the weight of the entire rim may be included. In either case the rim weight will be denoted by W_R and the weight of blades, shrouding, spacers, etc. by W_B. Taking r_o at the center of gravity of rim and blades is entirely arbitrary, but safe. It may be taken at the thinnest section of the disc where it joins the rim.

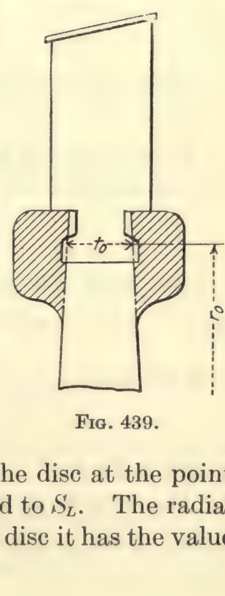

Fig. 439.

Were the rim revolving free it would have the stress S_H, and a correspondingly great tangential strain; but the strain in the rim must equal that in the disc at the point of attachment, and the stress will therefore be reduced to S_L. The radial stress in the rim may be considered as zero, but in the disc it has the value

S_C. If S_O is the assumed tangential stress at this point, the simple equivalent stress from Formula (6), Chap. XXI is:

$$S_E = S_O - \frac{S_C}{m}.$$

For equal strains in rim and disc:

$$\frac{S_L}{E} = \frac{S_E}{E} = \frac{1}{E}\left(S_O - \frac{S_C}{m}\right)$$

where m is the reciprocal of Poisson's ratio.
Then:

$$S_L = S_O - \frac{S_C}{m} \tag{3}$$

When S_O is finally determined (it will equal S_T),

$$m = \frac{S_C}{S_T - S_L} \tag{4}$$

This may be used as a check by comparing with values of m in Table 79, Chap. XXI.

If $S_O = S_C$ and $m = 3\frac{1}{3}$, $S_L = 0.7S$.

The fraction of the rim carried by itself under stress S_L is:

$$q = \frac{S_L}{S_H} = \frac{0.7S_O}{S_H}.$$

The remainder, $1 - q$, must be carried by the disc. Substituting the value of S_H from (1) gives:

$$q = \frac{0.35S_O}{\left(\frac{D_O}{10}\right)^2\left(\frac{N}{100}\right)^2} \tag{5}$$

Then the entire extraneous weight carried by the disc is:

$$W = (1 - q)W_R + W_B \tag{6}$$

From the first statement of Formula (3), Chap. XXX, the load per inch of periphery is found to be:

$$P = \frac{W}{22}\left(\frac{N}{100}\right)^2.$$

The required thickness at radius r_O is then:

$$t_O = \frac{W}{22S_C}\left(\frac{N}{100}\right)^2 \tag{7}$$

And from (2):

$$t = t_O\left(\frac{r_O}{r}\right)^n \tag{8}$$

at any radius r.

The total tangential stress at any radius is:

$$S_T = k_T S_H + c_T S_C \tag{9}$$

And the total radial stress:

$$S_R = k_R S_H + c_R S_C \tag{10}$$

where, as given by (1):

$$S_H = 2\left(\frac{D_O}{10}\right)^2\left(\frac{N}{100}\right)^2 \tag{11}$$

Stress S_T is generally the greater and is usually a maximum at the bore; but in some cases it is well to try for greater values of r/r_O, especially for larger values of n.

An inspection of the general stress equations makes it clear that for similar discs the stress varies directly as the square of the peripheral velocity (or, as $D_O^2N^2$); then the factors k_T, c_T, k_R and c_R are suitable for any disc, and are given in Figs. 440 and 441 for three ratios of d to D_O, and for various ratios of r to r_O. Values of k and c were taken from Martin's curves for $n = 0$, 1 and 2, and a smooth curve drawn between these points for several values of r/r_O; other values may be found by interpolation. It is probable that as great accuracy as the problem demands, or as is consistent with the practical accuracy of the analysis, may be obtained by the use of these curves.

In applying the equations and the curves, solve for (5), (6), (7) and (11), and decide upon suitable values of S_O and S_C. Take a trial value of n, select k_T and c_T at the bore and solve for S_T in (9). This should usually be equal to S_C, and different values of n should be tried until the right value of S_T is found. It may be safe to check the value when r/r_O is 0.9; then m may be checked by (4), which will be nearly as assumed if S_T for $r/r_O = 1$ is practically equal to S_O.

If all is satisfactory, determine enough values of t from (8) to outline the wheel section.

For a disc without a central hole, such as the wheel of the DeLaval simple impulse turbine, the formula is:

$$t = t_O e^{\frac{1}{S_C}\left(\frac{N}{100}\right)^2\left[\left(\frac{D_O}{10}\right)^2 - \left(\frac{2r}{10}\right)^2\right]} \tag{12}$$

where $e = 2.718$, the base of the Naperian system of logarithms. Then t_O may be found from (7) as before.

The stress is assumed to be uniform throughout the wheel, the tangential and radial stresses being equal. Jude says the reasoning upon which Formula (12) is based is open to grave doubt; however, practical results are obtained by its use.

If holes are bored in a disc for the purpose of equalizing pressure on the two sides, it may be assumed that the stress at the radius at which the

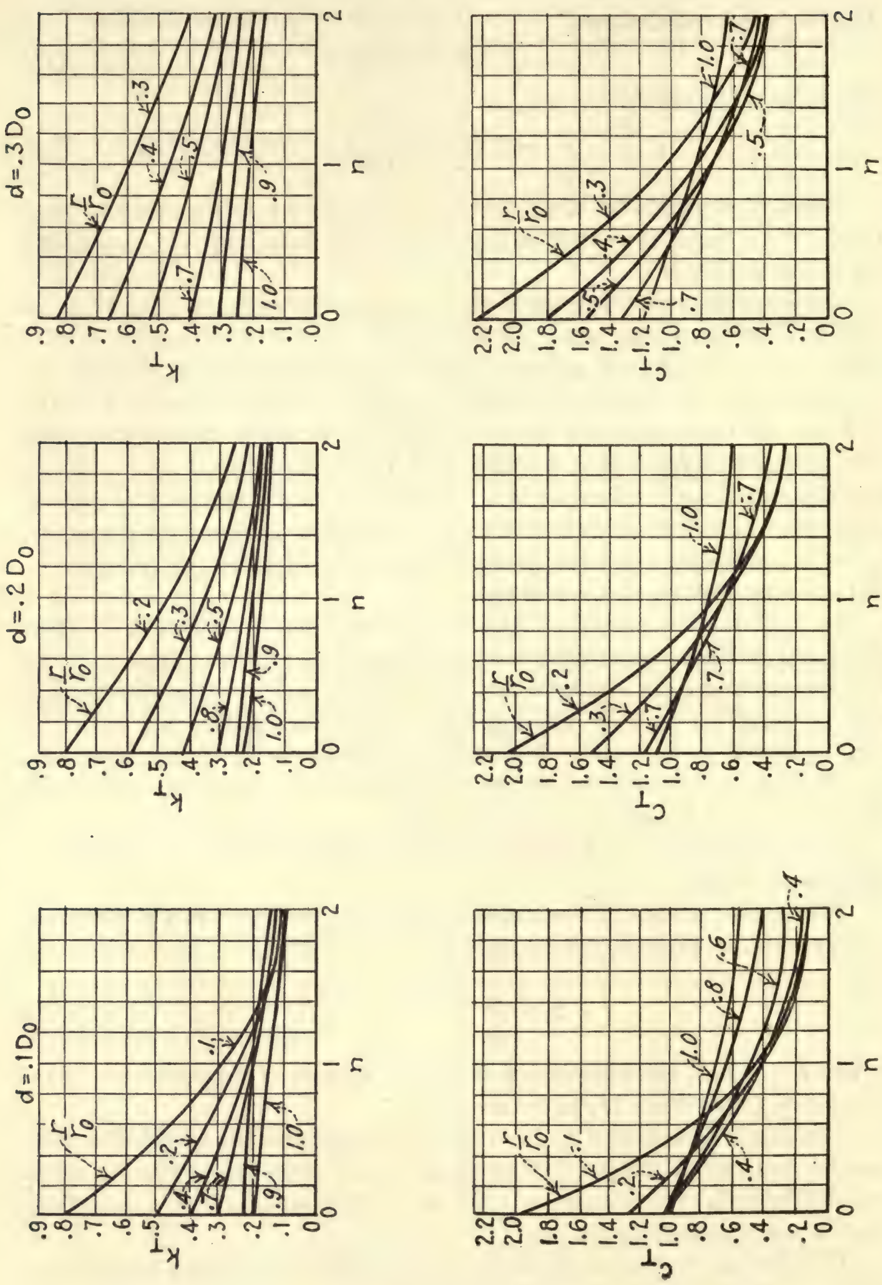

FIG. 440.—Tangential stress.

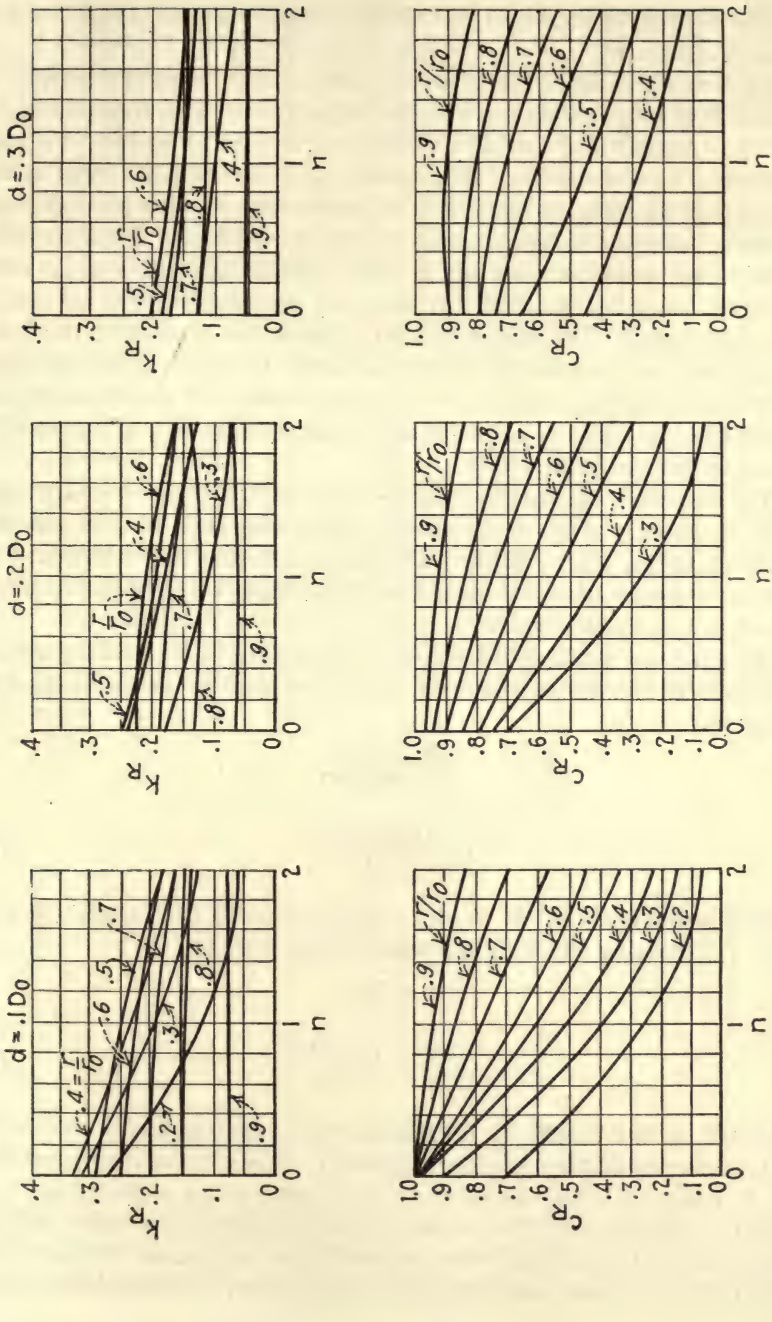

FIG. 441.—Radial stress.

holes are bored is increased in the same ratio that the circumferential section is decreased.

If the rim is wide, as when two or more rows of blades are carried, it is probable that the stress at the edges is greater than S_L, though still less than S_H. An extreme case is the drum of a reaction turbine with web connections to the spindle. The axial length between the webs or sets of arms may be so great as to receive practically no support therefrom, the stress being nearly equal to S_H due to the rotation of the drum itself. To this must be added the stress due to the centrifugal force of the blading, which may be found by dividing the centrifugal force by $2\pi a$, as previously stated. The area a may be taken as the product of the thickness of the drum and the distance from center to center of the two adjacent rows of moving blades. As the rim velocity of the drum type is usually less than 360 ft. per sec., there is little difficulty in keeping the stresses within practical limits.

217. Material for turbine wheels may range from ordinary open hearth machinery steel, to the alloy steels, and may be selected from Tables 73 to 78, Chap. XXI. Working stress may be determined by selecting a factor of safety from Par. 159, Chap. XXI, when the nature of the loading is determined.

Changing the notation of Formula (141), Chap. VI, and taking the distance from the center to where the load is applied in inches instead of feet, gives:

$$H = \frac{FrN}{63{,}000}$$

or,

$$F = \frac{63{,}000\ H}{rN} \tag{13}$$

where F is the force in lb. at any radius, required to transmit H at N r.p.m. A shearing stress S_D is produced over the area,

$$a = 2\pi rt$$

or,

$$S_D = \frac{F}{a} = \frac{10{,}000\ H}{r^2tN} \tag{14}$$

It will be found that S_D is insignificant compared to S_T, so that the live load due to driving may be neglected. The stresses due to rotation may be practically considered as static stresses and a standard factor of safety of 2 employed. Due to possible slight vibration, a factor of judgment of from 1.15 to 1.35 may be employed, an average giving a total factor of 2.5. In an example of disc design given by Martin, a working

stress of 16,000 is assumed for mild steel; a factor of 2.5 gives an elastic limit of 40,000 lb., which seems reasonable.

For high-speed wheels the use of high stresses is necessary, and it may be possible that the theory of rotating discs is more accurate with high stresses; at any rate low values of stress in the formulas result in absurd dimensions which are never found in practice. There can be little doubt that in a rapidly increasing thickness of metal the stress distribution is not that indicated by any theory that has any semblance of simplicity. However, judged by the usual standards of machine design, the methods used to determine the dimensions of steam turbine parts must be satisfactory when it is considered that proportionate to the period of its practical application, no other type of prime mover has probably had so few accidents.

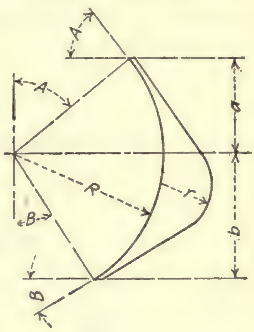

FIG. 442.

218. Blading Design.—The process of determining blade angles and lengths was discussed in Chap. XV. The determination of width is rather arbitrary and depends upon the length. The matter of angles and width being fixed, the radius and location of center may be found. The known quantities in Fig. 442 are the angles A and B, and the width $a + b$. It is seen from the figure that: $a = R \cos A$, and $b = R \cos B$. Then:

$$\frac{a}{b} = \frac{\cos A}{\cos B}$$

or,

$$a = b\frac{\cos A}{\cos B}$$

Then:

$$a + b = b\left[1 + \frac{\cos A}{\cos B}\right]$$

Or:

$$b = \frac{a + b}{1 + \dfrac{\cos A}{\cos B}} \tag{15}$$

And:

$$R = \frac{b}{\cos B} \tag{16}$$

The blade pitch and form of the back depend upon the radius r. As stated in Chap. XV, as large a pitch as practicable probably tends to reduce friction, and this is obtained by making r small. In impulse

blading the center of radius r usually lies within the blade section, sometimes on the curve of radius R. In Fig. 443, p is the maximum pitch which will—at least theoretically—direct the discharge steam at the angle B relative to the blade. It may be easily shown that:

$$\frac{p}{x} = \frac{\cos \frac{A + B}{2}}{\cos B}.$$

Or:

$$p = x \cdot \frac{\cos \frac{A + B}{2}}{\cos B} \tag{17}$$

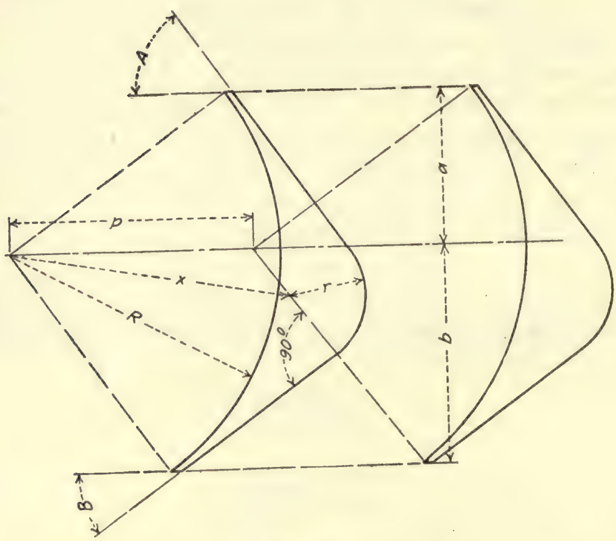

FIG. 443.

Good results will usually be obtained when $x = R$; then:

$$p = R \cdot \frac{\cos \frac{A + B}{2}}{\cos B} = \frac{b \cdot \cos \frac{A + B}{2}}{\cos^2 B}$$

$$= \frac{a + b}{2 \cos B \cdot \cos \frac{A + B}{2}} \tag{18}$$

Formula (18) may be used to find the pitch tentatively. If all spaces are to be equal a slight change may be necessary. There is no definite rule for spacing and it is probable that more or less latitude is permissible.

Strength of Blades.—Formula (24), Chap. XV, gives the tangential force f_W which must be divided among the blades taking steam at one time. For full peripheral admission this would be all the blades on a wheel. Formula (25) Chap. XV, gives the axial force f_F, this being usually small. If l is the distance from the center of the blade to the weakest section in inches, and n is the number of blades receiving steam, the bending moment is:

$$M = \frac{f}{n} \cdot l \tag{19}$$

where f refers to either f_W or f_F. If Z is the section modulus of the section considered and S is the stress:

$$S = \frac{M}{Z} \tag{20}$$

Should the section be irregular, as the blade section, Z may be found by the graphical method given in Appendix 1. Stress of the same sign (either tensile or compressive) may be determined by (20) at point of maximum stress by using both f_W and f_F, which act at right angles to each other; the sum will be the maximum stress.

Reducing S_C by making t_O greater, reduces the value of n, but the ratio,

$$\left(\frac{r_O}{r}\right)^n$$

will give greater values of t at all values of r, so that nothing is gained.

If certain values of t are desired at hub and rim, n may be found from (8), S_C from (7) and S_T from (9). Factor of safety and material may then be chosen. Unless an elaborate mathematical analysis is used, it is probably safer to cause intermediate values of t to lie on the curve plotted from (8). This is one of the cases where the addition of metal may be a source of weakness rather than strength.

219. Application of Formulas.—Assume a wheel similar to Fig. 444. Let the weight of the blades be 35 lb. (W_B) and the total weight of the rim 74 lb. (W_R). Also let $N = 5000$, $D_O = 25.125$ in. and $r_O = 12.562$ in. Let the material be nickel steel with an elastic limit of 50,000 lb.; then with a factor of safety of 2.5, $S_O = S_C = 20{,}000$. Then from (5), $q = 0.445$. From (6), $W = 76$ and from (7), $t_O = 0.73$.

Assume that $d = 0.2D_O$. From (1), $S_H = 31{,}500$. By several trials, solving for stress at diameter d, n was taken as 1.5. Then (9) and the charts give:

$$S_T = (0.355 \times 31{,}500) + (0.425 \times 20{,}000) = 19{,}700.$$

For $r/r_O = 0.9$, $S_T = 17{,}510$ and for $r/r_O = 1$, $S_T = 18{,}380$. At the outside of the hub where $r/r_O = 0.24$, $S_T = 18{,}400$. As the hub strength-

ens the disc in this region, n might have been less. The thickness at the hub where $r = 3.5$ in. is given by (8), and is 3.68 in. Values of t for various sections may be found by (8) and the curve drawn.

Taking $S_O = S_T$, and $S_L = 0.7\ S_O$, (4) gives: $m = 3.63$. This is some larger than the value assumed, and the value of S_L was based upon the other value, but the result is probably as nearly correct as the method in general.

220. Design from Practice.—Wheels are made of various forms, from a disc of uniform thickness to the disc of uniform strength of the DeLaval

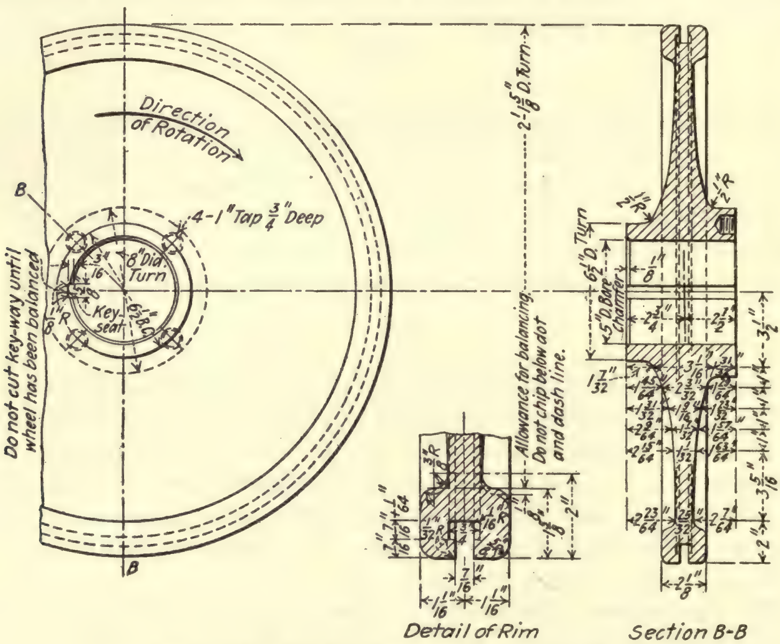

FIG. 444.—Southwark-Rateau turbine wheel.

Class A turbine. The latter is designed by Formulas (7) and (12). A disc of uniform thickness is expressed by (2) when $n = 0$. In some cases t_O is found from (7); then t at the hub from (8), so that S_T from (9) is within limits; then a straight line connects these two dimensions, making a disc easy to machine.

Figure 444 is a wheel designed to run 5000 r.p.m. by the Southwark Foundry and Machine Co., Philadelphia, Pa. The curve of this wheel does not check with (8); an approximation was probably made to simplify construction.

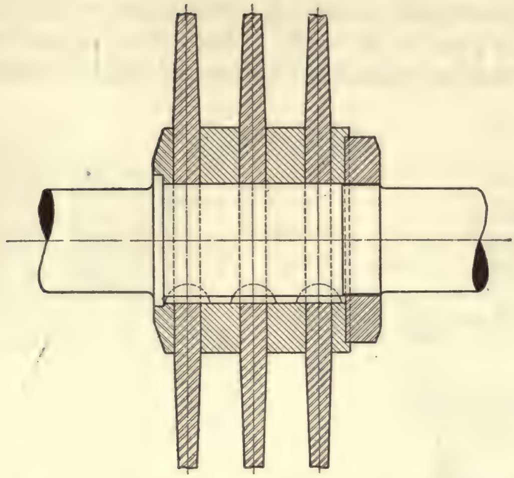

FIG. 445.—DeLaval Class C turbine wheel fastening.

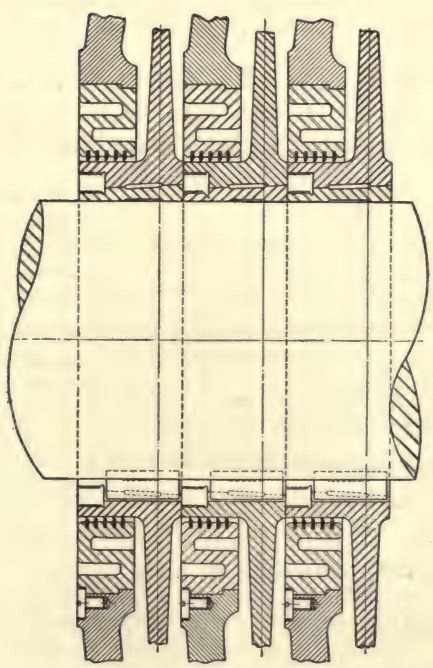

FIG. 446.—DeLaval multi-stage turbine wheel fastening.

Figure 445 shows the method of attaching the wheel of the DeLaval velocity-stage turbine to the shaft. Each wheel is held by a Woodruff key, and the wheels are spaced and centered by rings. All are held against

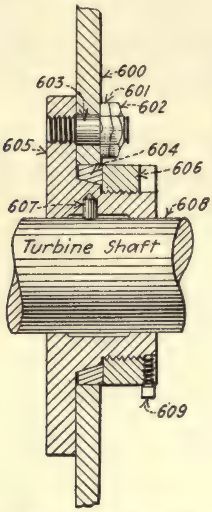

FIG. 447.—Kerr turbine wheel fastening.

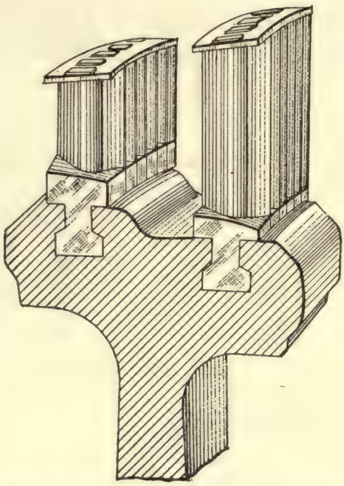

FIG. 448.—Rim of Curtis turbine wheel.

a shoulder on the shaft by a nut. Fig. 446 shows the method of attaching the wheels of the DeLaval pressure-stage turbine to the shaft. Tapered sleeves are keyed to the shaft, the wheels being drawn onto these by a nut.

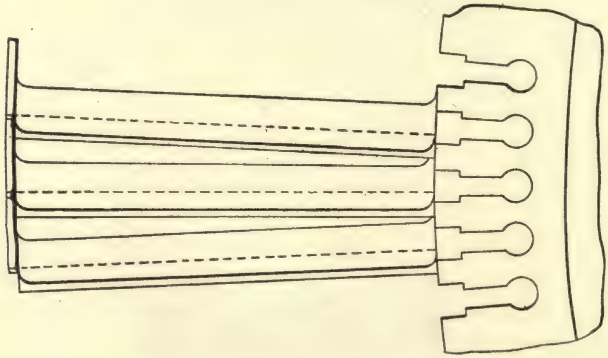

FIG. 449.—Ridgway turbine blading.

The labyrinth packing between the diaphragms and wheel is also shown.

Figure 447 shows the method of fastening the disc to the turbine shaft

used on the Kerr Economy turbine of the smaller sizes. The hubs are prevented from turning by dowels. The discs are of uniform thickness. In the larger sizes the wheels are made in one piece and held to the shaft by keys.

A section of the rim of a Curtis turbine wheel, built by the General Electric Co., is shown in Fig. 448. This is a 2-velocity stage wheel used in the first pressure stage. The blades shown are drop-forged with distance piece integral with blade. A projection on the blade passes through a shroud ring and is riveted. This strengthens the blading and forms one side of the steam passage through the blades.

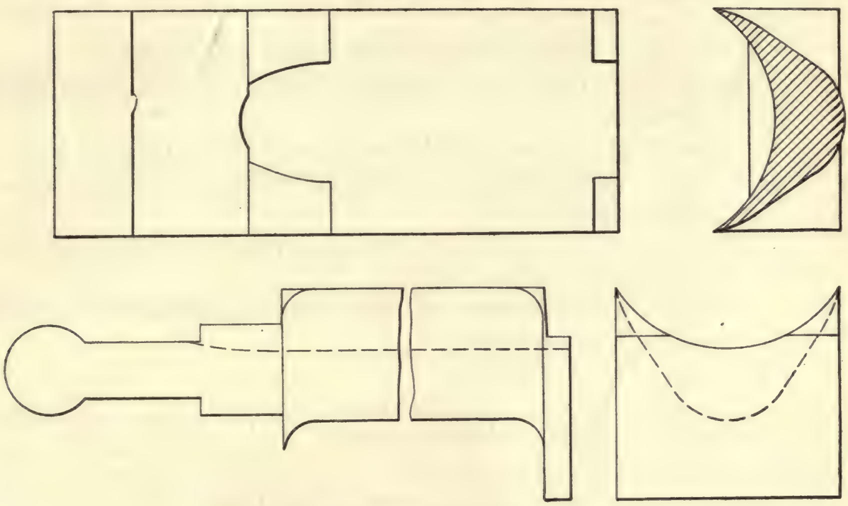

FIG. 450.—Ridgway turbine blade.

The method of fastening the blading of the Ridgway-Rateau-Smoot turbine is shown in Fig. 449, and a detail of the blade in Fig. 450. In the Ridgway blades no fillers or shrouds are used, as these are formed by the blades themselves. These blades are machined all over and usually made of monel metal. An advantage of the bulb-end type of blades is that in case of accident to any of the blades, any one may be removed without disturbing the rest.

References

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Blade fastening for steam turbines.........*Power*, July 20, 1915.
Strength on materials....................Prof. Arthur Morley.
Design and construction of steam turbines..H. M. Martin.

CHAPTER XXXII

TURBINE SHAFTS

Notation.

d = diameter of shaft in inches.
d_1 = inside diameter of hollow shaft.
d_B = diameter of journal.
l = length in inches from center to center of bearings.
x = distance to any point on shaft in inches; used in derivation of formulas.
a = length of section of shaft to be taken as isolated load $= q \cdot \Delta x$.
Δx = actual measurement in inches on bending moment diagram, corresponding to a.
h = pole distance in inches on vector diagram for bending moments.
k = same for deflection diagram.
z = actual measurement in inches corresponding to bending moment on diagram.
y = same for deflection.
p = weight scale = pounds per inch.
q = distance scale = inches per inch.
m = units of $z \cdot \Delta x / d^4$ per inch.
δ = actual deflection in inches under any load.
W = weight of isolated load in pounds.
w = weight per inch of length.
S = shearing stress in journal due to torque.
E = modulus of elasticity.
I = moment of inertia of shaft section.
M = bending moment at any point.
M_M = mean bending moment in length a of shaft, or in length Δx of diagram.
N = normal r.p.m. of turbine.
N_C = r.p.m. at critical speed.
ω = angular velocity at critical speed, in radians.
V = linear velocity in feet per second of isolated load W revolving at radius δ.
H = horsepower.

221. Nature of the Problem.—The rotative speed of the reciprocating-engine shaft is usually so low that vibration needs little or no consideration. In small gasoline engines with very high rotative speed the shaft is so well supported by bearings that vibration is not usually considered. With the shaft of the reaction turbine the drum construction is such that the required stiffness is attained easily for the speeds at which this type is operated. For the multi-stage impulse turbine with disc wheels, the shaft receives comparatively little stiffening from the hubs—and this is neglected in calculation—so that the shaft must maintain stability by its own stiffness. It must not run at a speed anywhere near its *critical speed.*

The determination of the shaft diameter is therefore an entirely different problem from that of the engine shaft, the latter being one of bending and twisting. These are both insignificant in the turbine shaft except at or near a certain speed—the critical speed.

For preliminary calculations it is convenient to have some formula to assist in fixing dimensions; then these may be checked by the methods of the following paragraphs.

If the shaft be assumed to transmit power, the smallest diameter, which may be taken at the journal, will be given by the formula:

$$d_B = 75\sqrt[3]{\frac{H}{SN}} \tag{1}$$

where H is the horsepower, S the shearing stress and N the r.p.m. The stress S must be taken low—2000 to 2500—to give practical results. Length of bearing may be determined from Par. 52, Chap. XI, and any necessary compromises made.

The shaft between the bearings is larger, sometimes having a number of different diameters increasing toward the center.

A tentative determination of an equivalent straight-sided shaft may be made by assuming a uniform load composed of all the wheels and a shaft of a greater diameter than that given by (1); then by the general principles of Par. 222 the following formula may be derived:

$$N_C = \frac{1850}{l^2}\sqrt{\frac{EI}{w}} = \frac{1850}{l^2}\sqrt{\frac{EIl}{W}} \tag{2}$$

where E is the modulus of elasticity, I the moment of inertia, w the load per in. of length including shaft, W the total weight in lb. and l the length of shaft in inches between bearing centers. For a solid shaft, taking E = 30,000,000:

$$N_C = 2{,}250{,}000\left(\frac{d}{l}\right)^2\sqrt{\frac{l}{W}} \tag{3}$$

N_C should be much greater or much less than the actual shaft speed, N. By assuming the number of r.p.m., (2) may be written:

$$I = \frac{WN_C^2l^3}{1850E} \tag{4}$$

Or from (3)

$$d = \frac{l}{1490}\sqrt{N_C}\sqrt[4]{\frac{W}{l}} \tag{5}$$

As actual conditions are usually not so favorable to stiffness, the actual critical speed will probably be less than that given by (2) or (3).

A good deal of information about the principles underlying critical speed may be found in Morley's Strength of Materials, to which the author is largely indebted for the ideas of the next two paragraphs. An absolutely correct solution is probably impossible from the practical standpoint, due to various influences which may not easily be taken into account, but a method will be given which is accurate in principle to within narrow limits; if this is carefully applied, practical results may be expected.

222. Critical Speed.—If a body is acted upon by a periodic force having the same frequency as the natural vibration of the body, the total energy will be increased at each application, increasing the strain energy and therefore the stress. If this continues until the elastic limit of the material is reached, distortion of the body will result. The frequency causing this is called the *critical frequency*.

When a body vibrates, the energy is of two kinds, kinetic and potential, their sum being constant. In the mean position the velocity is a maximum and the energy is all kinetic; at the extremes it is all potential.

The motion of natural vibration is simple harmonic, and therefore displacements may be treated as projections on the plane of vibration, of a body revolving with a constant angular velocity.

Assume the body in question to be a turbine shaft, deflected under its own weight and that of the disc wheels; with carefully balanced shafts this weight may be considered as the periodic force, the period being that of rotation. The force acts in but one direction, but the rotation of the shaft presents opposite sides alternately to the force, producing the same effect as forced vibration. If the rotative speed is the same as the natural period of vibration it is called the *critical speed;* then the potential energy required to force the shaft to its deflected position is equal to the kinetic energy of the entire mass, should its axis whirl on a surface of revolution developed by the center of the deflected shaft, the speed of whirling being the critical speed.

If the system is not in perfect balance there is a tendency to whirl at all speeds below the critical speed, the latter probably being lower on this account. As the speed is increased beyond this point the tendency to whirl is reduced, and the shaft tends to revolve around the axis of gravity of the system.

It is more convenient, and sufficiently accurate for practical purposes to assume each wheel or portion of shaft as an isolated load. Let these loads be W_1, W_2, etc., and the deflections under the loads δ_1, δ_2, etc. respectively. These deflections are radii of the circles the loads are assumed to revolve upon at the instant the critical speed is reached. The potential energy of these loads in ft.-lb. is:

$$\frac{1}{2}\left(W_1\frac{\delta_1}{12} + W_2\frac{\delta_2}{12}\ \cdot\ \cdot\ \cdot\right) = \frac{1}{24}\,\Sigma(W\delta) \tag{6}$$

The kinetic energy is:

$$\begin{aligned}\Sigma\left(\frac{WV^2}{2g}\right) &= \Sigma\left(\frac{W\left(\omega\frac{\delta}{12}\right)^2}{2g}\right)\\ &= \frac{\omega^2}{288g}\cdot\Sigma\,(W\delta^2)\\ &= \frac{\pi^2 N_C{}^2}{900\times 288g}\Sigma\,(W\delta^2)\end{aligned} \tag{7}$$

Equating (6) and (7) gives:

$$\frac{\pi^2 N_C{}^2}{900\times 288g}\Sigma\,(W\delta^2) = \frac{1}{24}\Sigma\,(W\delta)$$

From which:

$$N_C = 187.8\sqrt{\frac{\Sigma\,(W\delta)}{\Sigma\,(W\delta^2)}} \tag{8}$$

If in (8), load w per unit length were used in place of W, the expression would be:

$$N_C = 187.8\int_o^l\delta\cdot dx \div \int_o^l\delta^2\cdot dx.$$

Then from the equation of the elastic curve, if the value of δ in terms of x be substituted, the result for a shaft of uniform section, with uniform load, and for ends supported, would be given by Formula (2).

223. Deflection.—Before Formula (8) can be applied it is necessary to determine the deflection under each load W; this may best be done graphically. The underlying principles of what follows are thoroughly treated in Morley's Strength of Materials, but an attempt will be made to explain their practical use in such a way that they may be applied if this fundamental knowledge is lacking.

The general equation of the elastic curve, when the curvature is slight, is:

$$\frac{d^2y}{dx^2} = \frac{M}{EI} \tag{9}$$

Integrating gives:

$$\frac{dy}{dx} = \int \frac{M}{EI} \cdot dx \tag{10}$$

This gives the slope at a certain point in the deflection curve; another integration gives y, the deflection from some reference point.

Another fundamental beam equation is:

$$\frac{d^2M}{dx^2} = w \tag{11}$$

where M is the bending moment in in.-lb. and w is the load per inch of length. Integrating (11) gives:

$$\frac{dM}{dx} = \int w \cdot dx \tag{12}$$

This gives the slope at a certain point of the bending moment curve, and another integration gives the bending moment when certain conditions are known. Comparing (9) and (10) with (11) and (12), it is obvious that if a graphical method may be used to find M when the load curve (curve of w) is known, the same method may be used to find y when the curve of M/EI is known.

If the bending moment due to a continuous load—whether uniform or varying—is to be found graphically it is divided up into sections, each one of which is assumed equivalent to an isolated load located at the center of gravity of the section of continuous load. Let W be one of these equivalent isolated loads between x_2 and x_1; then:

$$W = \int_{x_1}^{x_2} w \cdot dx \tag{13}$$

the quantity in the integral sign being the area of the load curve between x_2 and x_1.

Likewise the quantity:

$$\int_{x_1}^{x^2} M \cdot dx$$

is the area of the bending moment curve between x_2 and x_1. If M_M is the mean height of this area and a the length to certain scales,

$$\frac{M_M a}{EI} = \int_{x_1}^{x^2} \frac{M}{EI} \cdot dx \tag{14}$$

Then from (10), (12), (13) and (14), if W is used for plotting the bending moment curve,

$$\frac{M_M a}{EI}$$

may be used for plotting the curve of deflection.

An important matter is the fixing of the scales, which may be taken as follows:

Let 1 in. $= p$ lb. $(= p$ units of $W)$.

Let 1 in. $= q$ in. (of measurement along shaft).

Before determining other scales the bending moment diagram will be plotted in Fig. 451. The loads, W_1, W_2, etc., may vary in amount and be distributed in any way. Lay these loads to any convenient scale

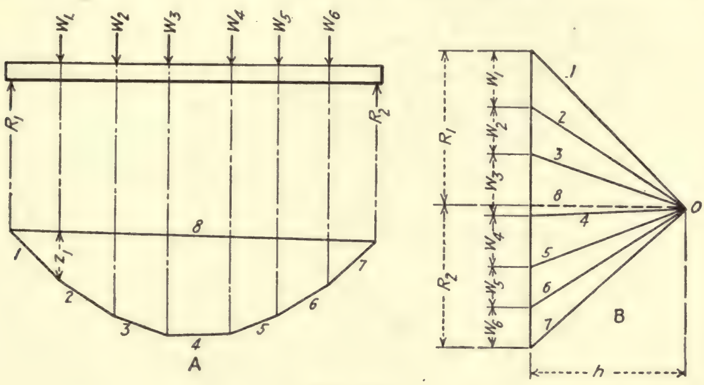

FIG. 451.

along the vertical line consecutively as shown in Fig. 451B. Take a pole at any point O, drawing rays 1, 2, 3, etc., to join the loads as shown. Starting at any point at the left reaction, draw link 1 parallel to ray 1 until it cuts the load W; draw link 2 parallel to ray 2, and so on until the right reaction is reached. Connect the two ends. A line drawn from pole O parallel to this determines the reactions, which may be used in determining bearing pressures.

The measurement z on A is distance, so the scale q must be applied to it; the measurement h is on the force diagram and scale p must be used. Let the quantities h and z be actual inches on the diagram. Then the bending moment at any point at which z is measured is:

$$M = pqhz \tag{15}$$

in which pqh is a constant.

In (14), E is constant; I may be constant, or in stepped shafts it varies. It will be treated as variable in order to be more general. To simplify explanation it will first be assumed that the wheel is located at the center of each step, the lengths of which are a_1, a_2, etc. In this case the weight of the wheel and the step are equal to W. The moment diagram in Fig. 452 was drawn as in Fig. 451. The quantity in (14), which is analogous to W may be reduced to constants and variables; the latter only need be plotted. Taking the value of M from (15):

$$\frac{M_M a}{E I} = \frac{pqhz\ q\cdot\Delta x}{E I} = \frac{pq^2h}{E}\left(\frac{z\cdot\Delta x}{I}\right) \tag{16}$$

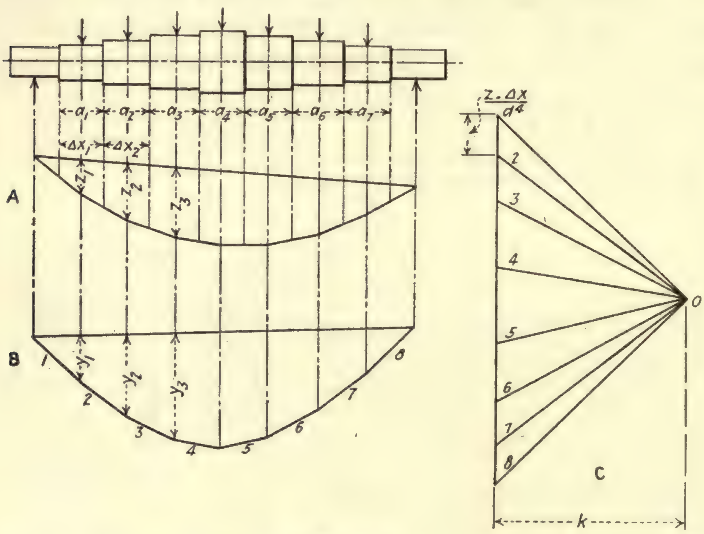

FIG. 452.

The quantity Δx is in actual inches on the diagram. In (16) the quantities in brackets are variable. For solid shafts:

$$I = \frac{\pi d^4}{64}$$

where d is the diameter of the shaft in inches. Should the shaft be hollow, d^4 may be replaced by $d^4 - d_1{}^4$, where d_1 is the inside diameter. As most shafts are solid, (16) becomes:

$$\frac{M_M a}{EI} = \frac{64pq^2h}{\pi E}\left(\frac{z\cdot\Delta x}{d^4}\right) \tag{17}$$

The quantity in brackets may be plotted. As the product $z\cdot\Delta x$ is small, it may be better to include part of the constants in the brackets in

some cases, especially if d is large. With a shaft of uniform diameter between bearings, d is constant and may be taken out of the brackets.

When it is decided what shall be plotted, the quantities may be laid off in succession as shown in Fig. 452C; a pole may be located at any point as shown, the rays drawn and the corresponding links laid off parallel to them on B. The quantities y and k are in actual inches on the diagrams; the former must be multiplied by the distance scale q and the latter by the scale for the quantity plotted; or:

$$1 \text{ in.} = m \text{ units of } \frac{z \cdot \Delta x}{d^4}.$$

In addition to this the constants of (17) must be used, as the entire quantity:

$$\frac{M_M a}{EI}$$

must be used to determine the deflection δ. The vertical intercepts of Fig. 452B are proportional to the deflection; then:

$$\delta = \frac{64mpq^3hk}{\pi E} \cdot y = Ky \qquad (18)$$

Should the wheels have offset hubs, and not be located at the centers of the steps, separate moment diagrams may be drawn, one for the shaft and one for the wheels, taking the loads at the center of gravity of wheel and step in each case. Should the steps be long they may be divided into two or more loads. These two diagrams may be combined; then dividing into suitable lengths, each one of which covers a portion of the shaft of constant diameter, the deflection curve may be plotted and the deflection under each load found. The shaft steps and wheels may be kept separate if desired in determining $\Sigma(W\delta)$ and $\Sigma(W\delta^2)$.

The deflection curve may be drawn more correctly by an *inscribed* curve touching the sides of Fig. 452B.

The dimensions h and k should be chosen so that the moment and deflection curves will not be too flat, as it is easier to draw a deeper diagram accurately.

224. Application of Formulas.—As an example a 13-stage turbine has been taken. The data was partly assumed and not very accurate. The diagrams were similar to Figs. 451 and 452 and will not be repeated. In plotting the deflection curve, a was taken instead of Δx; this removed one q from the constant in (18). The scales were: $q = 6$; $p = 300$; $m = 0.04$. Also: $h = 3$ and $k = 3$. From (18), $K = 0.00264$. The length from center to center of bearing is 67 in. and the total weight 1850 lb. The remainder of the data is placed in Table 102.

TABLE 102

No.	W	a	d	z	$\frac{az}{d^4}$	y	δ	$W\delta$	$W\delta^2$
1	80	10.80	4.30	1.42	0.0440	1.44	0.0038	0.304	0.00116
2	170	5.12	4.40	2.49	0.0220	2.20	0.0056	0.950	0.00532
3	100	2.80	5.30	3.40	0.0117	2.57	0.0068	0.680	0.00463
4	100	2.97	5.52	3.33	0.0106	2.74	0.0072	0.720	0.00520
5	145	2.97	5.67	3.64	0.0104	2.92	0.0077	1.120	0.00863
6	145	2.97	5.80	3.83	0.0100	3.02	0.0079	1.145	0.00905
7	145	2.97	5.93	3.94	0.0094	3.10	0.0082	1.188	0.00975
8	145	2.97	5.93	3.96	0.0095	3.12	0.0084	1.220	0.01025
9	145	3.24	5.80	3.90	0.0112	3.11	0.0083	1.205	0.01000
10	175	3.91	5.40	3.73	0.0172	3.04	0.0080	1.400	0.01120
11	200	4.60	5.27	3.43	0.0204	2.86	0.0075	1.500	0.01125
12	250	5.40	4.85	2.82	0.0275	2.48	0.0065	1.622	0.01060
13	80	10.80	4.30	1.52	0.0480	1.48	0.0039	0.312	0.00111

Taking $\Sigma(W\delta)$ and $\Sigma(W\delta^2)$, Formula (8) gives for the critical speed:

$$N_C = 187.8\sqrt{\frac{13.363}{0.0982}} = 2180.$$

If an equivalent diameter of $5\frac{1}{2}$ in. be assumed for a straight shaft, and the total weight taken as a uniform load, (3) gives: $N_C = 2800$.

The determination of N_C by either (8) or (3) should be greatly different from the actual turbine speed for safety—probably at least 100 per cent. greater, or 50 per cent. as great. It is practically impossible to consider all factors which enter into the problem, but with due allowance the method may be considered safe if the shaft is taken as a beam supported at the ends, and only loads between supports be considered.

Jude says that "the chief concern is whether the critical speed is (*e.g.*) 300 or 3000 r.p.m.; not whether it is 300 or 310."

Martin says it is not uncommon to run turbines of the disc type above the critical speed. "In that case, however, the efficiency of the turbine may be expected to diminish with time, since on every occasion on which the turbine is started up or stopped, the rotor has to pass through its critical speed and the consequent vibration gradually enlarges the fine clearances used where the shaft passes through the high-pressure diaphragms."

Jude further says: "It is found that if the speed be increased quickly, so that the critical velocity is of only passing influence, the whirl quiets down—so much so, in fact, that the stability of the system is in general greater than at speeds below the critical." He also states that the normal

speed should be as remote from the critical speed as possible, and that properly it should be above rather than below.

In the single-stage DeLaval turbine the speed is far above the critical speed; concerning the multi-stage turbine, however, it is said in Catalogue D: "The use in a multi-stage turbine of a shaft running above or near its critical speed is an error, not only on the grounds of safety and freedom from vibration, but also because any whipping or eccentric rotation of the shaft will require enlarged clearances where the shaft passes through the diaphragms. In other words, the leakage areas will need to be considerably increased. It therefore follows that the total leakage is reduced by using a shaft sufficiently large and stiff to suppress such vibration entirely, thereby permitting the radial clearance to be correspondingly reduced, although the larger shaft has a greater circumference."

The use of material for wheels which permit the use of high stresses will both lighten the hub and shorten the fit, making possible a shorter shaft. A straight shaft is stiffer than a stepped shaft of the same diameter at center, so if some method of mounting wheels be used which permits a straight shaft, it will increase the critical speed.

For turbines of the drum type the shafts are stiffened by the drum, and the critical speed is probably never reached. But high speeds are not possible with this type as may be seen from Chap. XXXI.

References

The design and construction of steam turbines........H. M. Martin.
Strength of materials..............................Prof. Arthur Morley.

CHAPTER XXXIII

TURBINE CASINGS AND DETAILS

225. There is very little calculation involved in the design of casings, which are practically the frames of turbines; therefore this chapter will be comprised of illustrations and descriptions of some of the details of several makes of turbine.

One of the principal problems in connection with casings is to provide suitable clearances for uneven expansion of casing and rotor. This becomes more of a problem in large turbines, so it has been customary to divide the turbine into high-pressure and low-pressure elements, sometimes on separate shafts, forming cross-compound turbines, and sometimes on the same shaft as tandem-compound turbines. Recently very large turbines have been placed in a single casing. An accident to such a turbine, doubtless due to expansion, is described in *Power*, March 19, 1918.

Some idea of turbine casings may be obtained from the illustrations of Chap. IV, and these may supplement those of this chapter. But few dimensioned drawings are given; the treatment is therefore more qualitative than quantitative. In some cases assembly drawings are given, the relations of the parts being better seen from these. No special order is adhered to, the parts of a given make being given together.

226. DeLaval Turbine.—A general view of the casing of the Class *B* turbine is shown in Fig. 453. The lettered parts are known as:

A. Wheel case.
B. Wheel case cover.
C. Turbine wheel.
D. Inner packing bushing bracket.
E. Inner packing bushing.
F. Outer packing bushing bracket.
G. Outer packing bushing.
H. Outer bearing bracket.
K. Insulating cover.
L. Exhaust flange.
M. Governor.
V. Turbine shaft.

Figure 454 shows a partial section of a velocity-stage Class *C* turbine. The steel retaining ring completely covers the wheels and serves to protect the cast iron casing in case of accident to the wheels.

Figure 455 shows the nozzles for this turbine, *A* being for the high-pressure condensing turbine, therefore having a great divergence; *B* being

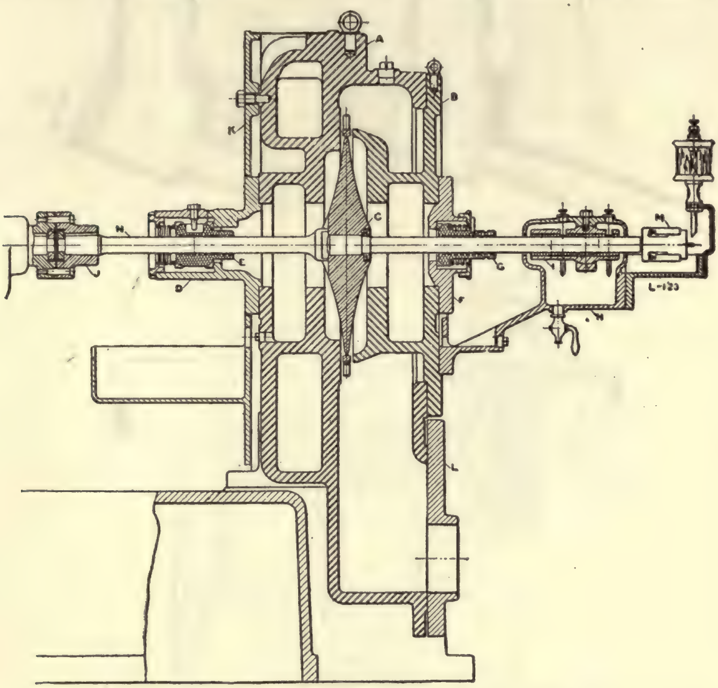

FIG. 453.—DeLaval Class B turbine casing.

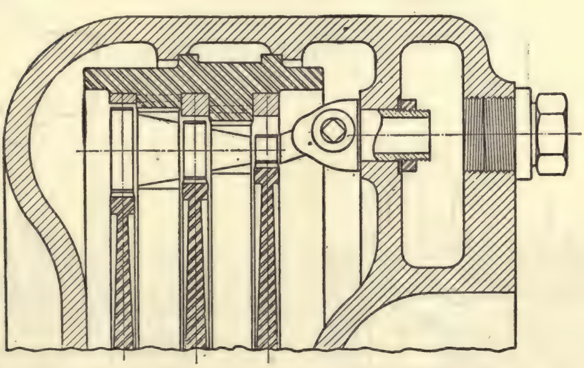

FIG. 454.—Section of DeLaval Class C turbine.

for low-pressure condensing, or high-pressure noncondensing turbines, has less divergence.

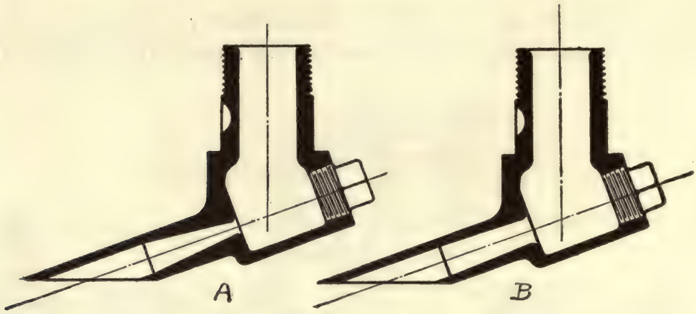

FIG. 455.—Nozzles for DeLaval Class C turbine.

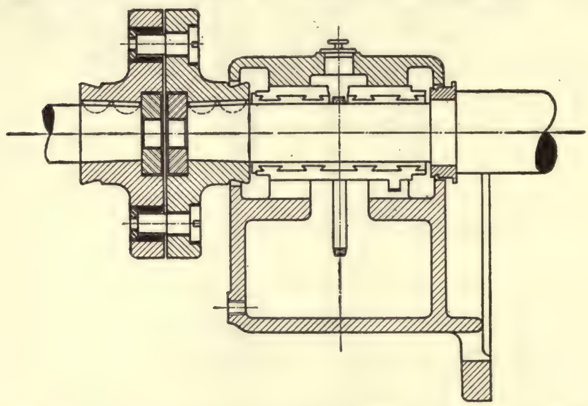

FIG. 456.—Outboard bearing bracket for DeLaval Class C turbine.

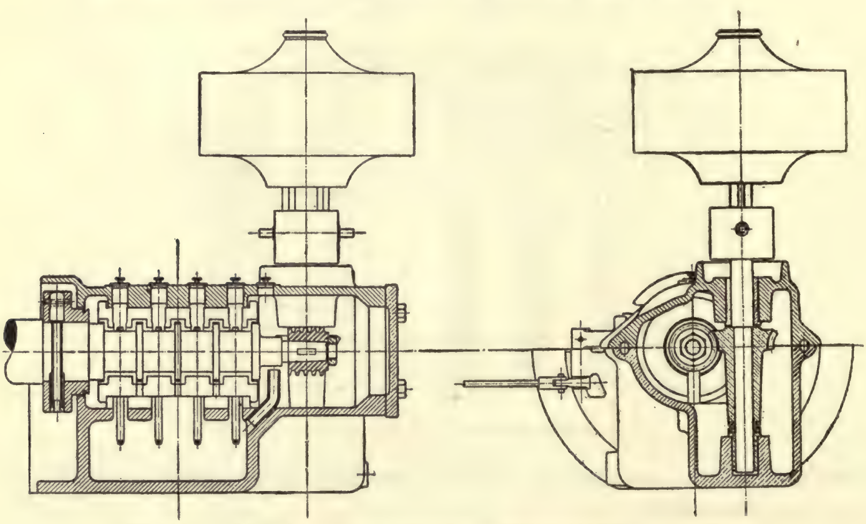

FIG. 457.—Radial and thrust bearing of DeLaval Class C turbine.

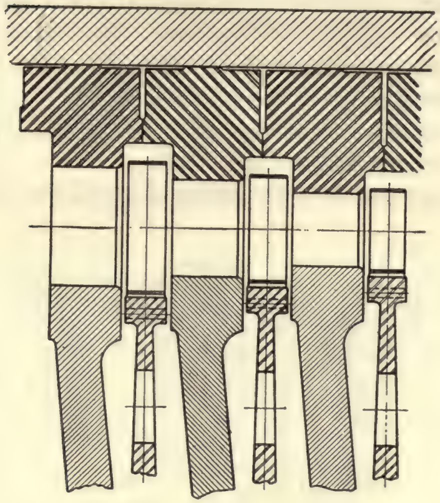

FIG. 458.—Retaining rings of DeLaval multi-stage turbine.

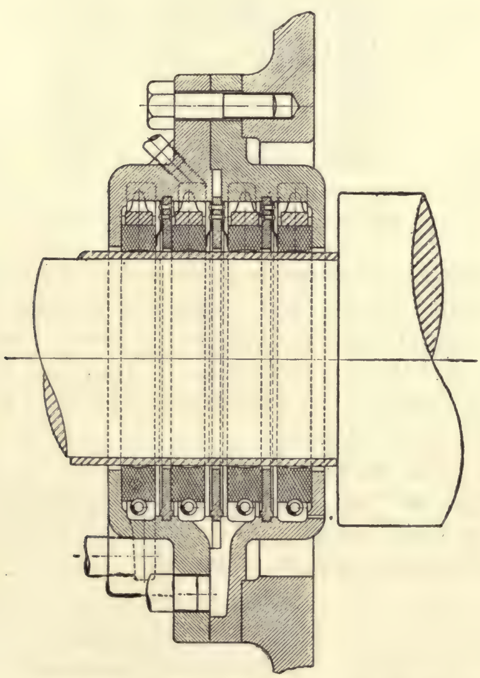

FIG. 459.—Carbon packing of DeLaval multi-stage turbine.

Figure 456 shows a sectional elevation of the outboard bearing bracket and flexible coupling of the Class C turbine. The bearings are ring-oiling; they are entirely separate from the stuffing boxes, so that no steam or water can reach them or enter the oil reservoir.

Figure 457 shows the combined radial and thrust bearing and governor drive of the DeLaval velocity-stage turbine.

Figure 458 shows the steel retaining rings of the DeLaval pressure-stage turbine.

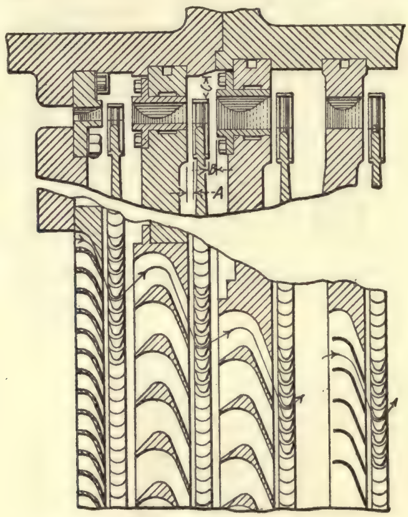

Fig. 460.—Clearances of Ridgway turbine.

The carbon packing at the low-pressure end of the DeLaval pressure-stage turbine is shown in Fig. 459. Provision is made for introducing live steam at a reduced pressure between the second and third rings, so that any leakage into the turbine will be of steam and not of air. The labyrinth packing between the diaphragms and hubs of the wheels is shown in Fig. 446, Chap. XXXI.

227. Ridgway Turbine.—Figure 460 shows sections of the first four stages of a Ridgway turbine. This shows the comparatively large blade clearances used in these turbines. The clearance A is $\frac{3}{16}$ in. or more; B is $\frac{1}{4}$ in. or more, and C is $\frac{1}{2}$ in. or more.

CHAPTER XXXIV

GENERAL ARRANGEMENTS AND FOUNDATIONS

Notation.

h = depth of foundation in feet.

d = diameter of foundation bolts in inches.

D_S = diameter of standard steam engine cylinder in in. (see Chapters XII and XIII).

W_F = weight of foundation in pounds.

W_E = weight of engine in pounds.

v = volume of foundation in cubic feet.

N = revolutions per minute.

228. General Arrangements.—For ordinary standard engines general arrangement drawings are not usually required. For special work or for some special arrangement of piping they are required. In any case, especially with large engines of the cross-compound, twin or duplex types, such drawings are a useful check and are valuable for the erecting engineer.

A simple layout of a Bass-Corliss engine is shown in Fig. 461. The engine is a 26 and 54 by 48 in. cross-compound, provided with a rope wheel. The wheel is in four parts, each having its own hub and arms and bolted together at the flanges Two wheels carry eight 2 in. ropes each and the others each carry nine $1\frac{1}{2}$ in. ropes.

The receiver and piping are under the floor, and so arranged that the connecting rod of either engine may be disconnected and the other engine continued in operation. Should the low-pressure engine carry the load, live steam which has been passed through a reducing valve is admitted to the receiver. Engines so built by the Bass Foundry and Machine Co. are heavier than the ordinary compound engine; the long-range cut-off gear is used so that one engine may carry the normal load continuously with a reasonable overload in case of accident to the other side. This is desirable in plants with few engines—perhaps only one—in which an accident requiring a complete shut-down would be serious.

229. Foundations are provided to secure the engine and to absorb vibration. The mass required for the latter depends upon a number of factors, but aside from cost, it is better to have too great than too small

a mass. In fact, greater cost in building the foundation may sometimes prevent far greater cost in corrections made necessary by excessive vibration.

A formula for extreme foundation depth devised by the author some years ago for Corliss engines is:

$$h = \frac{D_S}{4} + 3 \tag{1}$$

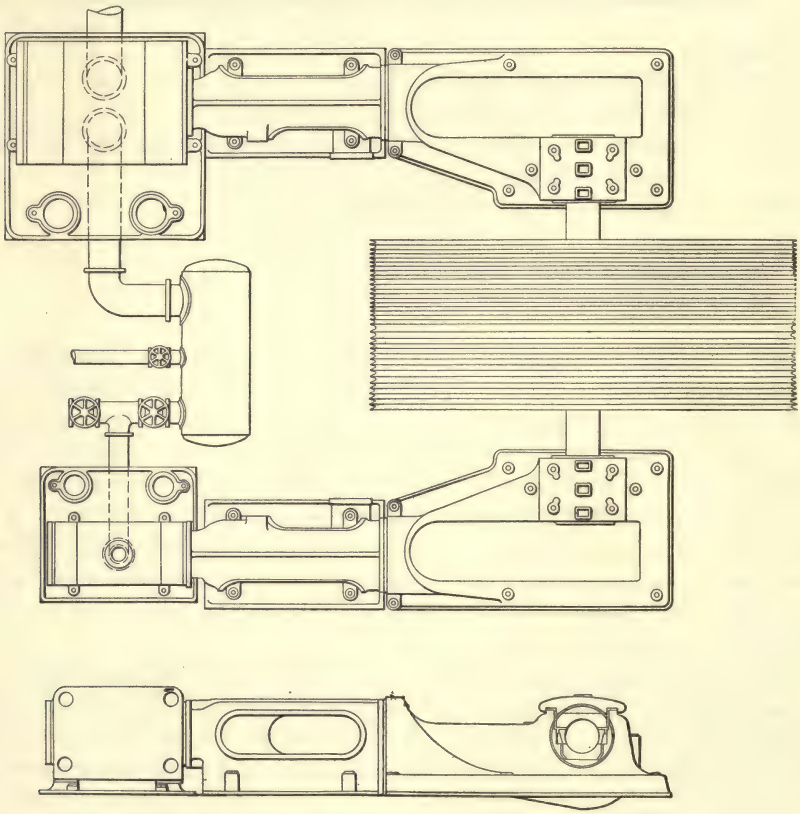

FIG. 461.—General arrangement of Bass-Corliss engine.

where h is the depth in feet and D_S the diameter of the standard cylinder used in Chaps. XII and XIII. This formula is entirely empirical and intended for ordinary conditions of soil, etc. It has been used for many large Corliss engines and so far as is known has proved satisfactory. This may be due to the fact that the proportions are rather massive, but concrete is comparatively cheap.

A formula of a more scientific form is given by E. W. Roberts in his Gas Engine Handbook, which, with change of notation is:

$$W_F = KW_E \sqrt{N} \tag{2}$$

W_F is the required weight of foundation in lb., W_E the weight of the engine and N the r.p.m. The value of K in the seventh edition of the handbook is 0.35.

After consulting with ten leading builders of stationary gas engines in this country, Mr. Roberts calculated values of K from their data for various types of engines. These values, with much other valuable matter relating to foundations was published in *The Gas Engine* for September, 1916. The volume in cu. ft. of a concrete foundation is:

$$v = kW_E \sqrt{N} \tag{3}$$

Mr. Roberts does not consider these investigations final, but at present they are the best effort that has been made toward the solution of the internal-combustion-engine foundation problem. The values of K and k proposed by him are given in Table 103.

TABLE 103

Type of engine	K	k
4-cylinder vertical gas engine	0.130	0.000975
3-cylinder vertical gas engine	0.150	0.001130
2-cylinder vertical gas engine	0.175	0.001310
4-cylinder vertical Diesel engine	0.177	0.001330
Single-crank, double-acting tandem	0.320	0.002400
Twin-crank, double-acting tandem	0.190	0.001430
Single-cylinder, horizontal semi-Diesel	0.300	0.002250
2-cylinder, horizontal semi-Diesel	0.240	0.001800
3-cylinder, horizontal semi-Diesel	0.230	0.001730
4-cylinder, horizontal semi-Diesel	0.225	0.001690
2-cycle horizontal semi-Diesel	0.230	0.001730

Material.—Foundations were formerly made of brick but concrete is now mostly used. A mixture of 1 part of fresh Portland cement, 3 parts of sharp clean sand and 5 parts of gravel or crushed stone is commonly used. In some cases a 1, 2, 4 mixture is used; this is stronger and may be better under certain conditions.

Bolts and Washers.—The determination of the size of foundation bolts

is arbitrary; however, if too small they break, causing a great deal of trouble. For Corliss engines the author devised the formula:

$$d = \frac{D_S}{16} + \frac{1}{2} \tag{4}$$

where d is the bolt diameter in inches and D_S the diameter of the standard cylinder as before. The number of bolts properly influences the diameter, but in (4) it is assumed that the usual number and arrangement of bolts are used. Square nuts are used on the lower ends of the bolts as these have a greater distance across corners, and do not require such small clearances in the lugs of the washer. Hexagon nuts are used at the upper end as they require less space and are more easily handled with a wrench.

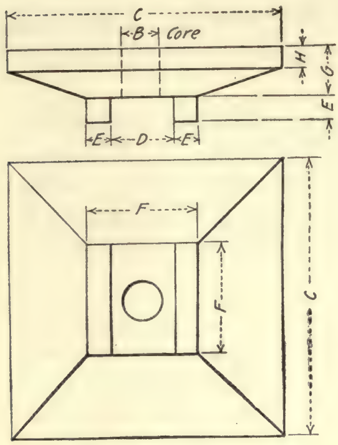

FIG. 462.

A simple foundation washer is shown in Fig. 462, for which the following formulas apply; the notation is given on the figure and d is the diameter of the bolt as before.

$$C = 6d + 1\tfrac{1}{2} \quad G = d + \tfrac{1}{2} \quad H = \frac{G}{2} \quad E = \frac{d}{2} + \frac{1}{4} \quad B = d + \left(\frac{1}{4} \text{ to } \frac{5}{8}\right)$$

$$D = 1.5d + \left(\frac{3}{8} \text{ to } 1\frac{1}{8}\right)$$

The dimensions are given in Table 104. Sometimes the lugs which

TABLE 104

d, in.	B, in.	C, in.	D, in.	E, in.	F, in.	G, in.	H, in.
1	1¼	7½	1⅞	¾	3⅜	1½	¾
1¼	1½	9	2⅜	⅞	4⅛	1¾	⅞
1½	1⅞	10½	2¾	1	4¾	2	1
1¾	2⅛	12	3⅛	1⅛	5⅜	2¼	1⅛
2	2⅜	13½	3½	1¼	6	2½	1¼
2¼	2⅝	15	4	1⅜	6¾	2¾	1⅜
2½	3	16½	4⅜	1½	7⅜	3	1½
2¾	3¼	18	4¾	1⅝	8	3¼	1⅝
3	3½	19½	5¼	1¾	8¾	3½	1¾
3¼	3¾	21	5¾	1⅞	9½	3¾	1⅞
3½	4⅛	22½	6¼	2	10½	4	2
3¾	4⅜	24	6¾	2⅛	11	4¼	2⅛

hold the nut are made deeper and are connected, forming a pocket. Then should a bolt break at the thread at the upper end it may be unscrewed from the lower nut and replaced. However, should it break well down in the foundation, the pocket would be a detriment as will be shown presently.

Construction.—Foundation plans are furnished by the engine builder. In some instances drawings are furnished of wooden templets which are located over the foundation and hold the bolts in position while the foundation is under construction. The foundation should be below the frost line if exposed, but they are often in a basement. In small foundations the bolts and washers are sometimes embedded in the concrete; but it is better practice to have the bolt in a pipe, or in a box formed by four boards. This box is smaller at the bottom than at the top and may be removed after the foundation is finished if desired. The pipe or box allows the bolt to be moved in any direction to allow for discrepancies in the engine bed holes.

In large foundations the washers are placed in pockets. Tunnels, through which a man may crawl, lead to these pockets. Such a foundation by the Bass Foundry and Machine Co. is shown in Fig. 463. This was built of brick for a 22 by 42 in. Corliss engine designed to run 100 r.p.m. The depth was made according to (1), and the foundation bolts of the frame by (4). The boxes for the bolts are not shown, but a note on the drawing calls for holes through brickwork 4 in. square. Should a bolt break the lower end will drop down in the pocket in the brickwork if no pocket is used on the washer. The nut may then be unscrewed by a man in the tunnel, and an ingenious mill-wright or engineer will find some means of removing it. By removing the washer a small chain may be let down from the top and fastened around the thread at the lower end.

Grouting.—A certain amount of space—commonly 1 in.—is left between the top of the foundation and the engine frame for grouting. The engine is supported on wedges by means of which it may be raised or lowered until it is properly leveled; then the grouting is poured in. This may be one of several substances. Sulphur has been used for small engines; and for heavy mill engines, iron and steel turnings are rusted together with sal-ammoniac. Cement is now the most common substance used. When this is well set, the wedges are removed and the nuts on the foundation bolts tightened.

The Soil.—Ketchem, in his Structural Engineers' Handbook, gives the following values of maximum allowable pressure on the soil in tons per sq. ft.:

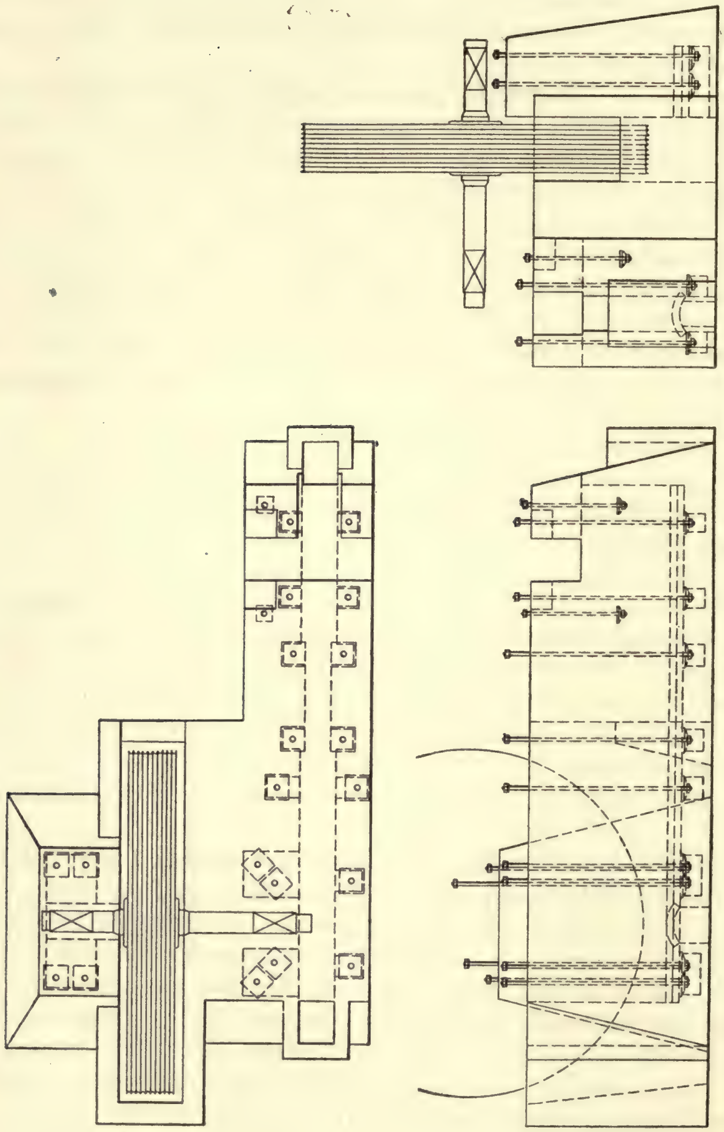

FIG. 463.—Bass-Corliss engine foundation.

Ordinary clay, and dry sand mixed with clay	2
Dry sand and dry clay	3
Hard clay and firm, close sand	4
Firm, coarse sand and gravel	5
Shale rock	8
Hard rock	20
All inferior soils such as loam	1

While rock is an ideal foundation bed in many ways, vibration may be transmitted through it to the walls of the building. It has been advised to make a pocket in the rock, in which a layer of sand is placed before starting the foundation footings.

Vibration may be transmitted through poor spongy soil, and if the plant is in a residence district this may be very objectionable. Increasing the weight of the foundation will help, but this will not always suffice. Balancing the engine seems to be the best remedy for this; in fact, much lighter foundations may be used with well-balanced engines in any case.

Horizontal vibration is usually the most troublesome. In a horizontal engine this may be remedied largely by balancing all of the reciprocating parts. In a vertical engine the revolving parts only should be balanced; but there remains the vibration due to the turning effort which can not be balanced.

It has usually been considered bad practice to build an engine foundation in contact with the walls of the building; but Roberts, in the article referred to, tells of a consulting engineer in Chicago who does this, his idea being to increase the mass that must be set in motion by the unbalanced forces of the engine. He further claimed that the results were entirely satisfactory.

The opposite practice of isolating the foundation by a bed of sand has already been referred to. Roberts tells of the use of cork for this purpose, and a cork sheeting is being used in this country. A concrete pit with a heavy bottom is first made; the sides and walls are lined with this sheet cork; then into this the foundation proper is poured.

The following foundation specification was furnished by the Buckeye Engine Co. for a 15¼ by 18 in. vertical steam engine. The depth is 4 ft., and about 15 in. margin is left around the bed casting. There are eight 1⅜ in. bolts and the allowance for grouting is 1 in.

SPECIFICATION FOR FOUNDATION

Build the foundation of a good concrete mixture as follows: 5 parts clean gravel or broken stone; 3 parts clean sharp sand, and 1 part of good grade Portland cement. Mix thoroughly and ram in. In building,

leave room around each bolt hole so that bolts can be moved ½ in. in any direction. This is best done by placing a pipe or a wooden box around bolts, which can be taken out after foundation is finished, or lifted up as the building of the foundation progresses. The drawings represent a sufficient amount of masonry to give the proper weight, but it must be on solid ground. If sufficiently solid ground is not obtained at the depth shown by drawing, the excavation must be continued deeper; the width must also be increased. The additional depth is to be filled with the same concrete mixture. In many cases however, sufficient solidity may be had by a thorough ramming of the ground.

The copings can be made of the same material or cut stone, and the whole should have at least two weeks to set before engine is placed. Observe closely the drawing of "templet" for placing foundation bolts.

References

American foundation practice .... *The Gas Engine*, Sept., 1916.
Exhaust vibration............................... *The Gas Engine*, Oct., 1916.

CHAPTER XXXV

DESIGN METHODS

230. Personal efficiency is one of the first requisites of the designing engineer. Many men are able to promote the business of others to perfection but are exceedingly negligent of their own. A designer should have a well-organized system of his own whether the concern by whom he is employed has such a system or not. He must be orderly, consistent and thorough, and must not take things for granted. His own records should be well kept as well as those he keeps for the company. He should have note and data books for different phases of his work. He should have the best handbooks and design manuals and be familiar with them. His working formulas should be well selected.

His drafting instruments should be kept in good condition, and although he attains to a position in which he is required to do little or no drafting, he should never feel above working over the board occasionally, for unless he has traversed this path he is hardly fit to direct the efforts of those who do this kind of work.

The designing engineer should never get beyond study. He should keep abreast of the times in his special work and should strive to broaden out some. Technical periodicals help in this, as does also membership in an engineering society—notably one of the national societies such as the American Society of Mechanical Engineers. He should be ready to lead in thought as far as he is permitted to do so and must not simply drift along in the regular routine of the office.

He should use a good slide rule such as the Log Log Duplex; he can then do more work with less fatigue and therefore of improved quality.

Some firms require important calculations to be kept in note books which are the property of the company. This is good practice, and if required should be attended to conscientiously. If not required it is well for the designer to keep such data himself.

If in charge of the department and responsible for shop orders, parts most needed or requiring the most time to obtain should be ordered first; this includes large or difficult castings, large forgings, and sometimes steel castings. It is sometimes difficult to do this as some of these parts are logically the last to be designed, but with proper system in designing this can usually be managed, so that the work may be started before the

design is complete. This is not the most desirable practice from the designer's standpoint, but the writer is sure that it may often be done to great advantage.

Printed lists are usually furnished for the parts built by different departments. If this is not done, a list may easily be made on tracing cloth with spots blocked for drawing or sketch numbers. In this way nothing will be overlooked. A list of Corliss engine parts prepared for the forge shop is given in Table 105. Space is left for the machine shop drawing showing the finished part as a matter of convenience to the office.

TABLE 105.—THE BASS FOUNDRY AND MACHINE COMPANY, ENGINEERING DEPARTMENT

Subject FORGINGS Order__________

Answering SMITH-SHOP Fort Wayne, Ind.__________

Name of part	Drawing No.	Sketch No.	Pieces
Crank shaft			
Connecting rod			
Eccentric rod			
Reach rod			
Piston rod			
Valve stems			
Crank pin			
Crosshead pin			
Crosshead key			
Hook bolts, main bearing			
T-head bolts, main bearing			
Hook bolts, outer bearing			
T-head bolts, outer bearing			
Extension bed links			
Girder bolts			
Flywheel hub bolts			
Flywheel rim bolts			
Flywheel links bolts			
Flywheel keys			
Rope wheel hub bolts			
Rope wheel rim bolts			
Rope wheel tie rods			
Rope wheel keys			
Gear bolts			
Gear links			
Gear keys			
Carrier shaft			
Foundation bolts			
Eccentric strap bolts			
Conn. rod wedge bolts			

The smith shop sketch numbers, number of pieces and the date of order are also given. A duplicate copy should be kept in the office, and when a piece is ordered, this may be signed by the smith shop clerk when the sketch is delivered. Similar lists may be made for other departments.

Standards.—Much work in heat engine design may be standardized, and this has been shown through the chapters on machine design. The application of standard tables to other pressures and to compound engines is explained in Chaps. XII and XIII. The main dimensions may be in-

TABLE 106.—STANDARD ENGINES

Name of Part		Drawing No.	
Cylinder			Size of engine............
Girder			Class of engine............
Guide barrel—No.			Main bearing—diam......, length......
Main bearing—No.			D_1 =
Outer bearing			
Cylinder heads			Crosshead { Screw & Locknut.......
Cylinder foot			Crosshead { Taper fit & key........
Piston			
Crosshead			Cent. of cyl. to cent. of shaft = ...
Crank .... {	Arm		Cent. of eng. to bottom of castings = ...
	Disk		
Wrist plate .... {	Single		Remarks:
	Double		
Carrier........... {	Single		
	Double		
Center foot			
Eccentric .... {	Single		
	Double		
Valves			
Bonnets			
Valve gear			
Dashpots			
Release rig			
Carrier pin stub			
Valve gear pin stub			
Governor			
Connecting rod.... {	Solid		
	Strap		
Eccen. & reach rods {	Single		
	Double		
Piston rod			
Valve diagram .... {	Single		
	Double		

cluded in one table for engines designed for a standard pressure, making it convenient in furnishing data for specifications. Horsepower tables may also be computed, but such calculations are so quickly made with a slide rule that this may be of little use.

As soon as an engine of any size has been standardized it is of great convenience to enter the drawing numbers in a blank similar to Table 106, which was prepared for Corliss engines. This greatly facilitates getting orders into the shop.

In furnishing data to salesmen for work not included in standard tables, the blank shown in Table 107 has been found useful.

TABLE 107

Subject Order
Answering

Fort Wayne, Ind.

Steam pressure.....lbs. R.p.m. { Maximum...... / Regular........ / Minimum........ } Piston speed........
Economical horsepower with...... cut-off in h.p. cyl........
Maximum horsepower with....... cut-off in h.p. cyl........
Run over or under.......... Belt forward or back ....
Diam. of wheel............. Width of face for belt..........
No. of ropes....... Diam......... Weight of wheel.........lbs.
Main bearing...... Outer bearing........ Counter shaft bearing ....
Main shaft diam ...... swelled to.........
Count. shaft diam..... swelled to.........
Guide barrel.......... Bed............
Diam. steam inlet to cyl. h.p...... l.p.......
Diam. steam outlet to cyl. h.p..... l.p....... Face of piston { H.p......... / L.p......... }
Diam. piston rod { H.p..... / L.p..... } Crosshead pin { Diam...... / Length..... } Crank pin { Diam ..... / Length.... }
Connecting rod diam. at neck....... At center.......
Arranged so that h.p. or l.p. engine may run alone and carry whole load.......
Further data:

Partial-assembly Sketches.—Sketches of groups of parts are of great convenience in design. They are also helpful in checking existing design, as any interference or other discrepancy is readily seen. These may be made on tracing cloth and a set of blueprints blocked for inserting dimensions used for each case. These may be kept on file for reference. If certain dimensions are not required a dash may be used. These sketches, with the standard tables previously referred to are useful in getting out large castings—such as the frame—much sooner than can be done by the ordinary process of design. Figs. 464 and 465 are part of a

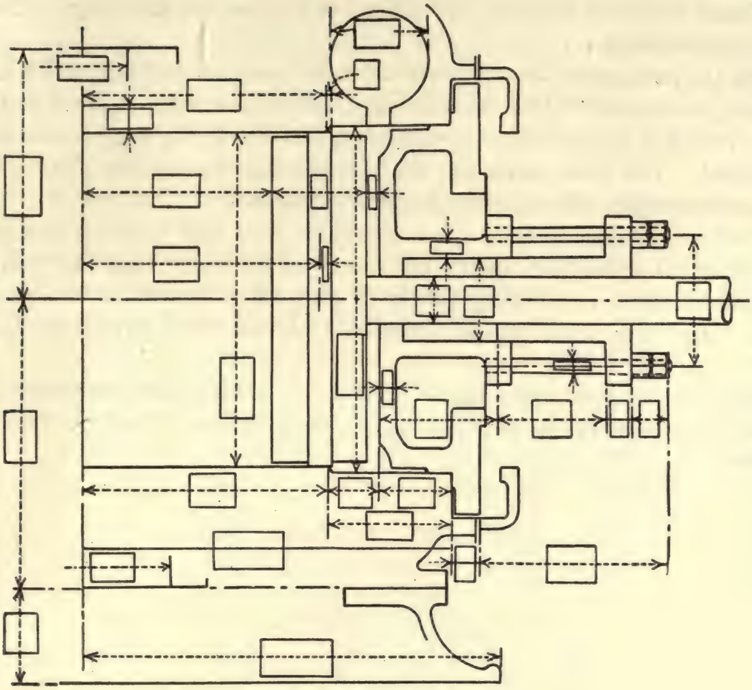

Fig. 464.

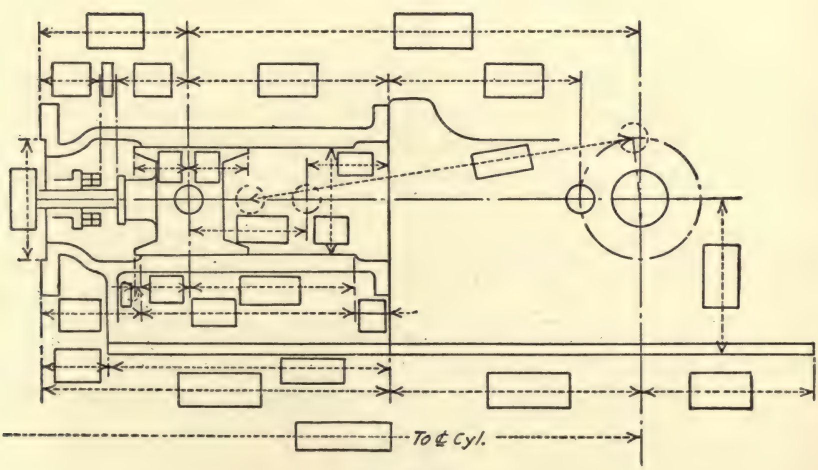

Fig. 465.

set of such sketches used by the author in Corliss engine design. They are self-explanatory.

The above suggestions are given as a few examples of how work may not only be expedited, but the liability of error lessened. A good system will be found in a large number of places, but if not, the suggestions may be helpful. The lists given are more applicable to smaller plants, but the same principle will apply to plants of any size.

If data is looked up for a given problem, it is well to put it in a data book in good form for future reference. The same may be said of formulas derived, care being taken to give all notation in the proper units. They are then readily available should such problems again arise.

Many other ways will suggest themselves if one starts to systemize, but all system should be as simple as possible so that it may not become a burden.

APPENDIX 1

MOMENT OF INERTIA OF IRREGULAR SECTIONS

A graphical method of finding the moment of inertia is given in Morley's "Strength of Materials," with a full explanation of the theory. The method will be briefly described here without proof.

In Fig. 466, the outside irregular line bounds the section of area A, whose moment of inertia is required. Draw any axis XX below the figure normal to the plane of bending (parallel to the neutral axis); also draw line SS any distance above the figure. Across the figure at any point draw line PQ parallel to XX. Where line PQ cuts the outline

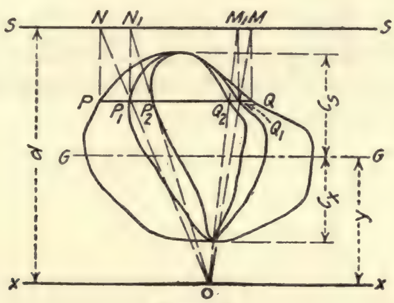

FIG. 466.

of area A, erect perpendiculars PN and QM as shown. Connect points N and M to any point O in line XX. Lines NO and MO cut line PQ at P_1 and Q_1, and these are points in an interior area A_1. By drawing a number of lines PQ, enough points may be found through which to draw the curve enclosing area A_1.

By erecting perpendiculars from P_1 and Q_1 and connecting points N_1 and M_1 with O, points P_2 and Q_2 are found, locating points in a third area A_2.

Having found the three areas, and knowing the distance d between lines SS and XX, we may proceed as follows: The distance of the center of gravity of the section (of area A) from line XX is:

$$y = \frac{A_1 d}{A} \tag{1}$$

The moment of inertia about axis XX is:

$$I_X = A_2d^2 \tag{2}$$

And about neutral axis GG:

$$I_G = I_X - Ay^2 \tag{3}$$

The modulus of section is:

$$z = \frac{I_G}{c} \tag{4}$$

where c may be either c_S or c_X; if the elastic limit in tension and compression is the same, the larger value of c must be used. If M is the bending moment in lb. in., and S the stress in lb. per sq. in.:

$$M = \frac{SI_G}{c} \tag{5}$$

and

$$S = \frac{Mc}{I_G} \tag{6}$$

The areas may be found by a planimeter. If not drawn actual size and one inch on the drawing represents n actual inches, d must be multiplied by n, and all areas found by the planimeter by n^2; then these values may be used in the formulas.

APPENDIX 2

Table 108.—U. S. Standard Bolts and Nuts
For Finished Heads and Nuts, Deduct 1/16 in. from Dimensions Given

Diam. = d, in.	Size of tap drill, in.	No. of th'ds = n	Diam. at root of thread, in.	Area at root of thread, sq. in.	Load in lbs. with stress per sq. in. of				Rough heads and nuts		
					4000 lb.	5000 lb.	6000 lb.	7000 lb.	Across flats, in.	Across corners	
										Hex., in.	Square, in.
1/4	3/16	20	0.185	0.027	108	135	162	189	1/2	37/64	23/32
5/16	1/4	18	0.240	0.045	180	225	270	315	19/32	11/16	27/32
3/8	9/32	16	0.294	0.068	272	340	408	476	11/16	51/64	31/32
7/16	11/32	14	0.344	0.093	372	465	558	651	25/32	29/32	1 3/32
1/2	13/32	13	0.400	0.126	504	630	756	882	7/8	1 1/64	1 15/64
9/16	7/16	12	0.454	0.162	648	810	972	1,134	31/32	1 7/64	1 3/8
5/8	1/2	11	0.507	0.202	808	1,010	1,212	1,414	1 1/16	1 15/64	1 1/2
3/4	5/8	10	0.620	0.302	1,208	1,510	1,812	2,114	1 1/4	1 7/16	1 49/64
7/8	23/32	9	0.731	0.420	1,680	2,100	2,520	2,940	1 7/16	1 21/32	2 5/64
1	27/32	8	0.837	0.550	2,200	2,750	3,300	3,850	1 5/8	1 7/8	2 19/64
1 1/8	15/16	7	0.940	0.694	2,776	3,470	4,164	4,858	1 13/16	2 7/64	2 9/16
1 1/4	1 1/16	7	1.065	0.893	3,572	4,465	5,358	6,251	2	2 5/16	2 53/64
1 3/8	1 5/32	6	1.160	1.057	4,228	5,285	6,342	7,399	2 3/16	2 17/32	3 7/64
1 1/2	1 9/32	6	1.284	1.295	5,180	6,475	7,770	9,065	2 3/8	2 3/4	3 11/32
1 5/8	1 3/8	5 1/2	1.389	1.515	6,060	7,575	9,090	10,605	2 9/16	2 31/32	3 5/8
1 3/4	1 1/2	5	1.491	1.746	6,984	8,730	10,476	12,222	2 3/4	3 11/64	3 57/64
1 7/8	1 5/8	5	1.616	2.051	8,204	10,255	12,306	14,357	2 15/16	3 25/64	4 5/32
2	1 23/32	4 1/2	1.712	2.302	9,208	11,510	13,812	16,114	3 1/8	3 39/64	4 27/64
2 1/4	1 31/32	4 1/2	1.962	3.023	12,092	15,115	18,138	21,161	3 1/2	4 3/64	4 61/64
2 1/2	2 3/16	4	2.176	3.719	14,876	18,595	22,314	26,033	3 7/8	4 15/32	5 31/64
2 3/4	2 7/16	4	2.426	4.620	18,480	23,100	27,720	32,340	4 1/4	4 57/64	5 29/32
3	2 5/8	3 1/2	2.629	5.428	21,712	27,140	32,568	37,996	4 5/8	5 11/32	6 17/32
3 1/4	2 7/8	3 1/2	2.879	6.510	26,040	32,550	39,060	45,570	5	5 25/32	7 5/64
3 1/2	3 1/8	3 1/4	3.100	7.548	30,192	37,740	45,288	52,836	5 3/8	6 13/64	7 39/64
3 3/4	3 5/16	3	3.317	8.641	34,564	43,205	51,846	60,487	5 3/4	6 41/64	8 1/8
4	3 9/16	3	3.567	9.993	39,972	49,965	59,958	69,951	6 1/8	7 5/64	8 21/32

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